Acoustics and Noise Control
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Module – 10
Lecture – 15
Flexural waves, evanescent waves

You will recall; in the last class, we started doing this example of oblique incidence of a plane acoustic wave in between 2 medium.

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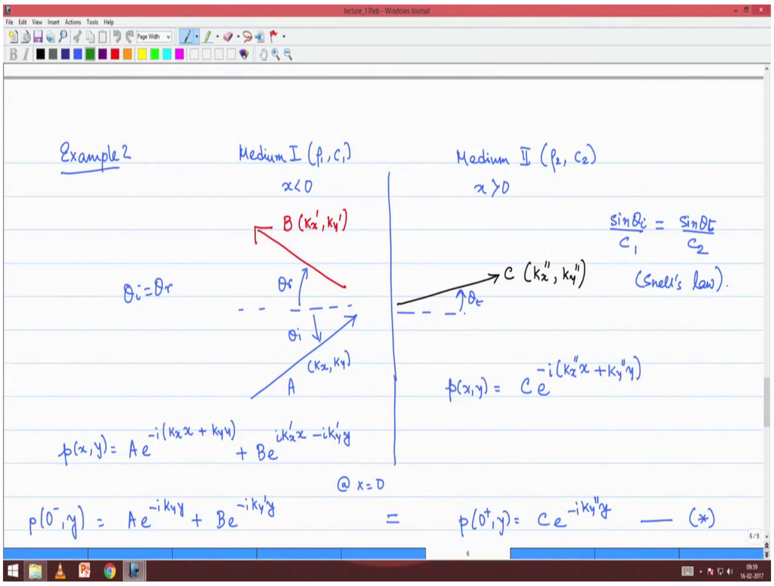

So, there was an incident wave a which was not normally incident between the 2 medium and the objective was to find what are the transmitted and the reflected waves in the last class what we did find was the angle of these 2 waves the in the reflected plane wave would be an angle theta r and theta i is the incidence angle. So, the angle of incidence was shown equal to the angle of reflection and also we were able to prove the snails law in the acoustic context here that is the angle at which the refracted ray or the transmitter bears with the normal to the 2 medium that is called as theta t and we had this relation which was shown to be the snail law.

So, today we will just complete that analysis by possibly determining the 2 amplitude. So, we are still to determine the 2 amplitudes the reflected wave amplitude and the transmitted wave amplitude. So, we will start again with this problem.

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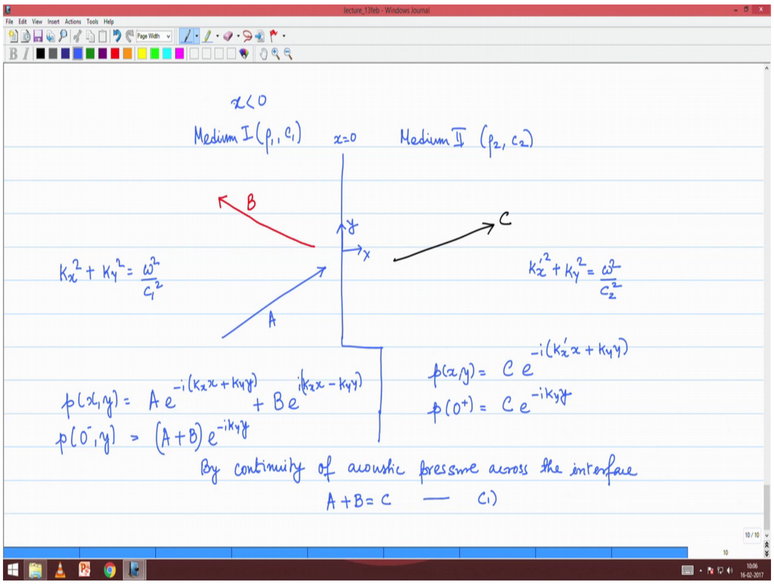

So, we had a medium one which is having the properties as rho one C 1 and medium 2 which is having the properties as rho 2 and C 2. So, you had an incident wave and the complex amplitude of that incident wave was given as a. So, what we have derived already is that the reflected wave will bear the same angle as the incident wave which means in the region x less than 0. So, this is the interface is marked as x equals to 0.

So, in this region which is x less than 0 you will have p x comma y given in the following fashion A e to the power minus i k x x plus k y y. So, both the components in the x direction as well as in the y direction, so, this is my x direction and this is my y direction along both the components, the wave is travelling in the forward direction what has been shown in the last class is that the B wave will also have the same amplitude of these wave numbers, but it will have a sin reversal as far as the k x component goes because the component of the wave travels along the negative x direction not along the positive x direction.

So, the B wave can be represented in this fashion B e to the power i k x x minus k y y. So, here you realize that between the A and the B waves what we have is the wave number component in the y direction as the same sin because both of them travels at least a component of these wave travels in the positive y direction where as the sin is flipped in the wave number component along the x direction k x the magnitude remain same because if the magnitude changes then the angle of incidence will not be equal to the angle of reflection the angle of incidence is equal to angle of reflection virtually gives us that the k x associated with the B wave will be same as the k x associated with the a wave there is just a flip in sin which shows that it is traversing in the reverse direction along x.

