Acoustics & Noise Control

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Module - 08
Lecture - 13
Acoustic mode shapes, Reflection

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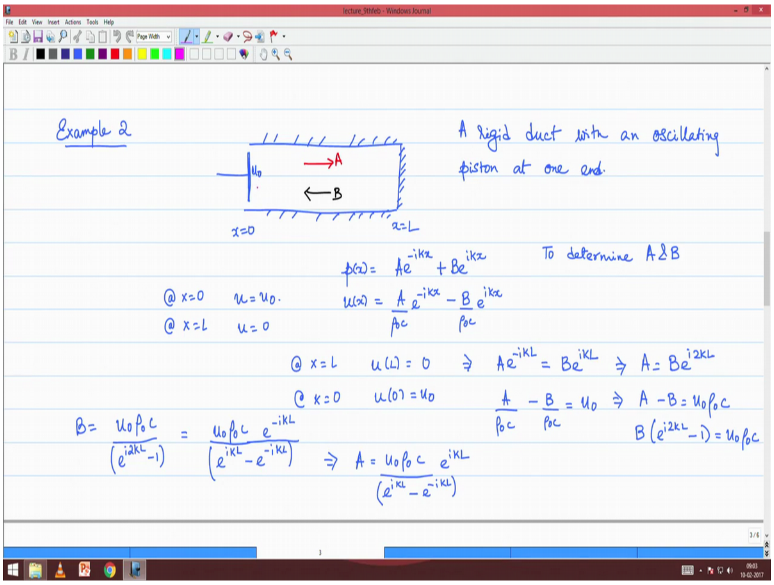

In the last class we took an example of a rigid acoustic duct wherein at one end of the rigid duct we had an oscillating piston, the oscillations where of harmonic nature.

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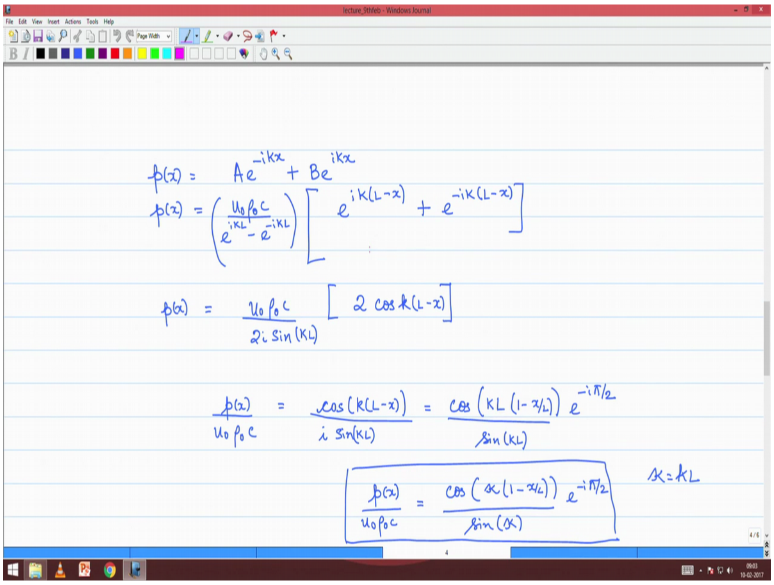

And we derived the pressure response over the entire domain of interest that is x going from 0 to l, from there we understood from this expression.

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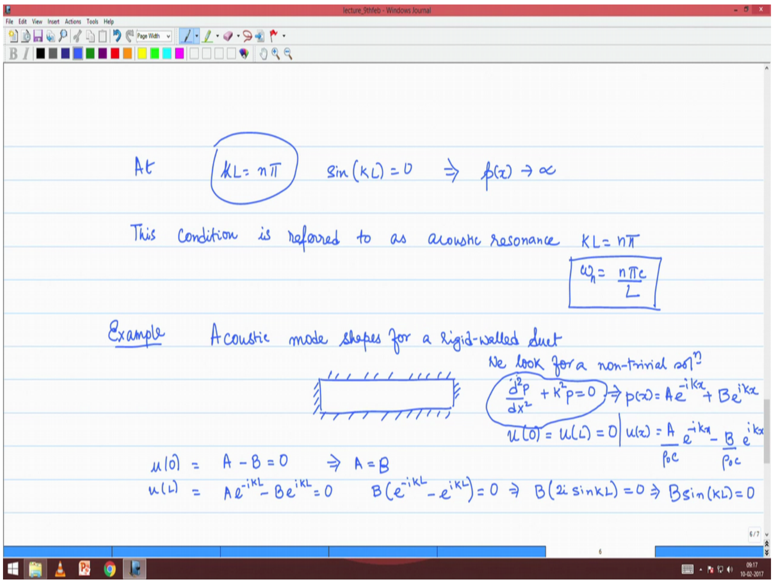

We understood that there is going to be trouble at certain frequencies which is omega n is equals to n pi C by L and those were the string is to be the acoustic resonances.

