Acoustics & Noise Control

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Module - 07
Lecture - 12
Travelling and Standing waves

Good morning friends.

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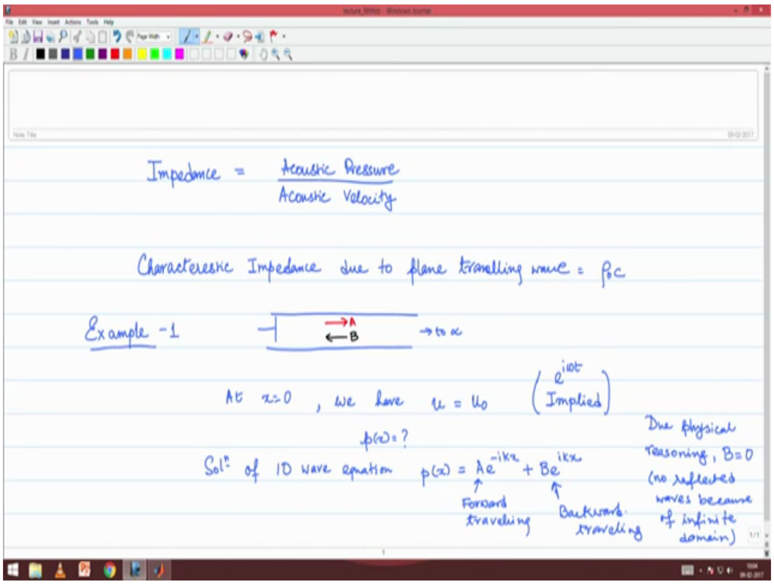

In the last class we talked about the concept of impedance, which is a very general concept. And by definition this is acoustic pressure divided by acoustic velocity at a given point. In particular, we found what is known as the characteristic impedance due to a plane travelling wave. And this was given as the ambient acoustic density multiplied by the sound speed. We understand that the velocity has to be they can take in the direction of the travel of the wave itself. Therefore, we will say characteristic impedance is rho 0 c with the understanding that the direction has to be direction of the velocity has to be in alignment with the direction of the wave in which it is travelling.

So, today we will use this concept of impedance to do a quite a few problems. So, let us look at the first example which is almost trivial. So, we will take a duct or a pipe if you want, which is extended to infinity, on one side and on the other side you have rigid oscillating pistol. So, this exercise; obviously, is sounding a little academic at this stage, but we will see how this little academic exercise if you may leads to more important concepts as we develop this. So, the point is the problem is follows at x equals to 0, we have the boundary condition given, u is given by u 0. Remember from here on I take e to the power i omega t as a time dependence implied. I am not going to explicitly write down this time dependence it is going to be assumed that all my quantities of interest they are dynamic quantities, and we are only going to look at a single frequency characterization of this

So, e to the power i omega t is implied. So, when I say the velocity is u 0 it essentially means u 0 e to the power i omega t the amplitude of which is u 0 the associated frequency is omega. Now the point is what is p of x: that is what we need to find out. So, we understand that the solution of 1D wave equation and this is 1D because we are talking about plane waves in a duct. So, we are looking for plane waves in this situation, nothing other than plane waves can exists by this geometry. So, we are looking for the solution for the 1D plane wave equation and that we know for sure can have this form which is a collection of a forward wave and a backward wave.

So, there can be a forward travelling wave of magnitude A and there could be a backward travelling wave of magnitude B or amplitude B right. We need to determine these 2 amplitudes. So, I will write that down as this is the forward wave travelling and this is the backward travelling, but then we realize that there is absolutely no reason why a backward wave can originate in this situation. That is because the backward wave is not possible here because there is no chance of any reflection by the geometry of this problem the cause of the wave is this source at x equals to 0. So, this x equals to 0 can lead to only an incident wave, which is going to travel in the forward direction, but then it is never going to come back because the boundary is never there the boundary is extended all the way up to infinity.

So, there is no boundary which means that the backward wave is actually not there. So, therefore, we might as well assume due to physical reasoning, we rule out the existence of a backward travelling wave. No reflected waves because of infinite domain. This is exactly what happens even in 3D, even in 3D when you are talking on an open atmosphere I mean to say you have just there is a sound source which is lying on the floor and on the top of this flow there is virtually no boundary. So, therefore, you do not expect any reflection to come from the boundary of this domain this is actually an infinite domain problem. And accordingly you need to rule out certain waves in one day case this is all very simple because there is only one dimensional wave propagation, there are only 2 waves one incoming and one outgoing.