So, that is as far as the x directional x lesser than 0 region goes in the x greater than 0, we said that there is a transmitted wave and we will denote this with c. So, the transmitted wave in this region can be represented as p x comma y as C e to the power minus i the k x here will be different because the material is different. So, i will call this as k x prime here plus k y y, we have already shown that the k y corresponding to the region x less than 0 has got to be the same as the k y in the region x greater than 0. So, the k y has to be same otherwise the continuity conditions will not get satisfied which was what was discussed in the last lecture also. So, the k y corresponding to this 2 regions is same, but the k x is different because finally, k x square plus k y square has got to be equal to a constant which is omega square by C square.

So, just to recapulate here you will have the condition k x square plus k y square is equals to omega square by C 1 square where as here you will have k x prime square plus k y square is equals to omega square by C 2 square right. So, the fact that C 1 and C 2 is different is the cause of k x and k x prime being different though k ys associated with these 2 regions are same. So, again we will appeal to the continuity condition at the interface between the 2 fluids, but in the region of medium one here we will get a plus B into e to the power minus i k y y right by substituting x equals to 0 right and on the from the other side what we will get is p at 0 plus is C e to the power minus i k y y.

So, that again establishes the fact that k y between these 2 regions has got to be the same which was even way proved earlier. So, now, if these 2 freshers have to be the same, by continuity of acoustic pressure across the interface what we have is A plus B must equals C this is our equation one this kind of equations we had already seen even for the case of normal incidence right. So, that part is nice and simple remember or objective is to determine B and C given A or the ratio of B by A or C by A.

So, towards that end we have got one equation we need one more. So, we will now appeal to the equation for continuity of velocities right and this time again it will be the continuity of velocity along the x direction right because velocity along the y direction can still be different because you are assuming the viscosity to be negligible in the case of acoustic derivations.

So, we will appeal to the continuity of velocity along x directions and you will recall we had already derived the relation between pressure and velocity along any specific direction which was something like this; this boxed equation.

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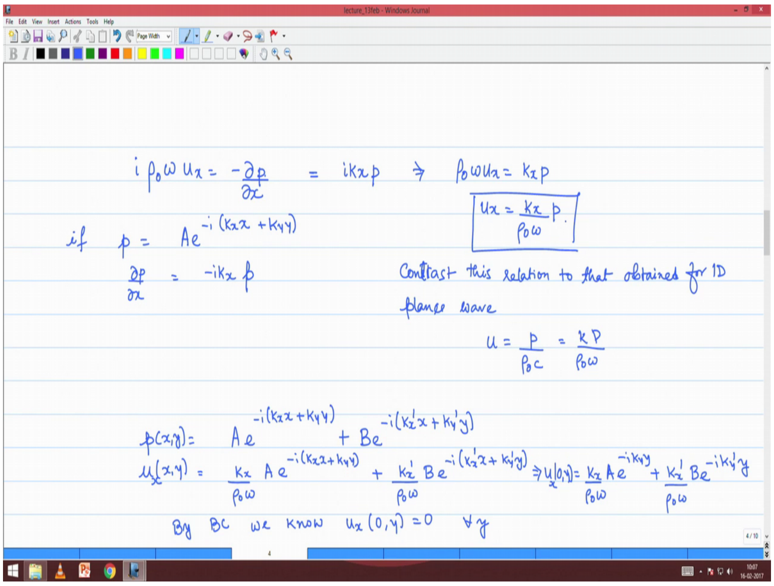

So, u x equals to k x divided by rho 0 omega into the pressure amplitude that gives us the appropriate direction of the velocity.

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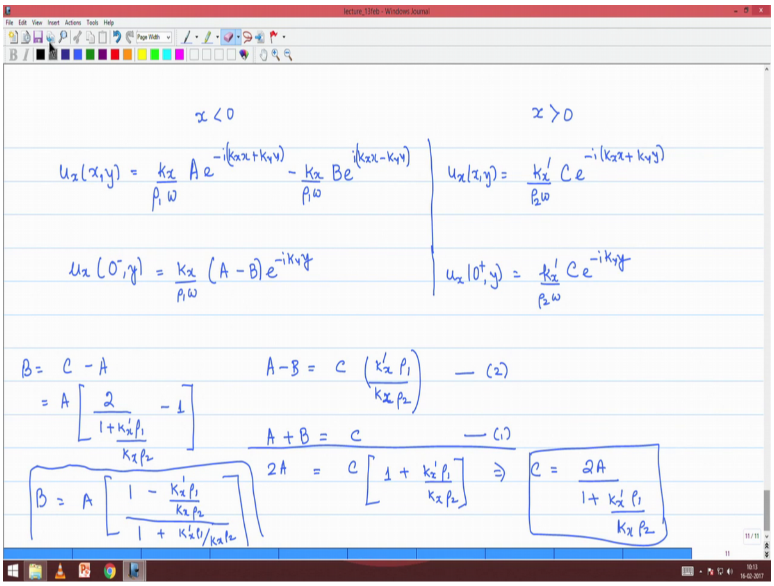

