Acoustics & Noise Control
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Module - 06
Lecture - 11
Harmonic Plane waves

So, good morning friends in the last lecture we talked about the stationary signals and just to give you an example we talked about this example

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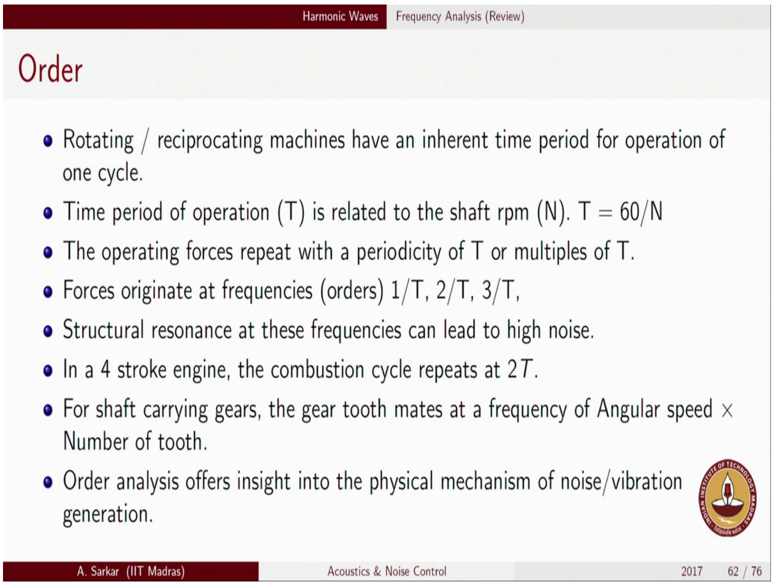

But we will recapitulate and the same that any rotating or reciprocating machines will have an inherent time period for operation of one cycle. For stationary signals we understood that different machine components do generate stationary signal.

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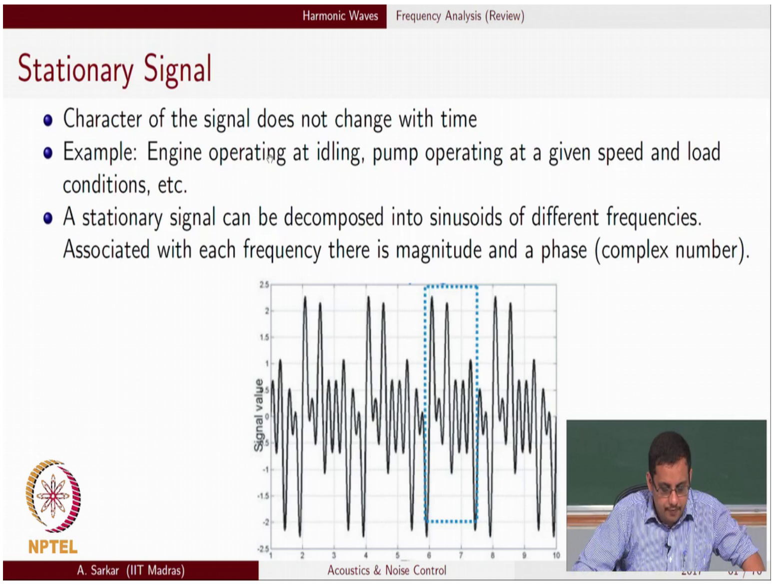

So, let us recapitulate that once more associated with the operation of such machines we have an inherent time period for the operation of one cycle. So, if the time period of operation is capital T that will be related to the shaft rpm as 60 by n.

So, in case of 4 stroke engine this will be. In fact, 120 by n, because 2 cycles of the shaft rotation will lead to one cycle of the combustion. So, the associated fundamental time period will be 120 by n. So, the operating frequencies will repeat with a periodicity of t or with periodicity of multiples of t certain processes like in 4 stroke engines you will have you know 2 cycles of the shaft of the crank shaft if once it completes 2 cycles of rotation only then the combustion will repeat. So, therefore, it will be multiplies of t, but then the forces which originate from the basic operations of the machine will therefore, have frequencies of the order 1 by t 2 by t and 3 by t.

So, they are all going to be integer multiples of each other or harmonics of each other that is the fundamental point. And if there is now a structural resonance occurring at any of these frequencies then you can expect large vibration and hence large noise. So, what we are trying to show is that 1 by t 2 by t 3 by t all of these are basically related to the engine rpm N right. So, N being the engine rpm the reciprocal of the time period is all related to the engine rpm. So, all of these forcing frequencies in other words are linearly related to the engine rpm or the crank speed fundamentally.

So, just as I said there is a the difference in 4 stroke engine as compared to the others is that the combustion cycle will repeat with 2 t and therefore, you will have 1 by 2 t has a fundamental frequency and all integer multiples of this fundamental frequency will be present in any noise and vibration signal. Similarly for the shaft carrying gears, when the gear tooths will mate then the fundamental frequency will be the angular speed into the number of tooth this is dictated by the time period that is taken for 2 consecutive engage engagement between 2 consecutive gear tooth pair this is called the gear meshing frequency.

So, the gear mesh frequency is given by angular speed times the number of tooth and these frequencies are in fact, colloquially termed as order. So, when we say that there are certain orders of your noise and vibration signature in the signal then we say that these are the fundamental frequencies and these are the harmonic frequencies which are present. So, in order analysis typically what we are interested is to find out what is the nature of the forcing that happens at different rpms, like the combustion process for example, again for different speeds of the crankshaft it is going to change. So, typically in acoustic analysis ordering order analysis is pretty important just a quick recap for the non stationary signals.