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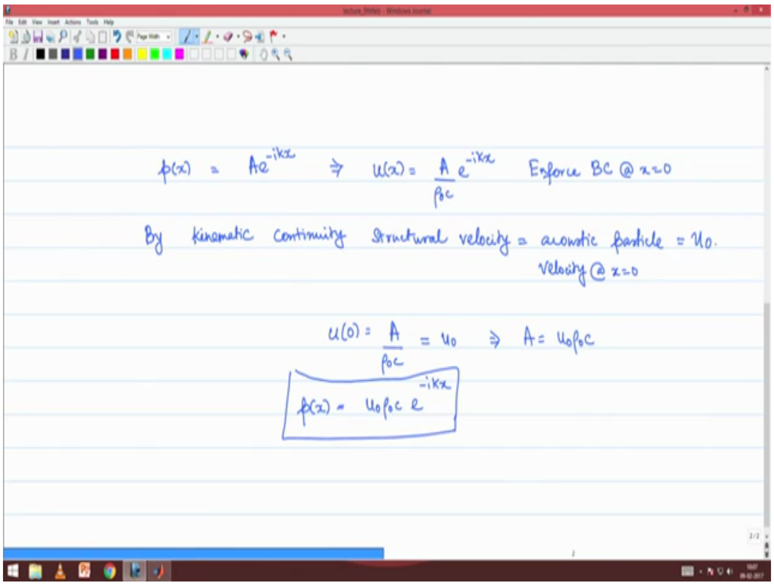

So there are no incoming waves, in other words possible for this geometry. So, that leads us to believe that p x has got to be only A e to the power minus i k x. And if p x is this then we know for sure u x is going to be A divided by rho naught c e to the power minus k x. Because rho naught c is the ratio between the pressure and the velocity which means the velocity is pressure divided by rho naught c. That is how you get this point. And also we know that we have to enforce the boundary condition at x equals to 0 which says that it at x equals to 0 since the piston is moving at a velocity of u 0. Therefore, we expect that the neighboring fluid particles the acoustic fluid particles which are just attaching onto the oscillating piston surface, which are just kissing the surface of this oscillating piston will have exactly the same velocity it cannot be anything else because there has to be kinematic continuity.

So, by kinematic continuity, structure velocity, equals to acoustic particle velocity at x equals to 0. And this by definition is u 0, which means u at x equals to 0 has got to be A by rho 0 c which is u 0 which implies A has got to be u 0 rho 0 c. Therefore, what is the final solution of the acoustic pressure profile it is p x equals to u 0 rho 0 c into e to the power minus i k x this is the pressure profile which is generated within this semi infinite duct.

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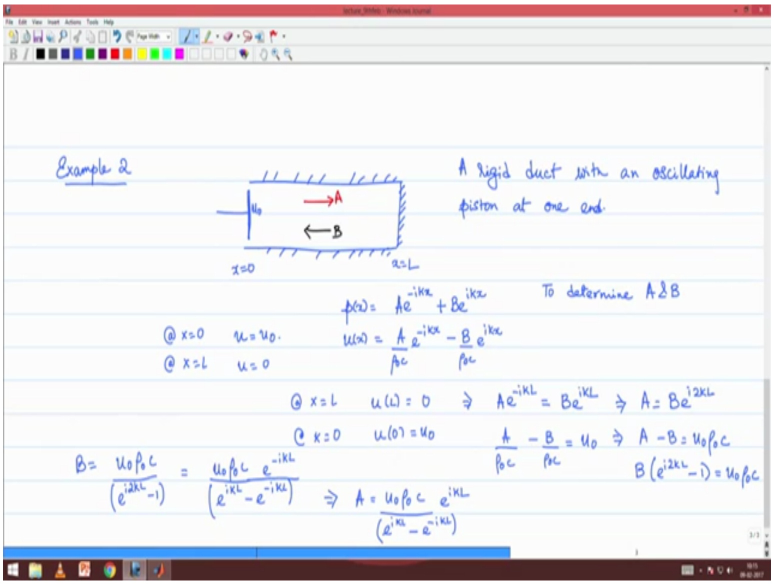

Let us now look at a more realistic problem where we have a finite duct. So, here we have a finite duct a tube pipe, if you may call it, and the duct walls are assumed to be rigid. So, it is a finite duct of length L again at one in x equals to 0 we have an oscillating piston and at the other end we have rigid termination x equals to L all the walls are completely rigid.

So, the problem description is as follows rigid duct with an oscillating piston at one end. So, again we are expecting only one dimensional wave solution in this case, I here we are limiting ourselves to one dimensional wave solution because everything is in 1D. So, excitation is such that all the wave all the fluid particles at the plane x equals to 0 will have identical motion which means that there cannot be anything other than a plane wave because all the planes parallel to x equals to some quantity will have identical measure right. So, therefore, we expect the solution to be in this form p x is A e to the power minus i k x plus B e to the power i k x.

Once you have one dimensional plane wave equation there cannot be any other solution other than these 2, this is what we have extensively studied. So, as usual there is an A wave and there is a B wave. Last time we exist we ruled out from physical arguments the existence of this B wave or the incoming wave because, there is no chance that there is such an invert travelling wave going to the fact that there is no reflection condition within that infinite duct problem. But now since it is a finite duct problem; obviously, the incident wave which is originated will reflect after back after sometimes and therefore, you expect both A and B waves should be there. So, A and B basically needs to be determined. So, at this point we do not know what is A and B.

So, the objective will be to determine to determine A and B right. So, how do you go about doing that? Please note the conditions are both the boundary conditions that we have is at x equals to 0, we must have u to be equals to u 0. So, again this piston is oscillating with amplitude of u 0, it is oscillating harmonically because everything that we study here on has harmonic time dependence alone. So, this is one boundary condition that we need to enforce that is at x equals to 0 the fluid particles which I have just attached to the surface of the oscillating piston, will have exactly the same velocity as the piston. There is no other way for the fluid particles to have any other velocity.