We could possibly do more and actually find the mode shapes also corresponding to these resonance frequencies. Again I would like to draw a parallel between the vibration analysis and the acoustic analysis in this case; in vibration analysis also associated with each natural frequency you will find a mode shape. So, essentially the idea is that at certain frequencies you can expect that there will be a non-trivial solution even in the absence of excitation.

So, in the vibration language it is called the free vibration response the response which happens even without the expenditure of any forces, similar things can be thought of even for acoustic. So, our example three would be the acoustic mode shapes for a rigid wall duct. So, we are going to have a rigid wall duct of finite length and every wall is supposed to be rigid right and we are looking for a non-trivial solution of this even in the absence of any excitation force or any excitation velocity in this case.

We understood that if there is an oscillating piston then obviously, there can be waves that is possible which is because of an expenditure of some kind of forcing, forcing in the acoustic (Refer Time: 02:38) refers to that velocity condition that we worked out yesterday. But here we are going to see whether there is a non-trivial condition possible this when there is a non-trivial solution possible for this equation, even in the absence of any velocity excitation. So, we are going to solve this is the same equation which is d 2 p d x 2 plus k square p equals to 0. So, the objective is we look for a non-trivial solution to this equation what is the trivial solution the trivial solution is p equals to 0. Obviously, p equals to 0 does satisfy this equation and that is called trivial solution because that is not of any interest in analysis.

But we have to solve this nontrivial solution subject to the condition that u at x equals to 0 has got to be same as u at x equals to L and both of them have got to turn 0 right because we have rigid ended terminations on both the sides. So, as usual we understand that this solution p x the general solution of this will be p A e to the power minus i k x plus B e to the power i k x.

So, this was like without any (Refer Time: 04:14) we can all whenever we see a one dimensional wave equation, the general solution is in this form which we have repeated time and again a and B are the undetermined constants. So, if we can could find a and B then we will we would have satisfied for trivial solution we would have a equals to B equals to 0 that is not of our interest.

So, if p of x is given in that form then u if x by the impedance condition would be written as A by rho naught C e to the power minus i k x, minus B by rho naught C into e to the power i k x right that again goes without saying this is simply derived from the condition that the impedance associated with each of these travel waves is rho naught c, and when you appropriately incorporate the sign associated with the forward travelling wave and the negative travelling wave you get a reversal inside.

So, now from this we get u at 0 will lead us to the condition that A minus B has got to be 0, rho naught C is a constant. So, that does not bother us and u at L that condition gives us A e to the power minus i k L minus B e to the power i k L that also has got to be 0. So, the point is from the first equation we get A equals to B and from the second equation we get B e to the power minus i k L minus e to the power i k L is equals to 0 and this implies B into 2 i sin k L with the negative sign is equals to 0, but since the right hand side is anyway 0 it does not matter right that minus does not matter. So finally, we would have even the i can be taken out. So, what the condition with which we are getting is B times sin k L has got to be 0.

So, this would mean either B equals to 0 or sin of k L equals to 0, but if B equals to 0 then A is also equals to 0 which means will end up with the trivial solution. So, the trivial solution definitely does come out as part of this derivation process which is intuitively pretty clear that it has to be trivial solution of A equals to B equals to 0, but the trivial solution is not to our interest so that means, this condition that there can be an acoustic pressure responsible even with the absence of any velocity excitation that is possible only if k L equals to n pi that is exactly what we derived even in the last if you recall we had the condition k L equals to n pi coming out of I mean the condition for acoustic resonance last class we give it as k L equals to n pi.

But that we derived from the premise of argument that it is the condition which leads to a blow up of the response, but here we are doing it from a completely different perspective here we are actually solving the free vibration solution and then we are led to believe that the freeway the free respond solution I should not use the word vibration, the free acoustic response is only possible under the condition k L equals to n pi.