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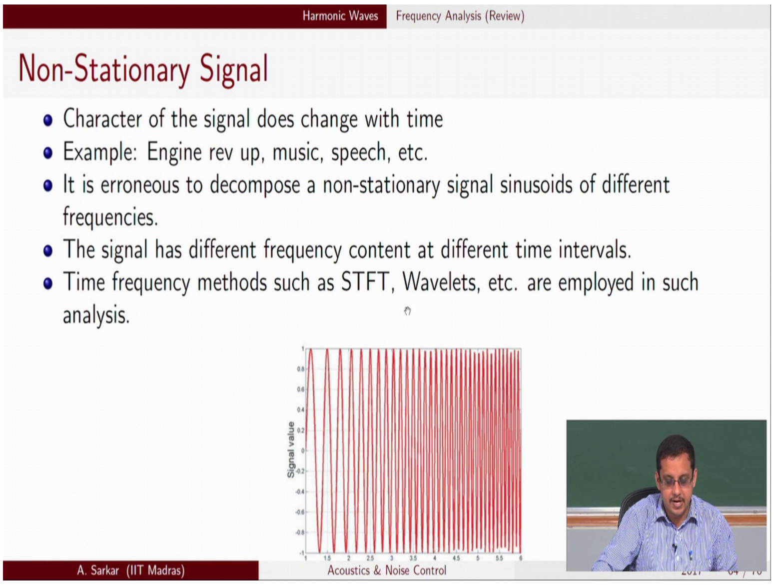

Also. So, as we said that the character of the signal does change with time and that is why it is called non stationary typical examples are music voice and engine rev ups that can happen.

So, in such cases it will be erroneous to decompose this non stationary signals into different frequencies. So, as a part of one of your assignments what you will be asked to do is you will be asked to take FFT and inverse FFT is so you will see that for a stationary signal if you take Fourier transform and then inverse Fourier transform it you will get back the same you will recovered the same signal whereas, that would not happen in case of non stationary signal. So, this is one way in which you can quickly identify the difference between stationary signal and non stationary signal. In other words, what; that means, is that when you are dealing with non stationary signal you should not use Fourier analysis technique without applying your mind. You can use a different versions of Fourier transforms like short term Fourier transform, but the routine vanilla application of Fourier transforms has to be avoided.

So, there are more specialized tools such that such as wavelet us and STFT which are employed and I think I talked about this graph in the last class also.

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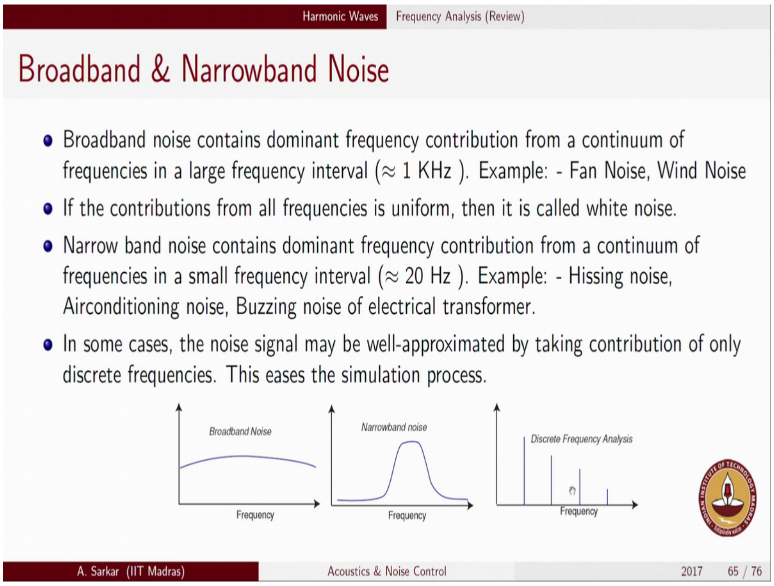

So, I will not recap that part carrying on we were till now talking about tonal noise; that means, single frequency type of situation or we also talked about frequencies, which are happening in an discrete manner when we talked about order analysis for example, there were multiple frequencies that we are talking about, but these frequencies where harmonically related. But then there are cases where we contain where this acoustic signal contains all the frequencies spill over a certain interval. We say it is a broadband noise if the frequency is contained in the signal is in a con is an interval of about one kilo hertz or so. Then we will call it as a broadband noise and the examples being fan noise and wind noise. If it is spilled uniformly from right from the limit is of audibility; that means, from about 20 hertz to 20 kilo hertz if you might wish then it is called a white noise.

So, white noise is an example where there is absolutely no correlation between the values of the signal, and the if there is no correlation then; that means, there is all the frequency contents are spilled on to the spectrum and that is why we call it as white noise right. So, therefore, this is an example of white noise, and typically these kinds of processors what we did the other day in the mat lab illustration process, if you remember when we just ask mat lab to generate us the a few samples of these random numbers, and we plate that out that was creating a harsh sound typical of you know let say the sound that your hearing of the ac right now.

It is also having a quite a dominant component of this white noise typically, when you have flows in without any resonance suffix not the whistling sounds, but any without any resonance effects if you have these kinds of flows you will have white noise. Whereas if you have some tonal noise embedded within the flow you will have a narrow band noise. For example, fluids when you have fluids depending upon the finger location of your finger, which hole you are closing you will have different resonance conditions. And associated with those different resonance conditions you will actually have a dominantly tonal noise, but yet there will be some frequencies filled around that tonal noise. And therefore, we would rather call it as a narrow band noise, but a dominant contribution coming from a tone.

So, these narrow band noise as opposed to broadband noise will have a very small frequency interval where in the frequency content is non 0 typically of the order of 20 hertz or so. Whereas when we say it is a broadband noise it is a much broader frequency interval wherein it has a dominant contribution. So, even the whistle that we blow out if you typically take sound signal and do a proper analyze experiment and analysis that sound will have a narrow band effect air conditioning saw, nows if you have here gone to the power distribution system place where there are lots of transformers you will say here a typical buzzing noise. So, that will have dominantly tonal impact, but then it is not exactly a pure tone there will be certain frequencies which is spill around that pure tone.