So, at x equals to 0 u equals to u 0, and we also have at x equals to L the termination at the termination u has to be 0. Because it is a rigid termination it cannot move. So, therefore, these 2 boundary conditions have to be satisfied. So, if p x is given in this form we will again adopt divide and rule policy. We know for the forward travelling wave the associated velocity particle velocity will be A by rho naught c, but for a backward travelling wave it will have the magnitude A by rho naught c, but in opposite direction. So, we will take this impedance approach to very quickly write down, the velocity profile inside this duct to be in this fashion.

Please note there is a minus sin associated with the B wave. And that accounts for the fact that the particle velocity now has to reverse in it is direction because we are talking about an invert travelling wave. So, this is the form of the acoustic velocity that we have supposed to get. Now at x equals to L. So, if we imply the second boundary condition at x equals to L we want u at L to be 0 and that implies A e to the power minus i k L must be equal to, B e to the power i k L this in other hand implies a equals to B e to the power i 2 k Laughter, that is fine.

The next one would be at x equals to 0, we wish to have u 0 to be 0 sorry u 0 to be u at x equals to 0 is u subscribed 0. So, that would give us I forgot an x here. So, that would give us A by rho naught c minus B by rho naught c should be equals to u 0 which implies a minus B has got to be u naught rho naught c fine. Therefore, if I now substitute A is equals to B e to the power i 2 k L, I get B into e to the power i 2 k L minus 1 is equals to u naught rho naught c. So, therefore, my answer for B looks pretty simple, which is u naught rho naught c divided by e to the power i 2 k L minus L. This I could make it a little simpler to look by multiplying and dividing with e to the power minus i k L.

So, that would lead us to u naught rho naught c divided by e to the power i k L minus e to the power minus i k L into e to the power minus i k Laughter; I am just multiplying and dividing by e to the power minus i k L. And if this is B a has to be e to the power i 2 k L times B which implies a has to be u naught rho naught c divided by e to the power i k L minus e to the power minus i k L into e to the power plus i k L, because I need to multiply the expression of B with e to the power i 2 k l.

So, therefore, the numerator will no longer B to the power minus i k l, but it will be e to the power plus i k L.

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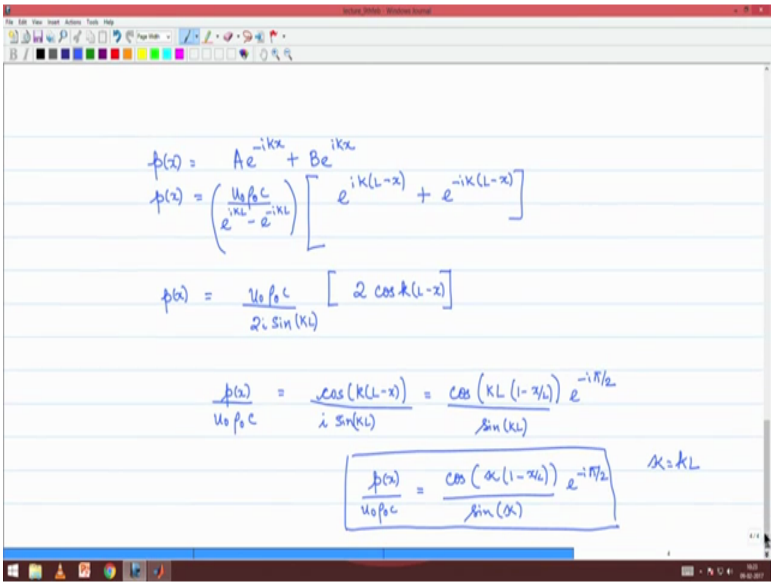

So, A and B have been found. So, therefore, we can as well right the pressure profile p x which we remember is e to the power minus i k x plus into A plus B into e to the power plus i k x, but then A and B both have been determined which is of that form. So, we can take u naught rho naught c divided by e to the power i k L minus e to the power minus i k L common. And with A we have and e to the power i k L with a positive sign. So, therefore, e to the power i k L minus x is what you get for the first term. And with the B what you get is you have e to the power minus i k L and there is e to the power i k x with the positive signs you taking care of all that you are going to get e to the power minus i k L minus x right.

So, this p of x: so employing the use of trigonometric functions now just to make the interpretation simpler. So, we understand the denominator will read as 2 times I times sin k L. And the numerator would read as 2 times cause of k L minus x. Therefore, this is p of x. In other words if I sort of non dimensionalize it p x by u naught rho naught c is going to read cos of k L minus x divided by sin of k L in the denominator you will have an i. I will make a few more simplifications. So, I will push pull out this k L factor and I will put 1 minus x by L instead of 1 minus x and then there will be sin of k L and the denominator of I could be put as e to the power minus I pi by 2.

So, this is the form of my non dimensional pressure, cos of I called this term k L as kappa, and 1 minus x by L divided by sin of kappa e to the power minus i pi by 2. I call kappa as k L. So, this is the final form of the response profile that I am looking. There is a very important difference between these forms of the solution with what we did at example 1. Let me explain this to you by a MATLAB plot. So, what I will do is I will plot this solution in MATLAB for you and I will show you how this solution is going to be different. So, what is the p x comma t that we need to plot out in MATLAB?