So, we will use this condition now to work out the velocities of in the 2 region. So, u x in the x y region corresponding to x less than 0 will be given as k x divided by rho 0 rho 0 here would stand for rho one because I said that the density in medium one is rho one right. So, rho one by omega into A e to the power minus i k x x plus k y y, but for the B wave it will be the other way round because the B wave is having the component along the negative x direction. So, that will have a minus sign. So, minus k x divided by rho one omega B e to the power i k x x minus k y y this is as far as x less than 0 region goes and for x greater than 0 this is u x x comma y is going to be k x prime this time because the expression involves k x prime the wave number associated in the x direction is denoted as k x prime for the region of x greater than 0 the right hand region if you might. So, wish to call it and the density in this region is rho 2. So, that changes things.

So, C in to e to the power minus i k x x plus k y y, so, again we will establish the continuity condition at x equals to 0. So, at x equals to 0 we could pull out k x by rho one omega a minus B into e to the power minus i k y y right x equals to 0 has been substituted and in the region x greater than 0 at x equals to 0 if you was wish to find out this value that also pretty simple k x prime by rho 2 omega C e to the power minus i k y y.

So, at the end of the day you need to equate these 2 which means A minus B must be equal to C into k x prime rho 1 divided by k x into rho 2 right omega is same between the 2 and thankfully we already have established k y between the 2 regions x less than 0 and x greater than 0 has got to be the same. So, that also does not bother us what we are left with is A minus B into k equals to C times this factor which is k x prime into rho one divided by k x into rho 2 and this is our equation 2 and if I look back at equation 1, we had A plus B equals to C right which is whatever equation one was. So, if we use these 2 equations we could easily relate C in terms of A. So, that reads as 2 A is equals to C into one plus k x prime rho one divided by k x into rho 2.

So, that implies C as 2 a divided by one plus k x prime rho one divided by k x rho 2. So, this is how we get the transmitted wave amplitude in terms of the incident wave amplitude you could also find B as C minus a and that is also not to difficult to work out. So, you can pull the a outside 2 divided by one plus k x prime rho one divided by k x rho 2 minus one and that comes out as a one minus k x prime rho one divided by k x divided by k x rho 2 and the denominator would read just the same k x prime rho one divided by k x rho 2. So, these are the 2 expressions for the 2 wave amplitude the reflected wave amplitude and the transmitted wave amplitude.

So, please recall when we did normal incidence between the 2 media we had the ratio in terms of rho 1 C 1, right, you might think why is that rho 1 C 1 ratio getting changed to this k x prime k x is coming instead of rho c. So, that is not difficult to understand if you recall that k x prime is omega by C 2, sorry, k x prime is I will do this in a different way. So, recall for normal incidence the reflection ratio was shown to be dependent reflection.

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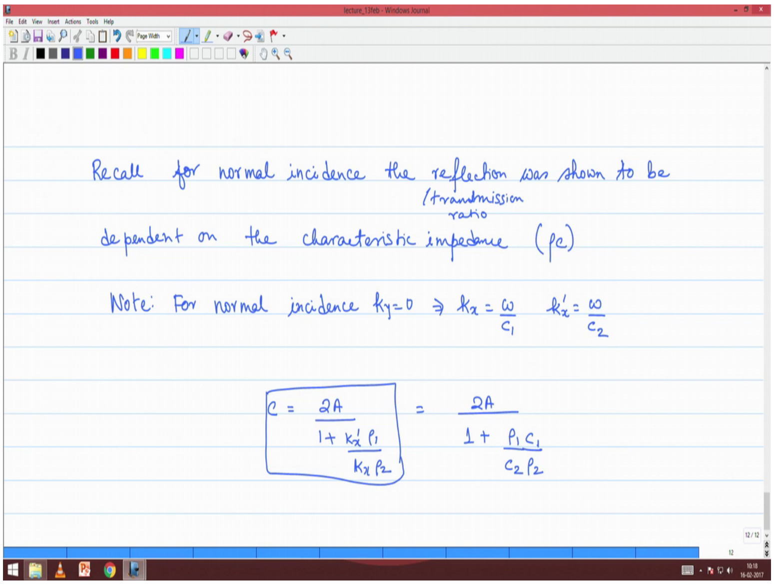

And the transmission ratio where shown to be dependent on the characteristic impedance this was the crucial quantity and you will recall characteristic impedance was given by rho C here what we are seeing is it is the ratio that comes back here for the case of oblique incidence is seen to be k x prime by rho 2 or k x by rho 1.

So, wave number along x direction divided by the density of the fluid medium is what seems to be thrown at us, but if you realize that for normal incidence, so, note for normal incidence what is the associated k y what will be k y for normal incidence if it is incident not obliquely, but directly in this form then what is the associated k y 0. So, for normal incidence k y turns 0 which implies k x for medium one is going to read as omega by C 1 and k x prime for medium 2 is supposed to be omega by C 2 because either ways k y is 0 and if you make this substitutions back into this expression. So, then again you will get to C that if I copy and paste this expression let say. So, this expression will be reading 2 a divided by one plus instead of k x prime I have to write C 2 rho 2 and C 1.