So, this is a broadband noise for you the contributions are unequal over the entire frequency level broadband noise does not mean that all frequencies are equally present that happens only in the case of white noise, whereas, in the narrow band noise he will see that the dominant region in the frequency spectrum happens to be located in a smaller interval as opposed to bro broadband noise. And when we do discrete frequency analysis we are just approximating individual frequencies we are not approximating all the frequencies, we are talking about each and every frequency probably multiple such frequencies can be analyzed, but we are not going to analyze a continuum of frequencies that is that is a crucial point of differentiation.

So, with this sort of precursor we are now in a position to get ourselves convince that it make sense to make the reduction from time to frequency domain it is actually a matter of simplification in terms of the mathematics of the problem, but at the same time we are convinced that there are lots of application areas in machine noise wherein we should be able to do a good analysis staying with a frequency domain based analysis.

So the way to go forward is that we will actually do the analysis for a single frequency, but then we understand if we can do it for one frequency, at a time we will just have to repeat for all other frequencies right. So, the discrete frequency analysis for example, is not a big deal we just have to will show you the procedure how to analyze for one frequency, but if you know what is the procedure to analyze one frequency at a time, then you might as well do it 5 times to do this analysis for 4 or 5 different frequencies. You have to simply write a loop in your computer program in your analysis methodology such that the loop repeats over the entire frequency points of your interest.

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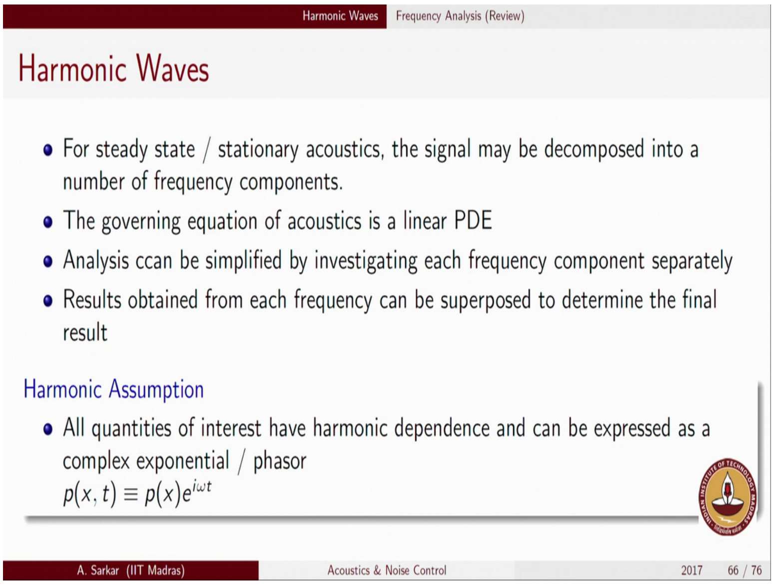

So, with that let us begin our analysis for harmonic waves. So, by now we are convinced that for steady state or stationeries acoustics the signal can be decomposed into a number of frequency components. And we also know that the governing equations of acoustics is a linear partial differential equation. So, therefore, we bring about the harmonic assumption which will come in a moment. So, the analysis can be simplified by investigating each frequency component separately rather than trying to analyze all the frequency component simultaneously, we will take the divide and rule approach wherein we will analyze each frequency component separately and then we will recover the total solutions simply as a superposition. So, that is the harmonic assumption for you. So, the harmonic assumption is hereby stated in the following manner.

So, all quantities of interest typically our quantities of interest are acoustic pressure acoustic velocity acoustic density and later on we will come across acoustic intensities also. So, all the physical quantities of interest from here on will be assume to have harmonic dependence and once you say it is a single harmonic dependence containing only one frequency then we can understand that it can be expressed in a complex exponential form or phasor form that is what we did in a few lectures back. So, from here on we will say that for example, our acoustic pressure quantity which we understand was a variable dependent on 2 independent arguments namely x and t. But from here on the time dependence will no longer be arbitrary the time dependence will be frozen to be that of complex exponential kind such that we have only a single frequency or a simple harmonic solution for p x same holds true for velocity same holds true for density whatever we wish, to analyze it will be analyzed in terms of these fundamental feature that any variable of our interest will be boiling down into this form.

The variable will be dependent on the special location, but the temporal dependence will be simplified in the form of a complex exponential that is the harmonic assumption for you. So, with this harmonic assumption, let us written to our acoustic wave equation as usual will start with the simple case of one dimensional harmo acoustic waves.

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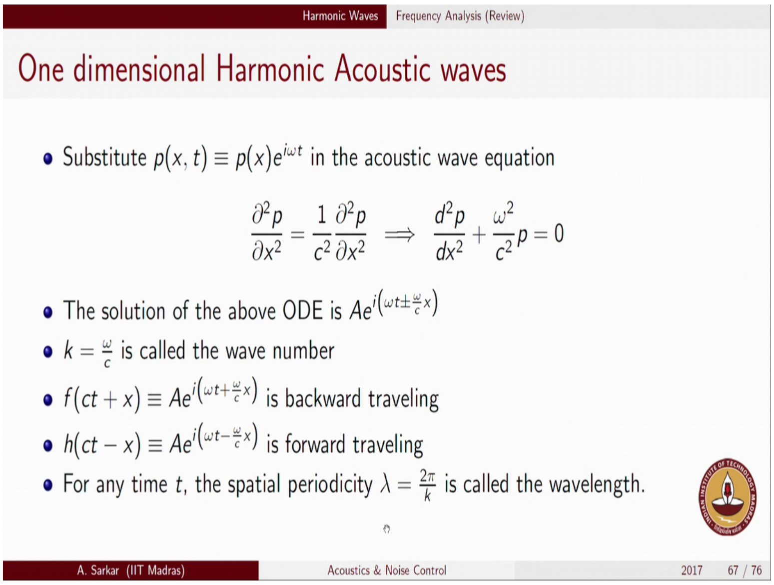