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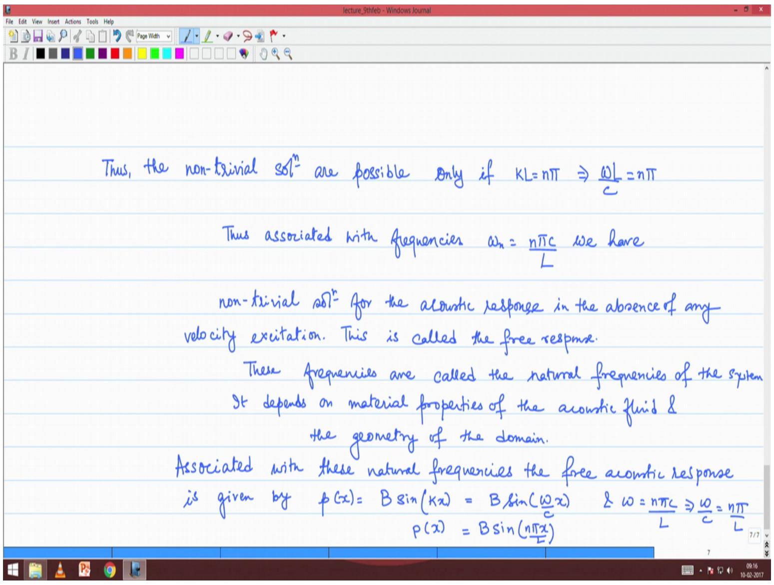

So, the associated frequencies will be called as natural frequencies. So, I will make this remark thus the non-trivial solutions are possible only if k L is equals to n pi; this in turn means omega L by C is equals to n pi. So, thus associated with the frequencies omega n is equal to n pi C by L we have non-trivial solution for the acoustics response in the absence of any velocity excitation this is called as the free response just in analogy with the free vibration response you are having a free acoustic response the response which is happening because a even with without the expenditure of any sort of excitation; excitation in the acoustics parlance I repeat is because of velocity conditions.

So, the moral of the story is this that associated with these very distinct frequencies which are given by the formula as n pi C by L, we are expecting a non-trivial solution to come out and these frequencies are called the natural frequencies. So, these frequencies are called the natural frequencies of the system, it depends on material properties of the acoustic fluid and the geometry of the domain. L is the geometric property and C is the associated with the material properties of the fluid.

Now, associated with these frequencies what are the solutions? The solution are exactly B sin k x. So, associated with these natural frequencies the free acoustic response is given by p of x is equals to B sin k x. Or in other words k can be written as omega by C sin omega by C x and omega is n pi C by L, which means omega by C is n pi by L right. So, therefore, p x could be written as B times sin n pi x by L right what about B we have not yet evaluated that undetermined constant, B should we look to evaluate that is it still remaining and unknown or you think that by the way we have already solved the equation. Please note that we are talking about a homogenous solution, this is a homogenous equation in absence of any right hand side.

So, therefore, any solution to this homogenous equation if it is multiplied by an arbitrary constant it remains a solution. So, by the nature of the homogenous solution we cannot expect that this solution will be completely determined up to the level of that constant right it has to be having an undetermined constant because by the nature of this equation. So, this equation if it has a solution let us say p 1 then 10 times p 1 is also a solution, 100 times p 1 is also solution. So, the solution scaled by any arbitrary number will remain a solution because the equation is homogenous the boundary conditions also are homogenous.

So, it is a perfectly homogenous system and homogenous system you cannot expect the solution to be completely determine this is true even in the case of vibration analysis where you get to see that the mode shapes can we only determined up to the level of one constant one constant has to be left undetermined.

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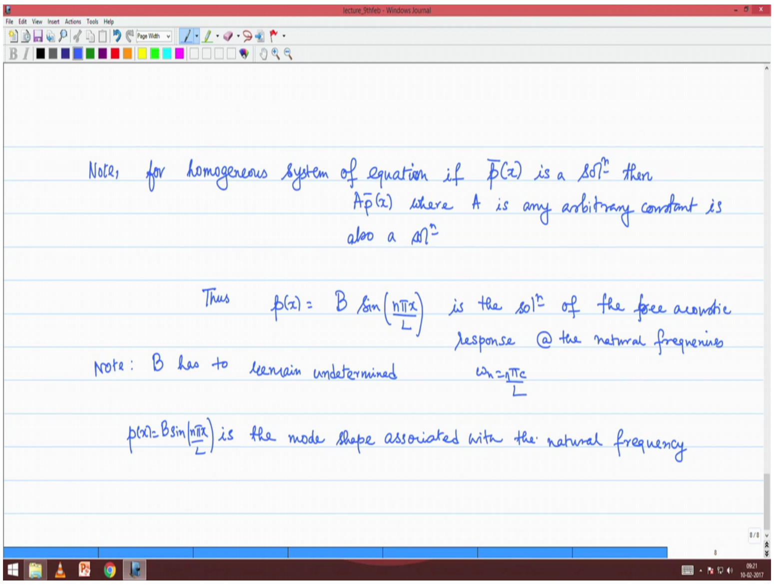