So, I have written instead of k x prime as omega by C 2 instead of k x i have written omega by C 1 and then the 2 omegas have cancelled each other. So, again we get back the ratio of impedances which is what we expect for a normal in incident case. So, we have also shown that from the oblique incidence you could recover back the results of the normal incidence because normal incidence is after all a special case of oblique incidence oblique incidence is more general also there are other features like total internal reflection which I think it is pretty easy to understand again you can looking at this relation you will be able to find out that this I mean going again using that argument similar to geometric optics from this relation it is clear that the transmission angle will prevail only if sin theta t is a number which is less than one right, but looking at this it is pretty clear that sin theta t will depend upon C 2 by C 1.

So, if C 2 by C 1 ratio is pretty high there is a possibility that the sin theta t terms turns more than one in which case there is no real theta t which means that there is actually no transmission that is happening is only and evanescent wave which is what we will describe. So, that aspect is called total internal reflection I will just leave with this comment that this aspect is just the same as you would have studied even in geometric optics right, but it is nice to see that we could derive these results which was sort of axioms in geometric optics from a very fundamental approach based on the mathematical theory of wave propagation of plane waves, right.

So, I think here will close our discussion associated with oblique incidence will move on to another crucial idea that of evanescent waves and to introduce that idea.

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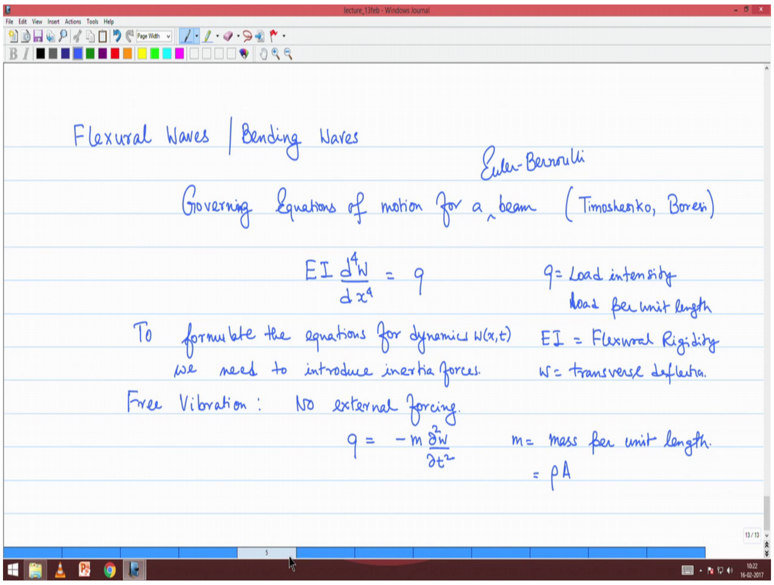

We will firstly, have to have a brief discussion about flexural waves or bending waves till now, we have been talking about acoustic waves. So, we will take a slight detour and talk about the waves in beam for example, similar waves will also exist for plates after all plates are the 2 dimensional counterpart of beams.

So, if you recall the it governing equations of motion for a beam equation of motion for a beam is given as E I d 4 w d x 4 equals to q where q is the load intensity or load per unit length E I is the flexural rigidity and w is the transverse deflection that we are interested in for a beam right this is called the Euler Bernoulli beam which is really good enough for our discussion right and in case you want to look up how this is derived look at any book in like such as Timoshenko book or Borisi book right these books will have the derivation of this equation. So, will pick the thread for our purpose from this governing equation we are not going to derive this governing equation because that I presume is almost an undergraduate affair.

So, we will pick up the thread from this governing equation this equation is the way it is written in here if it is applicable to statics, but to move from statics to dynamics is a very easy extension. So, to formulate the equations for dynamics we need to introduce inertia forces. So, the only change that we will make at this stage where in we are looking for the free vibration problem in absence of any forcing we wish to see whether there is a possibility of obtaining any solution for this equation right. So, the only force that we will include is the inertia force.

So, we are looking at the free for free vibration as we iterated a few lectures back for free vibration there is no external for saying that we need to account for the only forcing that has to be accounted for in this term q is going to be the inertia force and the inertia force will be minus m times del 2 w del t 2 right where m is the mass per unit length because as we said that q is the load intensity which is the load per unit length. So, similarly we will have to take out the inertia force per unit length which is why we have to take ma name as mass per unit length. So, this in terms of density would be the densities times the area of cross section.

So, this time we all we will start using partial derivatives because now we are saying that for dynamics we are going to have this w dependent upon 2 variables x and t for statics it is obvious w depends only on x there is no other independent variable arrive which is appearing in the statics problems, but for the dynamics problem there are 2 variables namely the special variable and the time variable which is what you have seen even for the acoustic pressure even for a one dimensional case you looked at that there was an acoustic pressure which we dependent on both on x n t.