So, the one dimensional acoustic wave equation, if you recall was derived to be this this a typo here it has to t on the right hand side. So, d 2 p d x 2 or rather del 2 p del x 2 is 1 by c square del 2 p del t 2. So, the t has to come in the right hand side, I had copy pasted it which is why the x has come back again. So, that is the acoustic wave equation if you recall and with the assumption, now for the harmonica assumption as we said, in the last slide we are going to replace this function p x comma t to be p x into e to the power I omega t and when you make that replacement del 2 del t 2 will be minus omega square p and you bring that on the other side and you get this form.

Please note that between these 2 steps there is a partial derivative sin or the del sin in the first step, but it has been reduced to d sin or the ordinary derivative. That is because after in incorporating the harmonic assumption there is only one argument to p that is x. So, therefore, there is no question of any partial derivatives there is only ordinary derivatives that appear in this equation. Note that this is a second order linear constant coefficient differential equation. You would have studied that in a course on how to solve this type of system in a codes on let say vibration and it is exactly the same way that you will solve this equations.

So, I am not going in pretty much details about this solution. So, the solution of this o d e will be of the form of e to the power I plus or minus omega by c times x e to the power I omega t has already been invoked. So, the time dependence is e to the power I omega t the special dependence has got to be square root of omega square by c square, which is plus or minus omega by c and it comes with a I sin because when you transfer it on the other side you will get a minus sin. So, I assume that you have a background in a vibration a theory. So, you will know how to solve this kind of second order constant coefficient differential equations.

So, this brings to the solution in this form. So, what we are seeing here is that the solution is given in it is functional form as e to the power I omega t plus or minus omega c times x right, omega by c times x. Together with an undetermined constants A, A e will be an undermined constant which has to e determined right, but the fundamentally if there are 2 solution e to the power I omega t plus omega by c times x or e to the power I omega t minus omega by c times x right. So, with this 2 solutions you can superpose or you can combined these 2 solutions in any combination and get this solution to the partial differential equation made possible.

So, therefore, this quantity omega by c is technically called the wave number it is actually single word wave number it is not I should not have given space for it k is called wave number.

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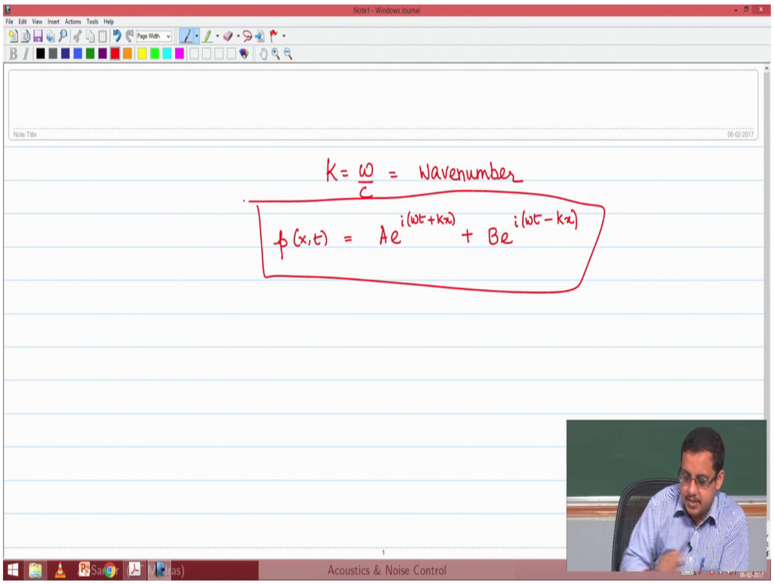

So, I will just make that correction here. So, k equals to omega by c is called wave number. And we have got p x comma t to be a e to the power I omega t plus k x plus b e to the power I omega t minus k x.

So, this is the form of the solution that we are having k being called as the wave number. So, you may recall that what we had earlier was that we had when we did not invoke the harmonic assumption, and we sold out the wave equation in it is complete arbitrary form. The solutions where f of tau plus x and g of tau minus x, but tau is c t, so we had the solution in the form of f of c t plus x, now f of c t plus x does confirmed to this form where you have e to the power I omega t plus omega by c x.

Please note the crucial thing to identify is that the coefficient multiplying the argument t should be c times the coefficient multiplying the argument x. So, we have exactly here the same attribute that is the coefficient multiplying x can be identified to be omega by c, and the coefficient multiplying t is identified as omega. So, therefore, the coefficient multiplying t is c times the coefficient multiplying the variable x.

So, f of c t plus x does conform to a functional form after invoking harmonic assumption as a where a is an undetermined constant, it does not depend on time or space or frequency or anything it is a it is merely a constant with the value of which has to be determined multiplied by a complex exponential, but the argument of the complex exponential has got to be exactly c t plus x, c t plus x and omega t plus omega by c x or omega t plus k x for that matter does not make any difference. We are identified f of c t plus x to be a backward travelling wave right. So, now, that f has got a special form which is complex exponential form. The physical character of backward travelling wave does not change it is just that the profile of the wave earlier we had illustrated this these functions like some triangular wave functions, but now we are specializing this to harmonic functions or sinusoidal function right.