So, I will put that remark that note for homogenous system of equation if p let us call this bar p bar x is a solution, then A time p bar x where A is any arbitrary constant is also a solution. Therefore, you cannot say that there is a unique solution in other words for a homogenous system that is possible this is much like the case of eigenvectors. Eigenvectors in matrix analysis as I as you know can be scaled by any arbitrary scaling factor. So, similar these functions which also the solution of a homogenous system of equation, it is just that this system of equations are ordinary differential equations as supposed to matrix equation, but then it is preserves this characteristics that it can be scaled by any arbitrary number.

So, thus p x is equals to B times sin n pi x by L, which was the solution that we derived is the solution of the free acoustic response at the natural frequencies omega n is equals to n pi C by L. And note B has to be to remain undetermined there is no way that you should say that the B is a perfect constant there is no way that you will get a perfect constant and you can say that the solution is unique the solution has to remain unique. So, in other words this p x equals to B sin n pi x by L is a solution for every possible value of B.

So, associated with that you will say that p x is the mode shape or I should write the full form of p x, p x equals to B sin n pi x by L is the mode shape associated with these natural frequency.

So it always appear, you have to find the natural frequency and the associated mode shapes. So, this is the associated mode shape and carefully note this term mode shape by the very usage of this term shape with we just mean to say that what is important is the nature of this function the value of this function is irrelevant, because the value of this function will keep changing with different these that you can choose. All of these any choice of B is perfectly correct in mathematical terms any B is possible. So, therefore, you should not associate any sacredness to the value of a mode shape you should look at it in terms of a shape.

So, just on a lighter way the idea of shape remains the same even though you scale it up. So, my figured as you see on a small television screen will be same as you see in a big screen. So, the shape of myself with my belly protruding out words and you know whatever I look that will the shape of myself will remain the same whether you look at my figure in a small mobile screen or in a big multiplex screen, I will not drastically turned to a six pack abs hero face profile, I will remain the shape of my figure will remain the same.

So, that the crucial idea that it is the shape which matters the value does not matters in the context of mode shape, and the plot of these mode shape should be pretty simple to do.

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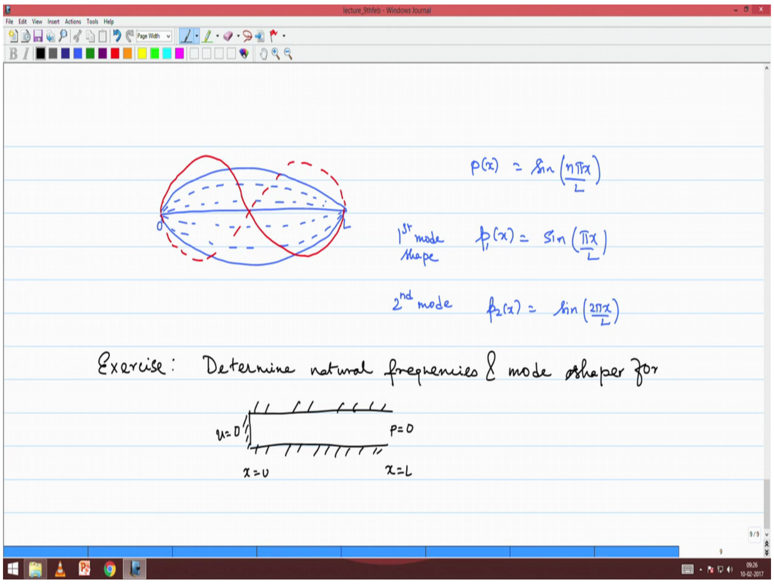

We have realized that p x is equals to sin n pi x by L right. So, let us look at the first mode shape. So, the first mode shape is p 1 x equals to sin of pi x by l. So, between 0 to L it will be sinusoidal, but it will cover half the wavelength. So, it will be like this. So, this is the first mode shape which covers half the wavelength; the within the span of 0 to L the sinusoid has covered half the wavelength, but remember it is a standing wave it is not a travelling wave that we are talking about now, because after superposition of the incident and reflected it has now become a standing wave and the manner in which it will vibrate it will it will change with time is shown roughly in this way.