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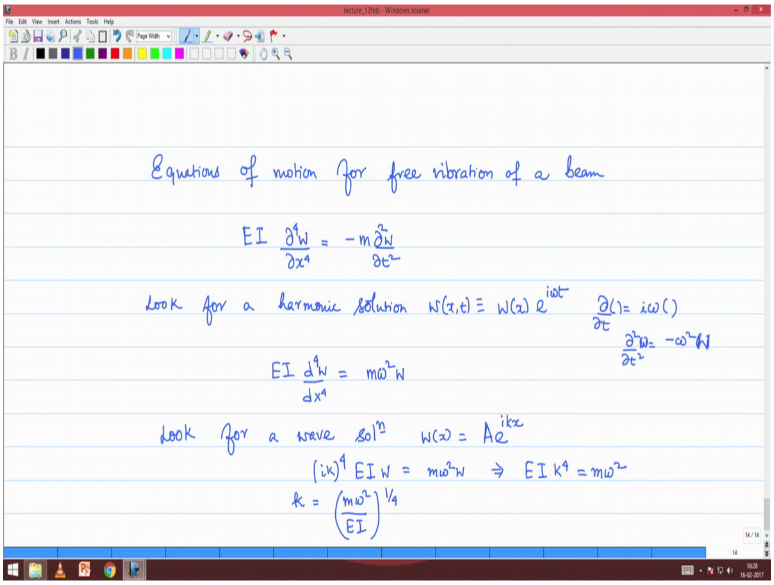

So, with that the equations of motion for free vibration of a beam is given as E I del 4 w del x 4 and I am changing to partial derivatives from the ordinary derivatives that will be minus m del 2 w del t 2 right there is no other forcing that we are interested in other than the inertia forces now let us look for a harmonic solution. So, we look for a harmonic solution just the same ideas as we did in acoustics. So, once we say that i am looking for a harmonic solution w x comma t will now take a firm w x into e to the power i omega t. So, omega is the associated frequency right. So, we are hoping that we can we are able to find some non trivial solution of the system which is of this form w x comma t is w x which is a function which is hesitate are known times e to the power i omega t right.

So, if you make such a substitution again then once you play your card of the temporal dependence to be a harmonic then they only variable of the only independent variable that is left in your problem is again x. So, there is no need again to go to partial derivatives because the only variable that we have is x. So, we might as well say that this is an ordinary derivative because this is the only independent variable that is left in the problem after invoking the harmonic dependence as e to the power i omega t and you know that once you have selected harmonic dependence you take derivative once it has got to be i omega times the function itself and if you take derivative twice it is i omega times i omega which means minus omega square the function itself right.

So, here what will happen is because of 2 time derivatives you are essentially going to get a product of omega square m times omega square the minus sign will cancel with the minus sign that is sitting here. So, m omega square w is what you will get. So, del 2 del t 2 of w is minus omega square w if you wish to call it that right. So, this is the equation that we need to solve at this stage we are looking for a harmonic solution and we are looking for a I mean we are looking for solution at each omega right. So, now, we make even one more mold assumption that we are looking for a wave solution for a wave solution.

So, what does that mean w x because we have already invoke the time dependence as e to the power i omega t therefore, w x also has got to be a complex exponential because only then it will remain a wave right as we have seen in acoustic also the wave solution essentially means the temporal dependence and the special dependence has to be of the same form. So, what we are looking is a function of this from possibly with some constants because is a linear equation into e to the power k s right or i k x **i** should say because e to the power i omega t is the complex dependence that we have taken for the time variable.

So, if we invoke this now what we get is i k whole to the power 4 E I which is the flexural rigidity multiplied by w must be equals to m omega square w and this implies that E I k to the power 4 must be equals to m omega square. So, just to recapitulate what we have done our objective was to find whether or not it is possible to get a wave solution or rather a harmonic wave solution to the beam equation right according the we invoke 2 substitutions one in time one in space in time we took the substitution a for the temporal functional dependence to be of the form e to the power i omega t in space we took the substitution as e to the power i k x and now what we are seeing is that yes if it is possible that you will get a harmonic plane wave solution associated with this beam provided the case and omegas are related through this equation, right.

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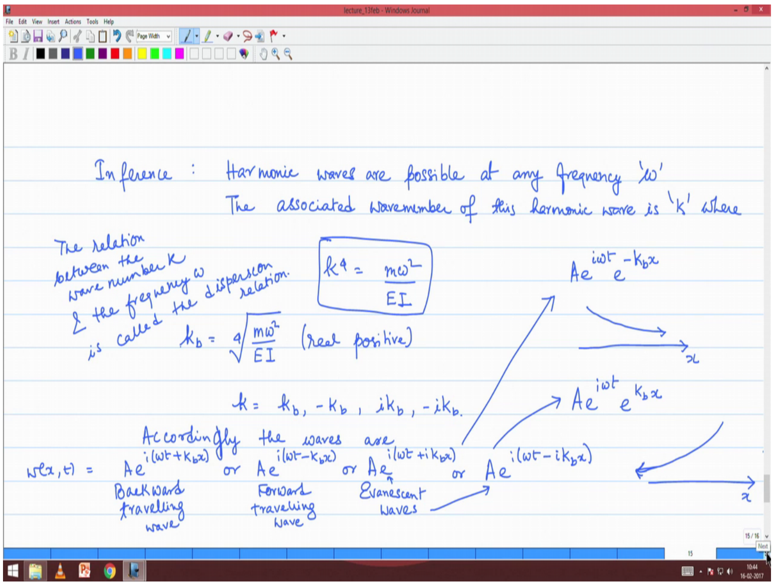

