Acoustics & Noise Control
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Module - 05
Lecture - 10
Frequency Analysis 2

In the last class, we talked about frequency analysis and steady state analysis. Here we will demonstrate through some MATLAB examples, what I really mean by a steady state sound and what is it that is meant by a single frequency sound.

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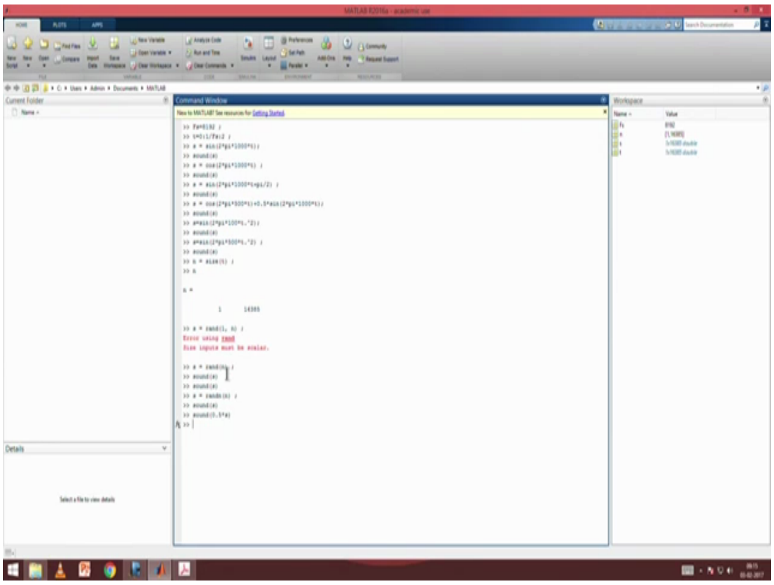

So, MATLAB offers as very many interesting commands. So, other than just plotting the signal as it is you can also here out the signal right. So, this is a very interesting tool that one can use for acoustic analysis. So, towards that n we will explore this facet of MATLAB which is to hear out given signal. The default sampling frequency that is assume is 8 1 9 2 hertz. So, this the default sampling frequency that is there. We need to create a time vector. And the way the time vector has to be created is that you have to specify the starting time which could be 0, you have to specify the ending time which could be anything of your choice. And the incremental times have to be 1 by the sampling frequency that is how signal processing works.

So, the time vector I call it t is 0, in increments of 1 by f s. And let us try playing out a 2 second sound or noise as the case may be. So, this is our time vector. Let us start with sinusoidal signal. So, s is let us says sin of 2 into pi into 1000 into t right. So, this would generate signal which is of 1000 hertz right as per our understanding, the sinusoidal signal can be represented as sin of 2 pi f t. So, I have used a value of f as 1000 here right. So, with this we have generated the s vector and now if we play out this sound this is how it plays out. So, it is a pure tone as you can understand right now if we redo this sound as instead of sin we take cosine, and again play out this sound it is actually the same right. So, you observe that the phase shift does not matter. Let us give a phase shift also now if you want.

So, let say s is equals to sin of 2 into pi into 1000 into t plus pi by 2 is the phase shift. And then again we play out the sound, the characteristics of the sound does not change right. So, this is a very important attribute and that is what we were discussing in the last lecture, when we said the characteristics of the sound does not change throughout it is duration, right. Let us take a little more complicated example probably. So, this time we will play out cos of 2 into pi or maybe yeah that is cos of 2 into pi into 500 into t plus sin of maybe will have a weightage of point 5 sin of 2 into pi into 1000 t.

So, now this is having 2 frequency components one 500 hertz and the other 1000 hertz. And just to sort of give you a feel I have down plate the 1000 hertz component. 1000 hertz component is point 5 times the 500 hertz component. So, let us see how a let us rather here how this plays out. So, this is distinctly different from the sound that was audible in the first case right. In fact, 1000 hertz the way it appears is it is perceptively most sensitive to our yards right. So, 1000 hertz the reason why I chose 1000 hertz was to give you a feel that that is how the human perception is most that is why that is a frequency where in the human perception is most sensitive. Now let us play out something different which is let say s equals to what would be this sound sorry, I first it to make a sin out of it.