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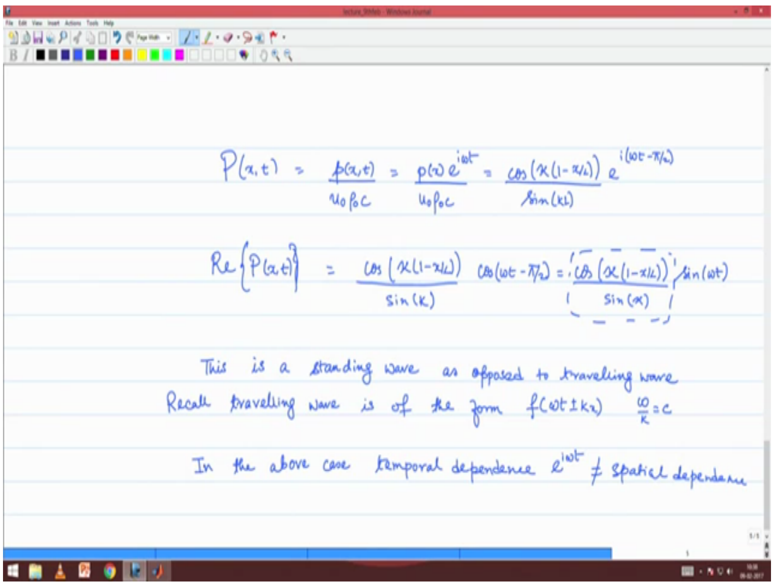

So, we will we will define this capital P if you may. So, like x comma t as p small p x comma x comma t divided by u naught rho naught c, which is p x into e to the power i omega t divided by u naught rho naught c. And that is cos of kappa 1 minus x by L divided by sin of kappa L. There is a u naught sorry there is no u naught 1 naught c (Refer Time: 20:44) and then there is e to the power i omega t minus pi by 2.

And we will plot only the real part of it, because we know the real part of it corresponds to the solution that we are interested in. So, real part of this capital P x comma t will be what, give this function the trigonometric function is bound to give us only real numbers. So, no issues with this part it just pops out, k L is basically kappa and then we will have to take the real part of e to the power i omega t minus pi by 2 which basically means we have to take cause of omega t minus pi by 2. And this is same as cos of pi by 2 minus omega. So, this is sin of omega t. So, this is given as cause of kappa 1 minus x by L divided by sin of kappa sin of omega t.

So, we will generate this plot for x at different time instance. We will generate this for omega t equals to let us say 0 for omega t is equals to pi by 4 for omega t is equals to pi by 2 and so on and so forth. So, for generating the plot we; obviously, need to choose some value of kappa, which is what we will choose what will be x by L, x by L will be in the range of 0 to 1, x cannot go outside L. So, therefore, that is how we will do it.

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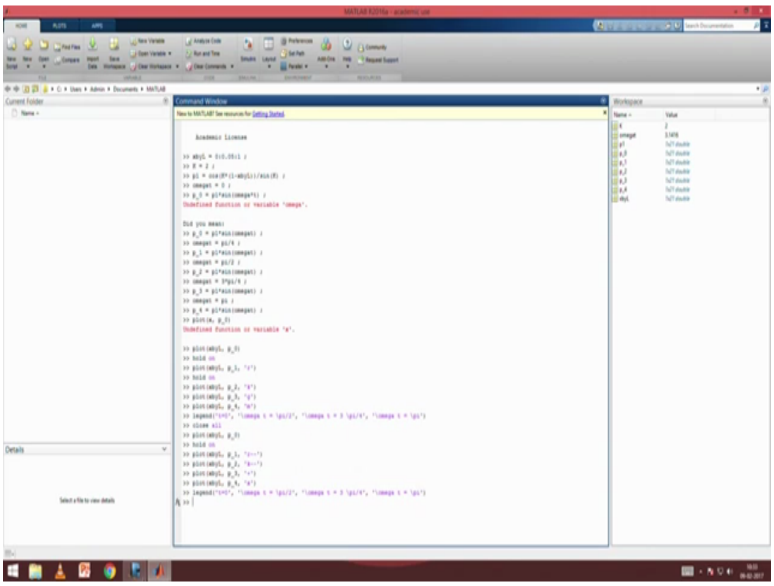

So, let us define this quantity x by L as 0 in steps of let say 0.05 up to 1. So, we have defined a 21 component vector and we can keep kappa or capital K for kappa as 2 for illustration we can try for others also, but keep it to for now and then we have to plot this p which is cosine of capital K into 1 minus x by capital L, divided by sin of kappa L, K L capital K, sin of k this is what will happen at this omega t equals to pi by 2.

So, let us let us take this as p 1. We will need to multiply this with sin of omega t. So, p or we will now take omega t as 0, this is the first situation and p will be p 1 into sin of omega into t, or this is p at 0 fine. Omega t, now we will do p at for will generate another p which is for omega t is equals to let say pi by 4. And this we will call as p for the next time step p underscore 1. Then we will generate for omega t is equals to pi by 2 fine. So, this will we will store it in p 2. Then we will come to omega t is equals to 3 pi by 4. And we will call this as p underscore 3. And let us settle with 4 of them.