So, in other words if k is m omega square by E I and fourth route of these numbers then you are going to get a wave right. So, let me write this note for you. So, inference harmonic waves are possible at any frequency omega provided or other instead of saying provided the associated once you say it is a wave there are 2 things that you must say one is the frequency and the other is the wave number. So, associated with the frequency omega there is a harmonic wave and the wave number of this harmonic wave is going to be k well k is given by that equation right. So, the associated wave number of this harmonic wave is k where k to the power 4 is m omega square by E I right.

The same thing we have seen in acoustics, but in acoustics the relation was associated with each frequency omega you are going to get a harmonic wave with wave number k where k is k square or k is basically omega by C here we are having k to the power 4 is m omega square by e i, but please note there is one start difference which I do not know whether you have realized this is going to give 4 routes this is not going to give 2 routes last time for acoustic waves we have seen that there are 2 wave number that is possible plus omega by C and minus omega by C and the interpretation of the 2 roots was pretty easy to understand that one of them travels in one direction the other travels in the other direction right, but here we have slight difference in this wave number equation and this difference is due to the fact that k comes with the power four. So, therefore, essentially there are 4 routes to this equation what are these routes. So, let us say that the forth let us call k b as the positive fourth root of m omega square by e i; that means, as you take the square root in your calculator if you take the square root twice the number that you get is k b.

So, then the roots of k will be k b minus k b, but also i k b and minus i k b right. So, the 4 roots of k would be in this form where k b is just the forth I mean square root taken twice and if k b is denoted by just the positive real fourth root of the equation then there are three other routes which is minus k b i k b and minus i k b. So, if we put each of these back into our system and let us try to interpret this solution, so, accordingly what are the waves the waves are w x comma t either is A e to the power i omega t plus k b x or you could have A e to the power i omega t minus k b x i think i better write this way or you could have A e to the power i omega t plus i k b x or you could have A e to the power i omega t minus i k b x it did not come out within the allotted space.

So, I would just need to shift this little bit. So, that was easy A e to the power i omega t minus i k b x right. So, this are the 4 solutions that are possible as per our derivation now let us look at the physical inference associated with each of these solutions the first 2 are pretty easy the first one is a backward travelling wave right because it has the same sign in time as well as the Phasor as a same sign in time and space. So, this is a backward travelling wave. So, how about this one the second solution that i have written down is a forward travelling wave that is also no big deal the third and fourth looks a little odd, but let us sort of open this up a little bit. So, what we have for this expression is the following A e to the power i omega t and e to the power minus k b x, right.

So, therefore, what we see is actually here we will have in if we plot out this in terms of x at any specific time we are going to see that it will decay as we plot in the positive x direction right. So, this is a decaying way where as what was what was the plot of this backward travelling wave and forward travelling wave showing it was showing that the amplitude remains constant with time as well as space right where is here we see that the amplitude actually falls off with space it is not the wave is getting generated at some point, but it is not being able to travel at that fixed amplitude it is rapidly decaying it is exponentially decaying in the positive x direction what is happening associated with this wave can be opened up as following this will be a positive k b x right minus i square here. So, minus i square is plus one. So, you will get e to the power i e to the power plus k b x and k b as per hour notation is a real positive number.

So, therefore, the plot of this if this is the direction of x it will actually decay in the negative x direction. So, that is actually natural justice if you take a wave component to be decaying in the positive x direction by symmetry there has to be a wave component in the negative x direction also right. So, instead of saying that it is growing in the positive x direction we would prefer to interpret this as decaying in the negative x direction and we will I mean if as the course probably matures you will see what are the applications of these decaying components right, but at present we are definitely convinced through our mathematical derivation that if we are looking for harmonic plane waves then not only is there a possibility of travelling waves which keeps on travelling at constant amplitude, but also there is a possibility that these travelling waves rapidly decay as they propagate right we will always interpret the direction of travel associated with the direction of decay right at least for now otherwise we may be mislead to think that some waves are actually growing in amplitude as their travelling that is physically not possible.

So, we will rather interpret the e to the power minus k b x as the one which decays in the positive x direction and e to the power plus k b x as the one which decays in the negative x direction instead of doing it the other waves. So, what we are going to call for these 2 waves is the following. So, this is an evanescent wave this is called an evanescent wave both of these are evanescent waves they are actually not traveling waves travelling waves as we have studied in great details will have this characteristics that it will have the same nature when it propagates the amplitude should not fall with distance whereas, here we are saying that the amplitude falls off with distance.