So, this is how the profile will change. So, it will wax and wane, but the shape of this acoustic pressure profile will remain the same write it will not change with time right and if you recall or if you have an idea of it this is exactly the same mode shape that you will have for a simply supported beam also, the simply supported beam will have a first mode shape which is this. Now for the second mode shape how will that look like p 2 x it is sin of 2 pi x by L so that means, it covers the entire wavelength within a span of 0 to L. So, this is roughly how it will look and it will wax and wane within this domain. So, this is the second mode shape and the third mode shape fourth mode shape all of it will come in a similar fashion.

So, I leave you with an exercise determine natural frequencies and mode shapes of for. So, this time I am going to have one end rigid and the other end open, it is again of length L, but one end is open to atmosphere which exactly means that the pressure equals to 0 here because it has to open out to the atmosphere and on the other end you will have the velocity to be 0. So, the boundary condition this time is one side you have velocity is 0 and the other side you have pressure 0 and you are supposed to work out this condition and compare the model characteristics the mode shapes that you obtain in this case with what I have done. The example that I have given is with both sides rigid both ends rigid you have to work out another example where one side is rigid and the other side is having a pressure equals to 0.

But essentially the idea is just the same you have to invoke these 2 boundary conditions and you have to work it out to see where you get to.

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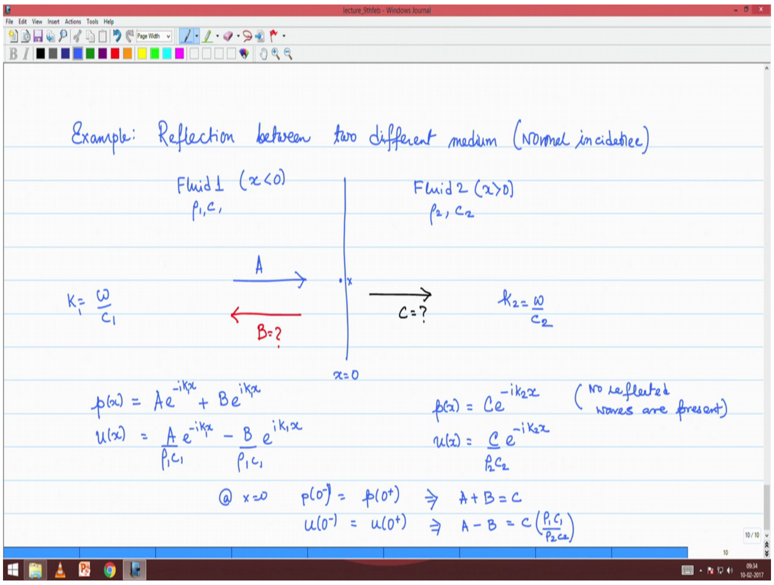

So, now will move to another example between 2 different medium and we are talking about normal incidence. So, I assume that we have 2 different medium let us say fluid 1 and fluid 2 and they are characterized by the properties rho 1 C 1 and rho 2 C 2.

So, normally incident wave plane wave are normally incident plane wave is incident from the fluid 1 side and the question is how much will be the reflected wave and how much would be the transmitted wave. Something will get transmitted to fluid 2 and something will get reflected back to fluid 1 right the question is to determine how much is the reflection and how much is the transmission from fluid 1 to fluid 2 and we are assuming at present a normal incidence, that is it is a plane wave such that it is the direction of travel of the wave is exactly perpendicular to the interface between these 2 fluids.

Later we will do an oblique incidence problem also where we will take that the plane wave is not normally incident to this interface, but it is coming at an arbitrary angle right, but at present will look at this problem which is what we define. Later we will also see that the ideas of this problem can be taken over even to the design of mufflers. So, we will see how that comes, but let us take it at this way now that you have to infinite domains one fluid 1 other is fluid 2, from fluid 1 side somehow an incident wave has been generated we will say the amplitude of that if A and we are interested to find the amplitude of the reflected wave and we are also interested to find the amplitude of the transmitted wave right and without loss of generality we will assume that this interface between fluid 1 and fluid 2 is exactly at x equals to 0, which means fluid 1 is in the region x less than 0 and fluid 2 is in the region x greater than the 0 right.

So, the pressure profile in the fluid 1 is definitely comprising of 2 waves: one which is an incident forward travelling wave and the other which is a reflected backward travelling wave there cannot be anything else right. So, we already know that the solution to the one dimensional plane wave equation has got to have these 2 components and therefore, we are happy to note that the pressure profile will be given in this form, there is forward travelling wave and there is a backward travelling wave, but for the reason x greater than 0 there is only a forward travelling wave there is no backward travelling wave. So, that makes it as C e to the power minus I just to do the bookkeeping, that fluid 1 will have a certain property and the fluid 2 we will have certain property the wave numbers associated which it will be different.