In the next time instant we will have omega t is equals to pi and then p 4 is equals to p or this. So, we have got the pressure profile for 4 different time instance 0 1 2 3 and 4. So, let us plot each of them one by one x comma p 0 that is anyway going to be 0 because of the sin omega t effect x by L, I am sorry x by L. This get 0 no surprises for that we will hold on and we will plot the next one in red.

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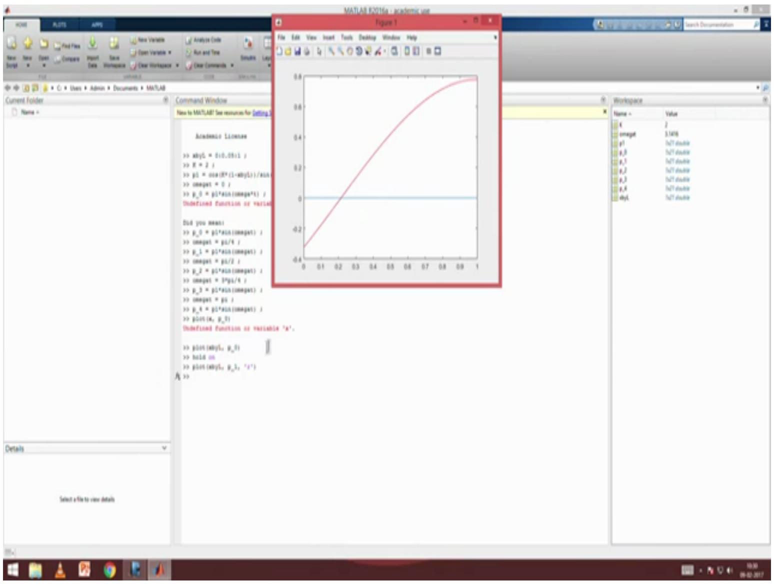

So, now you see this is how it looks like. Then we will hold on and plot for the next one plot p 2 in black colour. This is how it looks right after that plot p 3 in green colour. After that plot 4 in it is a magenta colour, all of these have been plotted. So, let me just put a legend. So, that I do not forget what is the ordering. So, the first one is at t equals to 0 small t is equals to 0, the second is at omega t is at pi by 2. The third is at omega t is equals to 3 pi by 4. And finally, the fourth is at omega t is equals to pi.

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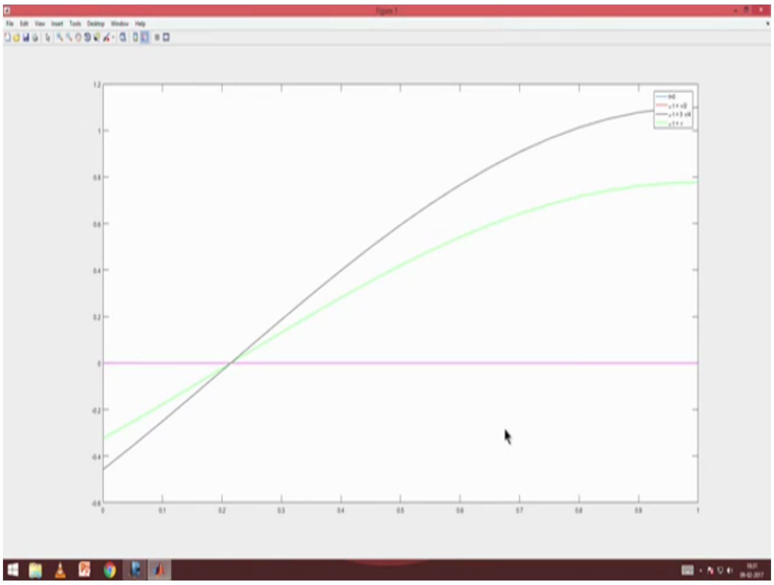

So, here we go. So, this is how the plot looks like what does happened is actually the red and the black has superposed. Similarly, the green and the magenta have superposed. So, maybe I can do it better by then I will just re plot with a different sin. So, that it is visible that it is x by L p underscore 0. That is the first one then we will have hold on and then we will have plot x by L, p 1 and that is in red dash line. Then we will have a plot x by L p 2 that is in black line. And then we will have plot in p 3, p 3 and I will use a marked this time which is plus, and then finally I will have a plot on p 4. And I will have a marked this time which is x and then legends.