So, this complex exponential form is going to lead to us to a backward travelling wave. And similarly you will recall that the other solution was g of x minus tau which could also be written as h of tau minus x. And now if you replace tau to be c times t you are going to get h of c t minus x as the other solution and this was identified to be the forward travelling wave solution. And this forward travelling wave solution h of c t minus x after invoking the harmonic assumption could be written in this other form.

So, if I now choose the minus sign associated with this solution, I get to see I have a time dependents coefficient which is c times the coefficient of time of space dependence which is omega by c. So, the plus sign is associate of this general solution is associated with the backward travelling wave, and the minus sign associated with this solution is associated with correspondingly the forward traveling wave.

We have understood in great details forward traveling waves and backward travelling waves even without invoking in the harmonic assumption, but now specializing for the harmonic assumption the physical picture remains unchanged it is just the form of the solution is no longer triangular, but rather it is harmonic or it is sinusoidal. So, I will show you a few form to illustrate the same, but before I do that you will note that when we had a complex exponential, we said that if omega is the speed of the phasor, then the phasor comes back after one cycle at after a time of capital T is equals to 2 pi by omega.

Now, we have the phasor to be dependent on both time and space that is the only complication we have. So, associated with the time dependence you have a time period of 2 pi by omega, associated with the space dependence of phasor you will have you will have a similar quantity which is called lambda or the wavelength, which exactly by the same analogy will be defined as 2 pi by k.

So, the time period is the time after which the quantities repeating, wavelength is the space after which the quantity of interest is repeating. So, the time period is 2 pi by omega because omega is the coefficient associated with t the coefficient associated with x is k. So, associated with that the periodicity in space this time will be 2 pi by k which is given the name lambda and technically qualified as wavelength. So, wavelength and wave number shared the same relation as time period and angular frequency.

So, in other words you now have a situation when we are dealing with harmonic waves where you have a phasor both in time as well as in space. So, that is a very crucial inside which we will elaborate upon, but before doing that let me just quickly plot, out a few a graphs to enable you to have a visualization of these harmonic waves. So, if I take for example, e to the power i omega t minus k x right and plot it for t equals to 0.

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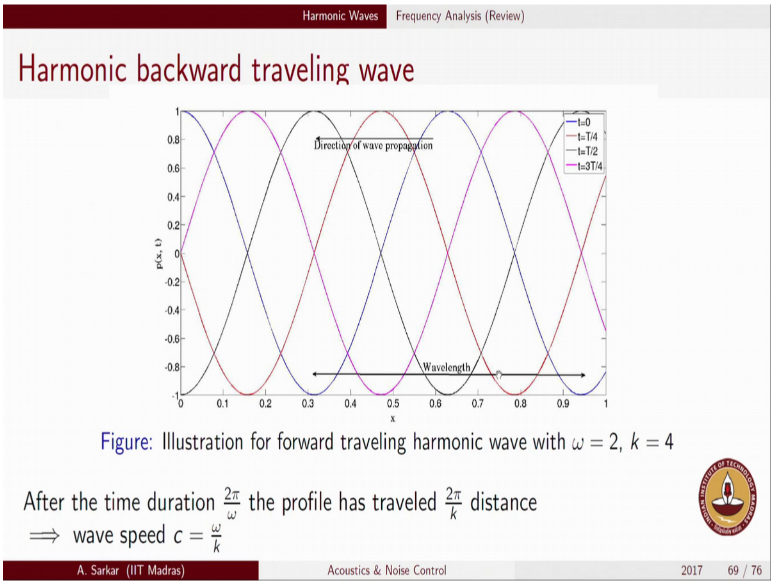

So, I am only plotting the real part. So, at t equals to 0 this fellow does not leave, you only have minus k x and then you take the real part of it. So, it basically means it is a cos k x type of a plot which is what this shows right. The blue curve that you see here is the response or the acoustic pressure profile at time t equals to 0 associated with this function right.

Now, if I increase the time by quarter of the time period right. So, it is the time has moved a head a little bit by quarter of the time period capital T in my notation denotes the time period. Time period is 2 pi by omega and quarter after a quarter period of time if I make the same plot, then I will see that I will I am going to get this red plot in other words this point has moved here the plot is always with respect to x. So, at consecutive time instance I have plotted in red in blue red black and magenta right.

So, what we see is that each of these points which are at what we say as constant phase points. So, the maxima point for example, is moving in this fashion. The minima point similarly from the blue plot if you can identify this is the point of minima. So, this minima point is moving in an identical fashion right. Therefore, visualize this if we now plot all these generate all these plots in a loop and have a visualization of this corresponding to an animation we will see that it is as if this profile is moving in the forward direction right.

But please note that at each instant of time there is a periodicity associated with the response. Whether you talk at t equals to 0 or at t equals to t by 4 or at t equals to t by 2 or finally, at t equals to 3 t by 4 every time you do get a repetitive plot right and this repetition in space is going to be called the wavelength right. So, I have just taken some arbitrary numbers for generating these plots which are given here. So, the point is that after a time duration of 2 pi by omega capital T that is which is not shown in this plot I have plotted only till 3 t by 4 I have left the last quarter I have not plotted it simply because I felt that the graph was getting 2 clotted.

So, the fact is that after a certain time duration what will happen with the time duration corresponding to the time period which is 2 pi by omega, if I generate this plot this blue form will exactly heat this form right. So, this point would come back here which means it will be exactly superposing. So, the profile would have traveled exactly a wavelength distance. Remember wavelength is associated with the snapshot at a fixed time. So, at time t equals to 0, if I see what is the wavelength this is from this point to that point right. What happens after a fixed after the time period capital T equals to 2 pi by omega this, point would have exactly come and it would have occupied the point here, which means the story is going to repeat somewhat like a mega serial it will keep continuing on the same track it will keep repeating the same story and it will be almost like a never ending situation.