So, therefore, I will put k 1 associated with fluid 1 and k 2 associated with fluid 2. Remember k 1 will be omega by C 1 where is k 2 will be omega by C 2 the omega is same we are talking about a single frequency analysis as we said in the case of harmonic assumption, but even for the same frequency because of the fact that the sound speed in these 2 medium can possibly be different will have to account for the fact that the wave numbers can be different. So, therefore, k 1 and k 2 could possibly differ between these 2 fluids. So, if this is p x the reason why we did not account for a backward travelling wave in the region fluid 2 is because we are not expecting any reflection in the region of fluid 2, there is no interface it is extending up to infinite. So, therefore, nothing comes back there is only a incident wave which is there.

So, no reflected waves are present and since there is no reflection, so therefore, there cannot be the any other component of this solution. What about the velocity components u as a function of x again will make use of the impedance relation A by rho naught C to the power minus k x by k 1 x and B by rho naught C into e to the power i k 1 x whereas, for the fluid 2 reason u x will be simply given by C by, I should also make my notations clear at this time it cannot be rho naught it has to be rho 1 C 1 right because that is how the density and sound speed was defined in fluid 1.

Similarly, the density and sound speed in fluid 2 is rho 2 C 2 capital C is the undetermined amplitude of the transmitted wave. So, that divided by rho 2 C 2 into e to the power minus k 2 x is the velocity that we will obtain in the fluid 2 rigid right. Now what we have to do is we have to look at the interface, at the interface the pressure from fluid 1 and the pressure from fluid 2 between two neighbouring particles has got to be the same because of continuity arguments right again the velocity also has got to be the same. So, we will simply have equate these 2 at x equals to 0.

So, at x equals to 0 the pressure coming from both the sides in other words p 0 minus must be equal to p 0 plus simply by continuity arguments and that should make it as A plus B equals to C, and similarly at x equals to 0 we should have u 0 minus is equals to u at 0 plus we should make it as A minus B is equals to C times rho 1 C 1 divided by rho 2 C 2 right. So, this is these are the 2 equations that we obtain from the 2 boundary condition we are required to find the values of B and C for a given value of A. So, basically this should suffice right because given an incident wave we wish to find what is the reflected wave and given an incident wave we wish to find what is the transmitted wave you cannot be reflected wave and the transmitted wave if you are not even given the amplitude of the incident wave.

So, A is known you wish to find B and C and looks like we have got 2 equation which should be solvable to obtain these 2 numbers B and C in terms of A.

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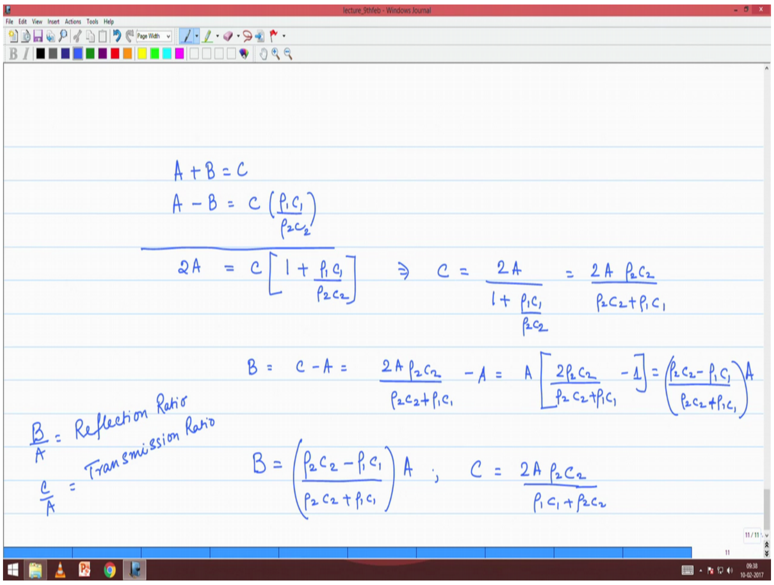

So, let us write this down once again what we are looking to solve is A plus B is equals to C, and A minus B is equals to C times rho 1 C 1 divided by rho 2 C 2, this is what we have written it once again. So, the solution of this fairly simple then it becomes 2 A is equals to C into 1 plus rho 1 C 1 divided by rho 2 C 2 right. So, that in turn implies C is equals to 2 A in divided by 1 plus rho 1 C 1 into rho 2 C 2. We could write this as 2 A rho 2 C 2 divided by rho 2 C 2 plus rho 1 C 1 and then B could be determined as C minus A. So, that becomes C plus, C is already found out C is already found out as 2 A rho 2 C 2 divided by rho 2 C 2 minus rho 1 C 1 sorry plus rho 1 C 1 and then you will have to put a minus A.