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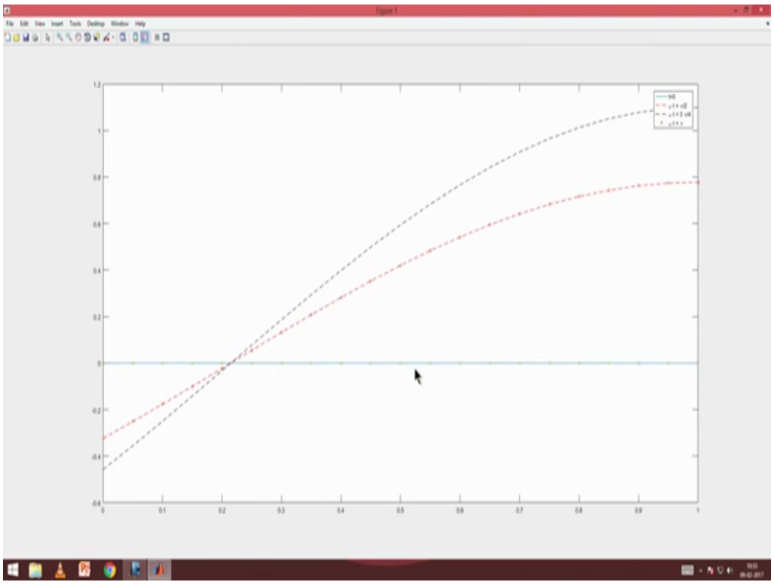

Let see how it goes. So, here you get to see this is how things look like. So, it starts with the blue colour, then it goes to the red colour and then it goes to the black colour which is at omega t equals to I think the legends also need to be redefined. It starts with the blue colour then it goes to the red colour then it goes to the black colour and then it comes back and what you get to see is the plus profile which is exactly the same as what happened in a few instance of time earlier also. And then finally, it comes back and goes red 0, what will happen in this is how the period remember we have done only from up to omega t is equals to p, right this is only half the period omega t equals to 2 pi will mean a complete cycle for oscillation.

So, in half the cycle what has happened is that the pressure profile has gone up even more up, then it has come down and down to 0 in the next cycle it will go in the other direction right. So, if you actually animate this way profile you will see that it is going up and down, but each of these particles will have amplitude of oscillation which is just this much right. Each particle will have in other words different amplitude of oscillation the particle at let say point 4, is going to oscillate at this level the particle which is at point 9 which is going to oscillate at that level right.

So, this is the waveform which is going to wax and wane it is not having the same response at all spatial locations. This is because what you have done now is that this is typically called a standing wave as opposed to a travelling wave right. Because 2 waves the forward wave and the backward wave have now superposed such that you are getting a pattern in space which is waxing and waning which is going up and down if you may. So, please, but the amplitudes across different points are different.

In fact, if you look at this point somewhere at 0.2 neighbouring 0.2 this point actually remains to have 0 pressure for all instance of time right. And this point will be called like it is like a node at this frequency which is corresponding to k L is equals to 2; obviously, this does not have much significance at other frequencies, but at this frequency this point does not seem to be moving at all it is having a 0 pressure all at all times right at some other frequency some other points maybe at this characteristics. So, this is an important aspect which we should be able to appreciate that what we are getting now is this is a standing wave as opposed to travelling wave.

So, recall travelling wave is of the form f of omega t plus or minus k x right. You have to have the same functional dependence in both space and time and the coefficient omega by k must be c right. To this form is defined by the expression that we have got here. So, in the above case temporal dependence is e to the power i omega t, and this is not equals to the spatial dependence the spatial dependence is some other function. The functional form is exactly given by this block and this functional form is definitely not the same as the tempo associated temporal function.

So, this leads to our first encounter with what is known as standing waves. Please realize that standing waves are produced in the case of a finite domain problem, whereas, travelling waves are produced in the case of an infinite domain problem. In the case of infinite domains you do not have any reflection and that is why there is only one propagating wave or one travelling wave.

But once you have a boundary reflection set in and because of that reflection you are going to get a standing wave as opposed to as opposed to the case of travelling wave. There is another important feature which I would like to talk about regarding this solution. If you had noted the denominator has the term sin k L or sin kappa L. So, there is a possibility that the denominator can actually go to 0, that and that happens at k L is equals to n pi right. So, at k L is equals to n pi, we have sin of k L going to 0 and that implies p of x will go to a very large number.

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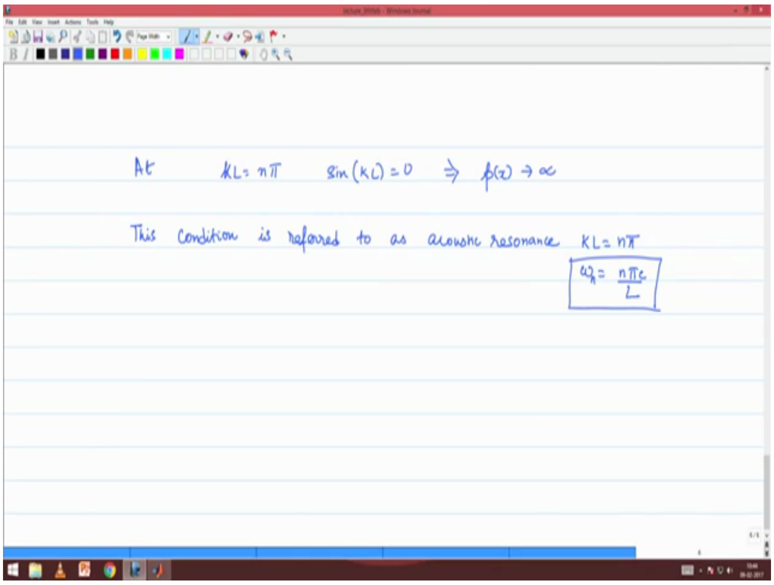

So, this is exactly similar to what you we call it as resonance in structures. In structures also when we dealt with vibrations some of you may have had the prior exposure to vibrations. In vibration we identify this condition to what is called as a resonance condition. In a resonance condition if frequency meets a certain criteria then you are going to have this condition which leads to a large response right; obviously, in reality you do not get infinite acoustic pressure just like in reality you do not get infinite structure infinitely high structural vibration the reason for that is there is always some realistic damping which keeps things into control and also non-linear effects kick in once the responses grow large and those sort of issues actually arrest the response to a more finite valued response, but even though it is finite valued definitely it is much larger. So, that is why we identify this condition as resonance.