So, after the time duration 2 pi by omega the profile would travel a distance which is equal to one wavelength. And therefore, in this fashion also we can c that the wave speed c we already realize that c was the wave speed even without harmonic assumption, but even with the harmonic assumption that idea is reinforced, that c wave speed is going to be omega by k. This is very important relation you should never forget that the angular frequency and the wave number that ratio of it is the wave speed right.

So, similarly you can talk about the backward wave this time. So, backward wave is obtained by plotting this form of the solution. So, again we generate the plots for different time instance starting with time instant 0 t by 4 t by 2 3 t by 4. What has not been done is what happens exactly at t at exactly capital T, time which is performing 2 pi by omega this blue point would have moved exactly here right. So, it is going to be exactly the same idea, but in progressive time instance this time if you plot the blue comes to red position the red comes to the black position, the black comes to the magenta position, and in the next cycle the magenta would again come to the blue position.

So, the entire story would be repeated after time of capital T, which therefore, it would actually overlay with this cap with this blue plot. So, there is no point in plotting that blue pot plot blue curved anyway. So, after the time duration again we will see that this profile would have travelled a distance 2 pi by k, which meant that the wave speed is again omega by k, but please note that this time the direction of wave propagation has reversed. The idea of periodicity of this profile remains the same it is the same periodicity I have generated the plot keeping those numerical parameters to be same the same periodicity is obtained which is what is called the wavelength. So, these are remember snapshots taken at a particular instant of time, at each instant of time there is a special periodicity which is the wavelength the value of which is 2 pi by k and it turns out it can be marked off in your figure to be this.

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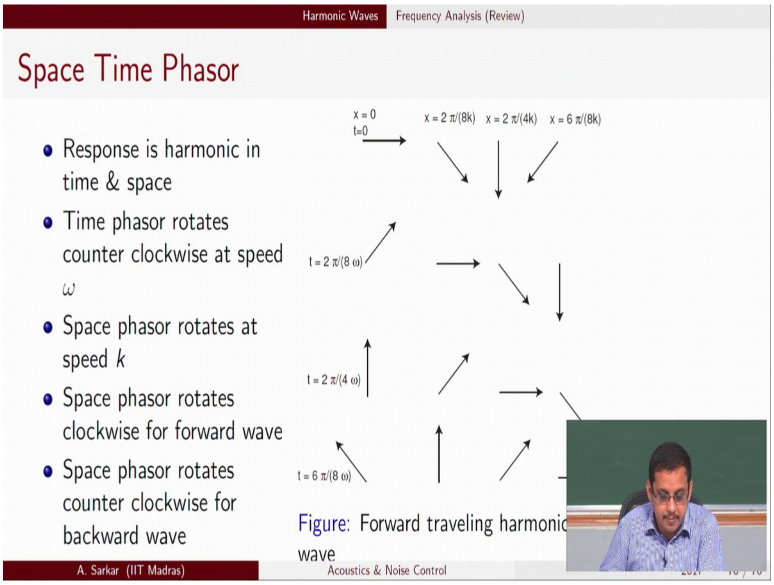

So, that is associated with the wavelength concept. So, carry on forward now what we introduce is this idea of a space and time phasor we already know what is a time phasor so if you look at any point. So, let us look at the first column of this figure right. So, this this plot is generated for x equals to 0 this is the phasor plot as opposed to a graphical plot. So, we already know that any point in space, will have only harmonic dependence with time. And that harmonic dependence is of the form of e to the power i omega t. So, therefore, it can be represented as a rotating vector the rotational speed of that vector being exactly omega being the angular speed.

So, if we plot out the form or in the phasor form if he depict this story at any point we will see that at consecutive instance of time it is going to occupy different positions and basically giving the feel that the vector is rotating in the anticlockwise direction with a speed of omega. This is true not only for x equals to 0, but any arbitrary x that you take right, but the only difference between the time direction and the space direction for a forward travelling wave is that these 2 phases as rotating in the opposite sense. And that can be understood from this equation also, because the sign associated with the special coefficient is flipped for a forward travelling wave you see the sign comes with a minus sign right.

So, omega by c for a forward travelling wave is associated with the minus sign whereas, the coefficient associated with time t always remains positive omega. So, therefore, the phasor associated with time rotates counterclockwise, but the phasor associated with space if it is a forward travelling wave will be minus k, and therefore, it will rotate at the speed k, but in the clockwise direction. If you have dealing with a backward travelling wave the situation is less complicated both the phasors are rotating in the counter clockwise sense.

The time phasor anyway always rotates in the counter clockwise sense, but this time the space phasor also associated with the phase phasor the coefficient is plus k and therefore, the space phasor rotates at k at a speed of magnitude k, but since counterclockwise. So, I have taken up the more complicated case here which is that of a forward travelling wave. So, in that case we see that the space phasor is rotating. So, in space what we take is that, now what we are dealing with is time remaining constant. So, the snapshot profile is as you would view in one row instead of having a column view point which is the viewpoint of what happens at different times at a fixed point if we adopt this view point, that it is a snapshot which means what happens to different points at the same instant of time.

So, at for the same same instant of time the different points will also be harmonically related which means that there will be a phasor, but the phasor this time will be rotating in the clockwise sense at a speed of k right. So, therefore, when it covers 2 pi by 8 k which means pi by 4 k it has gone 45 degree in the this time in the fourth quadrant right again in the next. So, in the next point in space, now I should not say time instant, but it is a next special point which is taken as x equals to 2 pi by 4 k in other words pi by 2 k the point would have the phasor would have come at minus ninety degree location.

Please understand if I view adapter view point in this direction, then I can feel that the phasor is rotating in the clockwise sense. Whereas, if I adapter view in this direction I would feel that the phasor is rotating in the counter clockwise direction right.