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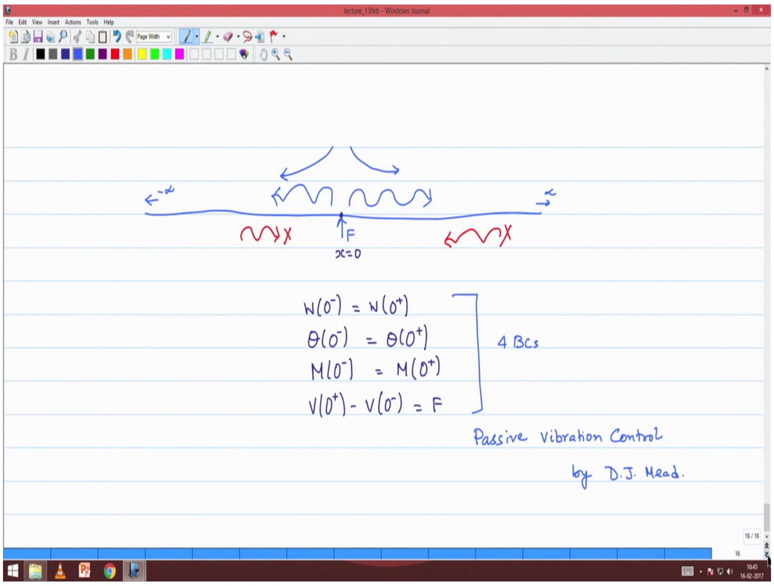

So, just to give you a feel of what will happen though will not do this derivation at least not at this stage if you have an infinite beam just like we studied the infinite acoustic duct at the first instance before going to the finite duct if you have an infinite beam system and infinite beam systems are not that in practical if you think of you know these transoceanic pipelines they are like infinite structures because virtually its very long.

So, if it is excited somewhere then; obviously, now at whatever frequency of excitation you Exide this you are going to expect that there will be some waves which are going to get generated right by physical possibility condition you are going to expect only a travelling wave in this zone which travels in the positive x direction you are not going to expect a backward wave in this zone because that is physically impossible there is no reflection that is going to happen the source is at the middle this source can at the worst create a forward travelling wave in this region to the right of the excitation in the region to the left of the excitation there will be only a backward travelling wave there cannot be a positive travelling wave because there is no reflection there is no excitation which is on the left side of it right, but then what you will see is that using just these 2 waves just this 2 travelling waves will not be able to satisfy the boundary conditions associated with this point what are the boundary conditions the boundary conditions is that there is jump in shear force of the beam and that jump in shear force has to account for this excitation force the shear forces as to change right because only then the jump in shear force the discontinuity in the shear force diagram is essentially equal to the applied force right because you have an applied force there is a jump in shear force.

So, there is a jump in shear force, but there is no jump in the bending moment because there is no applied moment and. So, you have to essentially satisfied three sorry 4 conditions at this point i am calling this point x equals to 0. So, at this point x equals to 0 you need w 0 minus to be equals to w 0 plus; that means, the displacement from the 2 size must match right you must also have the slope from the 2 slides to be matching you must have the moment from the 2 sides to be matching, but you must have a jump in the villas in the shear force to account for the fact that there is an applied excitation force which is f right. So, you would need to satisfy 4 boundary conditions in order to solve this problem using just 2 travelling waves you will not be able to satisfy all the 4 boundary condition right.

So, you have to satisfy the only way that is left and that is what comes out through our mathematical derivation is to account for these 4 boundary conditions with not just this 2 traveling waves, but 2 evanescent waves also right, so, these 2 travelling waves with the 2 evanescent waves. So, totally we are having 4 waves right. So, all these 4 waves will combine in the right proportion such that all this 4 boundary conditions get enforced because there are 4 waves 4 boundary conditions can get satisfied where as in the case of acoustic pressure you had a acoustic equation you had the case where you had just in the case of acoustic pressure you had a second order equation and because it was a second order equation you could not satisfy i mean you could satisfy only 2 boundary conditions for the acoustic case where as here for the beam equation is a fourth order equation and also from physical principle it is well known there are 4 associated boundary conditions and the 4 conditions can be satisfied only if there are 4 waves to choose from and over and above the 2 bending waves 2 travelling bending waves we get these 2 evanescent waves with sort of is a natures wave in which these boundary conditions can get satisfied right.

So, in case you are interested to look at this derivation you can look at d j mead book on passive vibration control this full derivation is given this is book is called passive vibration control by d j mead so, but as we said we are just going to look at the bending waves because we need to understand one crucial these aspects even in the acoustic parlance which is what we are going to head towards. So, what we understand is that there are 4 wave numbers which will get triggered at every frequency and this equation by the way which relates the wave number to the frequency is called the dispersion relation. So, the relation between the wave number k and the frequency omega is called the dispersion relation sometimes in some books it is interpreted as a phase velocity with respect to speed, but even the wave number and the display frequency is also a dispersion relation there are many ways in which we can interpret this. So, I am not come at as A to phase velocity concept, but will do that in the moment.