So, I can take this A fellow outside and then what comes is 2 rho 2 C 2 divided by rho 2 C 2 rho 1 C 1 minus 1 and that in turn is rho 2 C 2 minus rho 1 C 1 divided by rho 2 C 2 minus rho 1 C 1 into A right. So, therefore, the moral of the story is the reflected wave amplitude is rho 2, C 2 minus rho 1, C 1 divided by rho 2 C 2 plus rho 1 C 1 into A and C the transmitted wave amplitude is 2 A times rho 2 C 2 divided by rho 1 C 1 plus rho 2 C 2 right. So, please note that what matters in terms of reflection and transmission is the quantity rho C, rho C we have already identified as the characteristic impedance of the media.

So, what you see here B by A is called the reflection ratio and C by A is called the transmission ratio. Both these quantities depend on the characteristic impedance of both the fluid, it does not depend upon density or sound speed rather it depends upon the product of the 2 right it depends upon the product of the density and sound speed which is the characteristic impedance.

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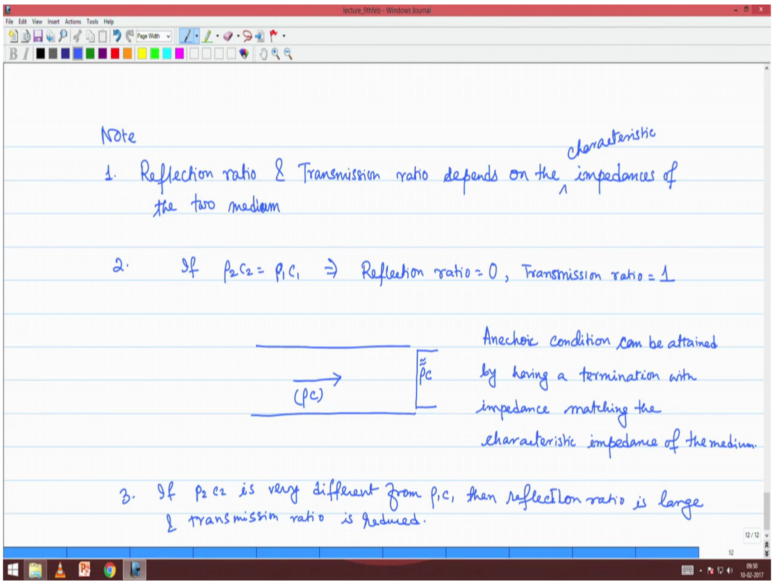

So, that is the first observation reflection ratio and transmission ratio depends on the impedances of the 2 medium or rather I should call it as characteristic impedance because there are many other forms of impedance that we will see, the reflection ratio and transmission ratio depends on the characteristic impedance of the 2 media right that is a very crucial observation.

Another important observation falls out from the fact that if you have the 2 medium to be different, but none the same the characteristic impedance is the same. In that case you will have rho 2 C 2 is equals to rho 1 C 1 even though rho 2 may be different from rho 1 and C 2 may be different from C 1 their products might give a possibility that they are equal. So, in that case as we see then the reflection ratio is going to be 0, none of it will be reflected. So, that is actually a very crucial idea that in case you find a material which is having the same impedance as the characteristic impedance of the medium in with the sound is moving, then virtually it will not lead to any reflection right I will explain that better.

So, if rho 2 C 2 is equals to rho 1 C 1 then reflection ratio is 0 how about transmission ratio? If rho 2 is equals to rho 1 C 1 then C is equals A. So, therefore, transmission ratio becomes one which is very nice to see because you know that everything is getting transmitted none of it is none of the no part of the wave is actually sorry if rho 2 C 2 is equal to rho 1 C 1 reflection ratio is 0 and transmission ratio is 1. So, what we observe is that if the entire incident wave is normally incident between 2 fluids such that the characteristic impedance of the 2 fluids are held equal, then there will be no reflection and complete transmission 100 percent transmission will happen. This idea is very important in the design of absorbing material, because by now you know that if there is any impedance then or if there is any boundary rather if there is any boundary then there will be reflections.

But the condition of obtaining 0 reflection is the same as the condition of having an impedance which is same as the impedance of the other medium right. So, the incident side whatever impedance you have even if you have a finite domain, but that the interface of that finite that crossover you have the impedance quantity to be the same as the characteristic impedance of the incident side, then you are going to have 0 reflections. So, virtually you are going to have a non reflecting condition that is created. So, I will just explain myself a little bit better.