So, using that very same analogy we will refer to this condition as the acoustic resonance condition. So, this condition is referred to as acoustic resonance, which basically means k L has to be some integer multiple of pi. In terms of frequencies it means omega has to be n pi c by L. So, the resonance frequency is omega n can, will be harmonics and they will depend only on the geometry and the material property of your fluid of your acoustic fluid. So, these are the natural frequencies that we are getting which essentially means that you have a very large acoustic pressure will be build up even with a very small velocity of the oscillating piston.

Again this simple problem, as I said is scalable to higher dimensional cases also. The important take away is that if you have a finite domain acoustic problem for example, if you are interested to know, the response in this room which is actually a finite domain abounded domain, then you can expect that there are resonance is in this room just like there are resonances in a structure. But if you are interested to know how let say and engine is radiating into the open atmosphere or how a vehicle is behaving during the time of pass by noise during the time of pass by noise measurement the vehicle is supposed to be in open atmosphere; obviously, ground it. So, there is one boundary which is the ground plane there is well understood.

But other than the ground plane the acoustic domain is not bounded. So, therefore, it is also a semi infinite domain. In the case of the pass by noise application you do not expect any resonance, but if you are looking for an in cab noise application with the window shutters completely closed then the acoustic space within your car is going to be converted into a finite domain space. And then you will have acoustic resonance you can do this experiment in a car if you want to keeping it in neutral keeping your car in neutral just keep revving up your engine.

At some rpm you will see suddenly a very high noise is coming. So, that noise is associated with the acoustic resonances of your cavity. I mean one possible source of that large noise at that rpm is because of the acoustic resonance of the cavity. So, these cavities these are all call cavity resonances or acoustic modes associated with the cavities.

So obviously, the 3 dimensional problems is more complicated then can then can be done in pen and paper calculations, we need specialized commercial softwares to calculate these resonances for us in case of a 3 dimensional problem, but the concept of resonance can is more is what I am trying to explain it out to here. You should understand this the concept of resonance which I tried to explain with this simple one dimensional case, the same concept of resonance applies even for a more complicated problem for which; obviously, union numerical simulations as opposed to close from analytical expressions.

So, I will just generate another plot what we have done presently is that we have generated a plot against x for this function right. What we could also do we can plot this amplitude against kappa for a fixed value of x, and let see how it how that one looks like.

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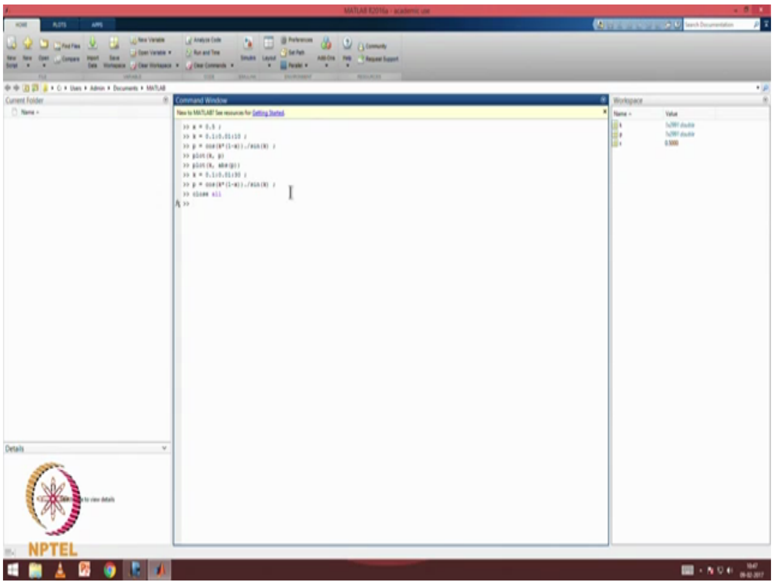

So, we will close this figure and also we will clear all the variables we will clear the screen for us. And this time we will keep x by L value let us say at 0.5 at the midpoint. And we will loop around the frequency variable k or k L rather from let us say we do not want the 0 value to come in. So, point one in steps of let us say point 0 one up to 5 maybe 10 little bit fine. So, what we want is to find the pressure at different values of the frequency.

So, what is the expression for that cos of k 1 minus x by L divided by sin k? So, we will do that. So, the pressure that we are looking for is cos of k into 1 minus x element wise we have to make the division divided by sin of right. And then we can plot out k divided with respect to p.