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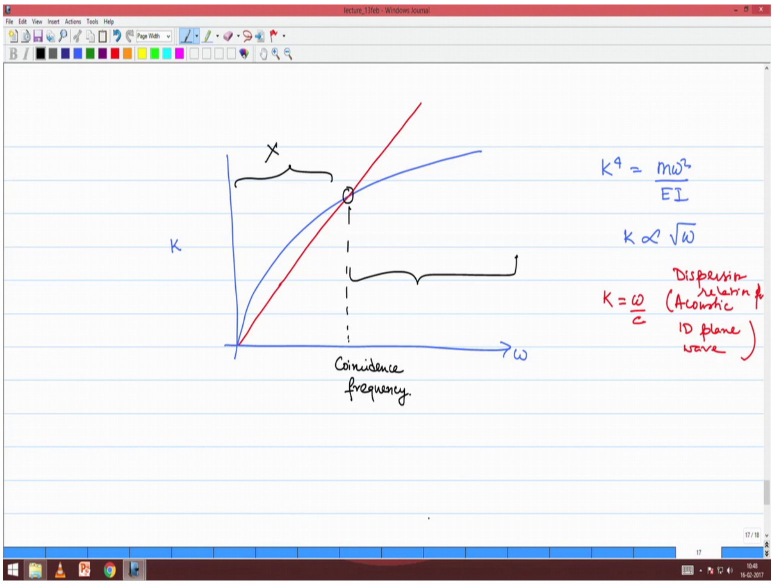

So, the dispersion relation will present us in the form of a graph which is omega by k and what we have just now seen for a flexural wave is that we will have k to the power 4 is equals to m omega square divided by E I which means k is proportional to square root omega right. So, this is just the real part real positive wave number that I am plotting similarly the others can be plotted right. So, k verses omega plot is what we have arrived at just to reinforce what we had for acoustic for acoustics we had k equals to omega by C right. So, k equals to omega by C is what happens for acoustics and that dispersion line is pretty simple to draw. So, this is what we will get for acoustic. So, I will call this dispersion relation for acoustic 1 d plane wave this is what has already been derived.

Now, this crossover point is very interesting and this frequency is called coincidence frequency and it turns out and will do that derivation in the next class that sound radiation actually happens only for these frequencies whereas, in these frequencies sound radiation is almost negligible the actual sound radiation happens only for these frequency will come to why probably in the next class, but just before we end this class a short story to motivate you the idea that we are coming to you have all learnt about this bell jar experiment right.

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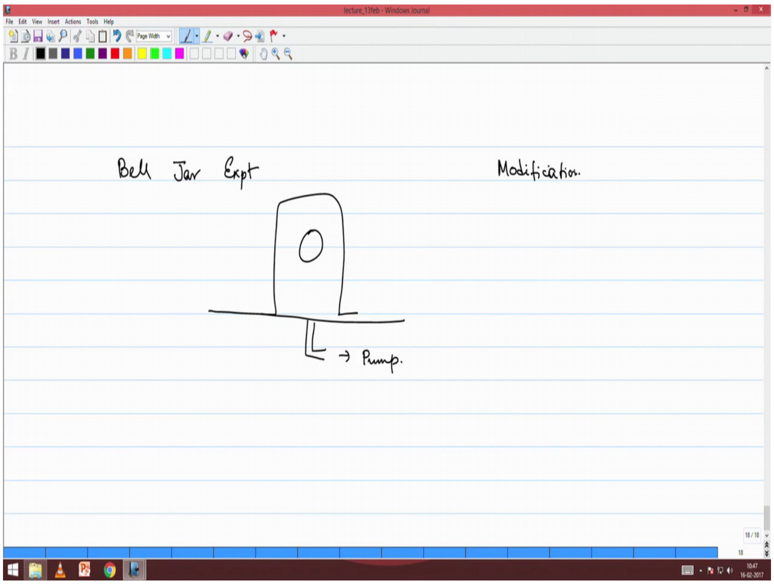

So, in the bell jar experiment you had some sound source which was enclosed over a bell jar and then progressively it was evacuated right by using a pump right and then it was told to you that as the evacuation is happening the material is being withdrawn which means that the sound is growing feeble and that is what was observable, right.

So, that is well known, but in a modification to it what we do is we withdraw the air, but we replace the air with let us say hydrogen and this experiment was actually conduct it, I will give you the references, I will give you the paper also. So, if you put back hydrogen now instead of air and put this back put this hydrogen back at the same pressure same atmospheric pressure. So, you cannot now say that the chamber is evacuated the chamber is having the same pressure as the air should have, but just that the medium has changed from air it has become hydrogen what actually happens is the sound grows even more feeble right and that is the reason for all this to be happening is because of that coincidence frequency.

So, as has been reported in the literature this is what we call the coupling between the vibration and this sound. So, as I said without proof though at this stage that the vibration happens only for frequencies beyond the coincidence frequency sorry the sound radiation for a given vibration happens only for frequencies beyond the coincidence frequency. So, once you replace this 2 medium it does happen that this coincidence frequencies guess gets changed and only the components of waves which are supersonic which are having wave numbers below the acoustic wave number. So, these are the acoustic wave numbers only radiating components of the structure will be in this zone which is below the wave numbers associated with the acoustic wave numbers only those wave numbers will radiate others will not will come to the proof of this aspect in the next class again and again I will come back and explain this observation that why is it that hydrogen will not be able to radiate sound into the air despite holding the atmospheric pressure at the same value as it is with air, but for now will stop here.

Thank you.