So, let us say you have a plane wave which is traveling let us concentrate within a small zone, and now the associated impedance in this region is some rho C. Now as what is seen visually is that this domain is actually terminating in a wall, but this wall is made of a very special material and the special material has this property that the impedance at exactly the same interface is at least approximately equal to rho C, then what we will find is that though this is a bounded domain, but since it is being bounded by a wall where the impedance matches with the impedance of the incident side there will be no reflection.

So, virtually this is an infinite domain, for our calculation purpose this is as good as an infinite domain because none of this part of the wave is going to get reflected back everything is going to sort of actually what is going to happen is it is going to get absorbed with in this medium or the absorption is made equivalent to the transmission process in the 2 examples that I have been talking about, but whether it is absorption whether it is transmission the point is this that there will be no reflection right. So, this no reflection condition is called the anechoic condition. So, anechoic condition as you know it echoes happen due to reflection. You could prevent an eco to happening to be happening or you could prevent the reflection by having termination impedance which matches with the characteristic impedance of the fluid.

So, anechoic condition can be attended by having a termination with impedance matching the characteristic impedance of the medium, and this is a very crucial logic that whenever you have an impedance match there is no reflection or anechoic condition. If you look carefully all these the studio around you are having specially designed walls these are perforated walls, and these perforation and behind these perforated walls as you see with your naked eyes there are absorbing materials.

So, the purpose is to get to an anechoic condition. So, the impedance associated with these wall conditions hopefully will approach this impedance of the fluid around us and as a result you are not going to get an eco, because if eco happens in the studio then is going to be very disturbing for the recording. Similarly there are other instances where you do not like in auditorium a very good auditorium you do not want to have any echoing effect. So, you would like to have an anechoic condition, another place where anechoic condition is very useful is in the testing chamber because typically as per the standards you have to for example, if you are dealing with past by noise you have to measure the noise in the open atmosphere.

So, if instead of measuring the engine noise in the open atmosphere you have to measure it even in the closed chamber then; obviously, because of the reflection associated with the walls of the of your room, there will be additional reflected waves over and above the incident waves this will; obviously, lead to misleading results. So, you have to create a condition within a closed chamber such that you mimic the anechoic conditions of the open atmosphere, that can be created by again putting appropriately designed acoustic absorbers and the way to qualify whether the acoustic absorber is doing it is job, is by calculating or rather measuring it is impedance if we do see that the impedance across the frequencies of interest are falling near about the characteristic impedance then we will sort of believe that it is doing a reasonable job otherwise we will have to rework these calculations.

So, impedance is a very important quantity, and in the third comment I will just invert the argument the other way round; just like we see that if there is an impedance matching condition no reflection is created conversely if there is an impedance mismatch then there will be large reflection right. So, that can be again easily seen from here if you have rho 2 by C 2 to be very different from rho 1 by C 1, then you will get B by A to be large. So, you if you create a large difference in impedance between 2 neighbouring fluids, then you can cause a large reflection right and this idea is again going to be exploited in muffler design.

In muffler design when you have this expansion chamber kind of muffler, this is exactly what happens that you are creating 2 zones with 2 different impedances just that impedance within a muffler has to be defined in a little different way, but the idea remains the same that when you have the sudden expansion the impedance suddenly changes and the wave sees a impedance mismatch and the wave does not like it to see to it to the fact that there is an impedance mismatch, a part of the wave turns backwards.

So, what happens is that the part which goes outside your exhaust is diminished, a part of it goes backwards right up to the engine maybe the engine noise someone sitting inside the engine will feel the noise as increased due to the muffler, but why should we care about that. We should care about what is the noise which is going past the exhaust not what is the noise which is felt inside the engine I mean when I say inside the engine I actually mean inside the combustion chamber of the engine. So, this muffler as actually what it has done by creating reflection is that it has partially reflected back the noise back to it is source, and that is not a problem because the receiver is getting saved. The receiver is having lesser noise to deal with the philosophy is you reflect back the noise to it is source and thereby at in your objective.

So, that is the principle of design of reactive muffler. So, I will just we will come to mufflers in great details later, but I just wanted to make this comment. So, we will quickly conclude with this note that if rho 2, C 2 is very different from rho 1 C 1 then reflection ratio is large and transmission ratio is reduced. So, as I said there are applications of these observations in the fact that you have coming in the fact of muffler design which we will do in greater details later on in this course.

Thank you for today.