But please note that once, I have any arrow given I should be able to generate the plot consecu at consecutive instance of time at as well as consecutive special positions. For consecutive instance of time I have to rotate the phasor in a counter clockwise direction for consecutive instance of space I have to rotate the phasor in the clockwise direction at a speed k. Now let us see how therefore, to generate the location the phasor orientation for this point, all that I have to do for different time instance is that I have to rotate it counterclockwise. So, at the next instant of time it will be rotated counterclockwise again the time has been. So, chosen that it will rotate by 40 degree, but this time it is in the counter clockwise, since which is why this position will come to this position in the next instant of time it will come to this position and finally, here.

So, this story keeps happening in all of this arrows right. So, what finally, happens therefore, or the moral of the story is this, if you look at any diagonal position such as this right. So, you are going to see that the orientation of the phasor is unchanged right. For example, this orientation or this orientation, so all of this space time points. So, whatever happens at x equals to 2 pi by 8 k t equals to 0, same is the orientation of this phasor at x equals to 2 pi by 4 k time is equals to 2 pi by 8 omega, so the orientations being identical. Same thing happen here this x and this t will have the same orientation so; that means, the point is going to travel each of these distances at these time instance right. So, the nature of the solution is going to just get communicated progressively at different spatial points as the time evolves, right.

So, that is that is the important point that is what we understood even without the assumption when we generated different plots or through the method of characteristics, we understood, that whatever happens at a particular x and t gets conveyed along the characteristic lines. So, exactly the same picture is again reinforced by this phasor plot wherein you see that all these based time points have identical phasor representation which means the phase associated with this wave remains identical at different space time x comma t. So, this is what is called as space time phasor. So earlier we where only drawing phasor with time now we are drawing the phasor with respect to both space and time and this is the very interesting inside and you do well to learn this aspect.

So, with the time phasor rotates at counterclockwise speed at bias space phasor rotates at speed k, but in in the clockwise sense I should have written that space phasor rotates at speed k, but in clockwise yeah that is an next bullet space phase rotates with clockwise sense, but the forward wave, but or the backward wave it will rotate in the same counterclockwise direction, and I have just explained to you the reasons why we are differentiating between these 2 different kinds of wave that is the forward wave and the backward wave.

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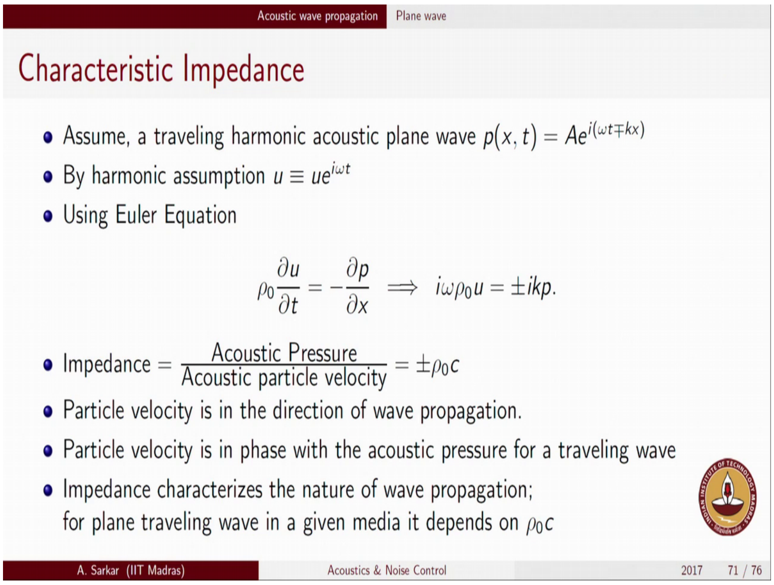

Carrying on, now we will now introduced to you another important concept of impedance and in particular this is called characteristic impedance associated with the traveling wave. So, by now we are convinced that harmonic travelling wave can be represented in this for, it has an undetermined constant and together with a complex exponential the argument of which is omega t plus or minus k x.

The minus sign is associated with the forward wave the plus sign is associated with the backward wave. So, I doing this problem together for both forward and backward wave right. Now as we set our harmonic assumption at that time we said that all quantities of interest have got to depend harmonicon right. So, not only the pressure the velocity is also will depend harmonically. So, therefore, the velocity has also now been reduced to this harmonic forms. So, the temporal dependence is no longer explicitly made the temporal dependence is implied to be complex exponential. In fact, what we will do probably from the next classes that we are not going to write this explicit temporal dependence. Here after all the time dependence is assumed to be e to the power I omega 2 right.

So, temporal difference will be implied to b complex exponential. So, that part does not change the spatial dependence is what we are going to solve out. And we are going to write that explicitly anyway at this point we are witting it in full details. So, the velocity acoustic particle velocity u can be written in this form as u times e to the power i omega

t. Please remember this is the particle velocity this is not the velocity of the way. Remember each particle simply oscillates about it is own position at a certain velocity, but the wave keeps moving forward in with the velocity c.

So, if you recall the Euler equations the Euler equations connects the acceleration of the fluid particle with the pressure gradient for a one dimensional situation, the pressure gradient is just del p del x and the acceleration is rho 0 del u del t we have already seen that the convective acceleration term, does not bother us because we are dealing with extremely small oscillations or extremely small fluid flow velocities, and therefore, these thinks will just be the convective acceleration terms will just get reduced to 0.

So, all that we are left with is just the time derivative of u, and now since we have assumed the form of u to be the form of u in time to have a dependence of e to the power i omega t; that means, when you take the time derivative of this function u all that you are going to get is an i omega popping out of the u itself right. So, again that is something you would have seen even in vibrations class that I omega times u is d u d t. So, that is what I have written here, and similarly del p del x, now can be worked out and when you work out del p del x for the forward wave it in it will be minus, i k x right and when you work out del p del x for the backward which comes with the positive sign it will be plus i k x right, but together with the minus sign that is sitting here the signs will now be reversed. So, for the forward wave it will be plus, i k p for the backward wave it will be minus i k p right.