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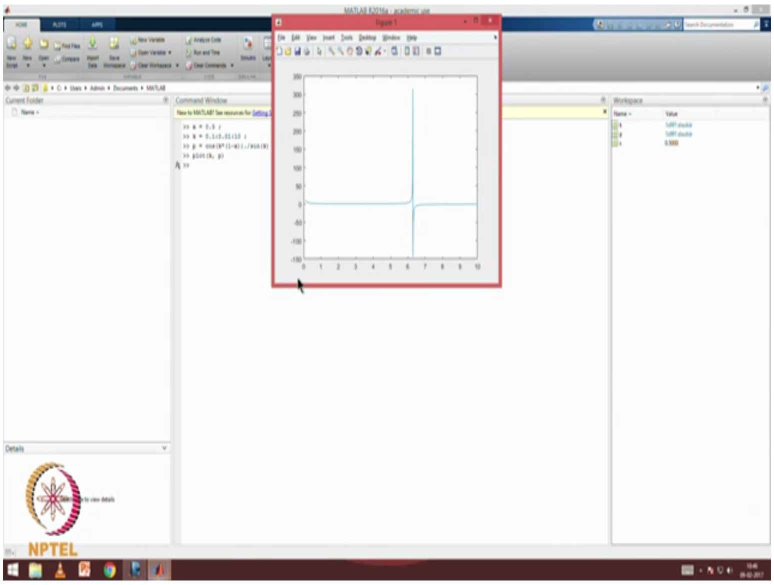

This is how it looks, we can n fact only plot the magnitude of it because we are looking at the amplitude k absolute value of p.

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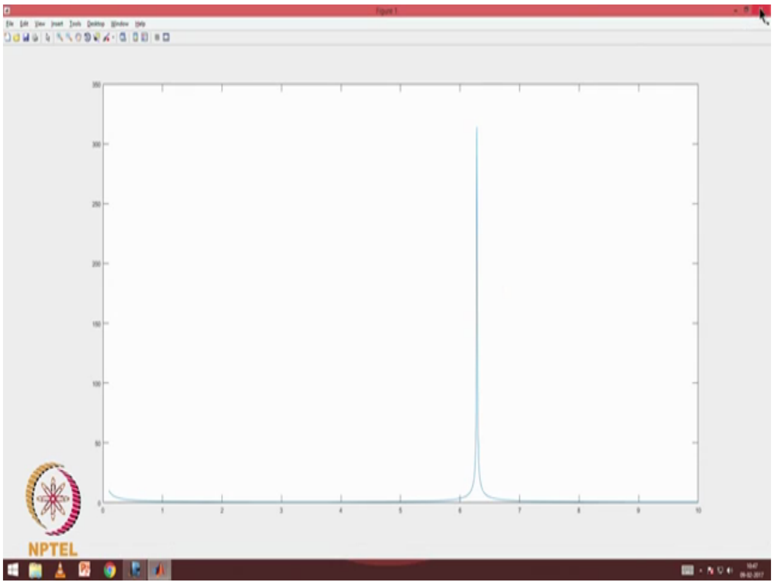

You get to see a very sharp peak here this is exactly the same f r, f peaks that you would have observed even for vibration.

You will keep getting many more such peak. So, if I extend this k up to let us say 30 would be good enough to show you multiple peaks, then again I calculate p, which is cos of something and then again, if I make that plot, but before that I will close the existing plot and I will plot it again.

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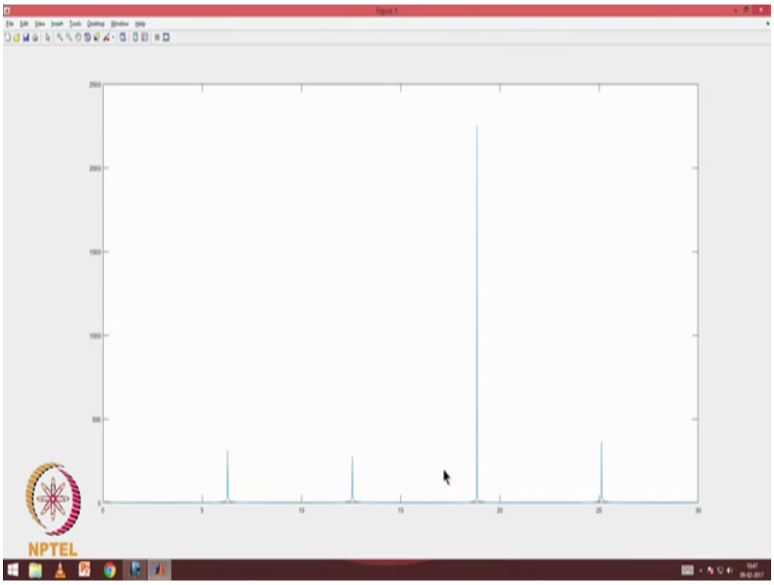

So, you will get to see multiple peaks that is because like vibrating system this acoustic system is also a continuous system and all continuous system will have infinite number of natural frequencies and they are all harmonically related. Because we saw the integer multiples are in the natural frequencies are impulse. So, this is what I had in store for you today we will take it up from here in the next class.

Thank you.