So, we have go this relation relating pressure and velocity for both forward travelling plane wave and backward traveling wave. So, we defined a quantity which is called impedance and the impedance as per definition is acoustic pressure divided by acoustic particle velocity right. So, what we have is for a forward travelling wave which comes with a positive sign in this equation you will have this quantity to be rho 0 c, and for a backward travelling wave we will have it to be minus rho 0 c right. And the fact that there is a flip in sign should not sort of bother you that is because for a forward travelling wave the direction of wave propagation is opposite as compared to the backward travelling wave and as a result the direction of the particle velocity will also be opposite.

So, the particles will not travel in an identical fashion between the forward wave and the backward wave. The particles will have exactly the opposite velocity. So, the pressure to velocity relation will be the in magnitude it will remain the same, but in sign it will change right. So, if you have a forward travelling wave the pressure and velocity will be both bearing the same sign because the wave is going in the forward direction. So, velocities or will also be in client in the forward direction when the pressure is positive right when the pressure is negative, remember we are talking about acoustic pressure which means we are talking about pressures over and above the mean atmospheric pressure right.

So, wherever there is a region of buildup of pressure there we have a buildup of acoustic velocity also. So, the velocity is directed in the positive direction. So, in other words with the particles are trying to go in the positive direction there is a pressure buildup right similarly when the particles are for a backward travelling wave when the particles are trying to go in the negative direction there is a pressure buildup which is, but natural because we have these 2 waves travelling in the opposite direction.

So, accordingly impedance for travelling waves will be is found right now which is of the value rho 0 c, but the sign of the impedance term has to be carefully understood the sign for the positive travelling wave or the forward travelling wave will be plus whereas, the sign of impedance associated with the negative travelling wave will be minus. And in particular just to we will keep saying that the impedance will this term impedance will appear in very many different context right, but remember the impedance calculation that we are doing on this slide is associated with the impedance of a travelling wave.

So, to make sure that we are talking about the impedance of a travelling wave we call this impedance as characteristic impedance. So, that is why this rho 0 c is actually the characteristic impedance. So, as I argued that particle velocity will always be in the direction of wave propagation. I mean the relation between the particle velocity and the direction of wave propagation is what clauses this change in sign.

Please note that let say for a forward travelling wave particle velocity is always in phase with the acoustic pressure right. Wherever there is an acoustic pressure buildup there is a velocity buildup also. Similarly wherever there is an acoustic pressure buildup there is a negative velocity that is coming out through this mathematical formula, but you will identify that this negative velocity is actually in the direction of the wave propagation right because you are dealing with a backward travelling wave for backward travelling wave it comes with the minus sign. So, when you see that there is a pressure buildup this formula tells you that the velocity is going in the negative direction.

But the negative velocity is actually the direction of the travel of the wave. So, that is the direction of travel. So, in other words if you interpret this formula physically what it means that always the particle velocity is in phase with the acoustic pressure for a travelling wave right. And this is a very important result this will not happen for other kinds of waves once you start getting superposition of waves you are not going to get this situation. So, that is why we say that this quantity impedance is actually what characterizes the nature of the wave right it depends only on 2 quantities which is the medium rho 0 and c.

But please note that it is not c alone which when we when we talked about waves in general without building on to this harmonic assumption we said that the wave speed is dependent on c which is a ratio of gamma p 0 by rho 0 right, but now what we are saying is that we are talking about ratio of pressure and velocity and that ratio of pressure and velocity depends on this quantity rho 0 c which, In fact, is square root of gamma p 0 times rho 0.

So, as long as this quantity is same we are going to see that wave propagation effects such as reflection rare fraction snails law all of this depends on this quantity rho 0 c rather than anything else. So, therefore, this quantity rho 0 c which is basically a property of the medium is characteristics of a travelling wave. We will see that once you have superposition of waves we have standing waves for example, then you will not have the impedance given to be in such an simple formula the impedance will depend upon special locations for example, the impedance will depend upon frequencies, but it is only for travelling plane waves do you have such a simple formula for acoustic pressure divided by acoustic particle velocity.

Not only that we will just preempt a few results that will come much later even for other more complicated wave such a spherical waves or cylindrical waves or any other fancy kind of 3 dimensional wave, under suitable approximation which is to say that if you are really far away from the source you can say that this impedance term you can see to it and we will derived that result later on that. For example, in a spherical wave if you are far away from the source you will get to say that this impedance of this spherical wave we will tend towards this number rho 0 c and that is why we will say that the spherical wave will approximate the plane wave condition. And that is why we study plane wave in great details because at least in some approximate sense you can be very sure that this plane wave theory applies to lot of other more complicated situation.

But I will end today's lecture with this with emphasizing this fact that, rho 0 c is the characteristic impedance of a plane wave you should associate more importance with the value of rho 0 c. And the sin of rho 0 c you should be able to interpret depending upon forward or backward. Please remember mathematically it may seem that they are very different because of the plus or minus sign, but if you think in physical (Refer Time: 48:25) it is actually making sense that the velocity seen the particle velocity will actually change it is sign. Because the direction of the wave has change, it is sign the final point is the pressure and velocity in the direction of propagation of the wave if you take that to be the definition then you will see always it is rho 0 c right.

But usually it is bests mathematically it is best understood as acoustic pressure divided by acoustic particle velocity that is the formula that we will pick up from here the concept of impedance actually get pretty complicated once you have a superposition of different waves. So, we will leave it here for the present.