So, I have now kept a sinusoidal signal which is sin of 2 pi 100 into t square. So, basically I am having a frequency which is changing with time right. So, let see how this one plays out. So, this is a non stationary sound maybe I can do a little better also. So, you distinctly feel the sound characteristics is changing the this is what is called a charpi sound, that the frequency of the sound is changing with time. A very obvious example of this is if you stand in the platform and you find and train that is approaching the frequency of the sound perceptibly changes right. That happens because of Doppler effect and this is basically a demonstration of Doppler effect where overtime the frequency of the sound is changing.

A similar example as I gave in the last lecture is that of revving up of an engine. The engine as it revs up even in stationary condition you will hear something like this. Another example I would like to give you let see whether it works which is that of random signals. So, will, firstly, I have to determine in which is the size of the t vector that happens to be 8 1 1 6 3 8 5. So, we will create this time a random vector of size n. And then we could play this out. This looks more this sounds more like this sounds more like a machine noise something is running right. So, this is where all the components are present. In fact, this is a uniform random noise this is not rand n we could play out a different noise, but there are issues associated with this command. Maybe you can explore what would be the. So, this is a like a machine noise that is playing typically you would expect all the components will be that. This is also very typical of flow induced noise, what I can possibly do is I can play this with an lesser amplitude.

So, you get to see here all the components certain aspects of this sound reproduction is not correctly done because you are running it through the computer hardware and depending upon how accurate this computer hardware is. There may be certain differences, but basically it gives you a feel that if you have a single frequency component you hear a single frequency or a single pitch sound where as if you have a random noise of this kind it is basically broadband right. It has all the frequencies split into it and therefore, you hear a completely different characteristics of sound.

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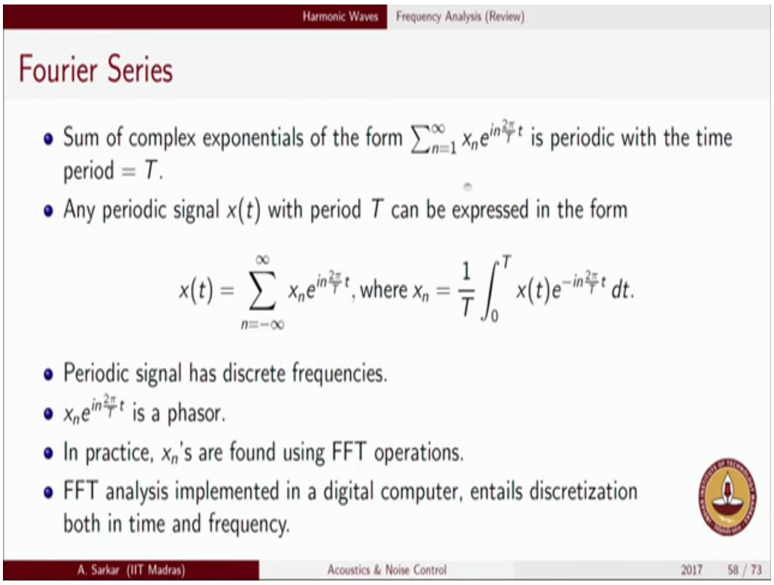

So, with that demonstration let us get back to the lecture material. So, we will just quickly review the Fourier series issues which hopefully you have learnt in your earlier courses also, but therefore, it is a quick review. So, what we are trying to do is that we are trying to break up any periodic signal in terms of summation of this form. We already I have identified that each of these complex exponential forms are associated with a phasor, what we know for sure is each phasor is herb is harmonic. And each harmonic component is a periodic component also, but what Fourier series gave gives us is that for every periodic signal, with periodicity capital T, we could break up that periodic signal into some of harmonic components which is this. And x n are called the associated amplitudes and x n can be determined through this formula right.

The derivation and the other issues associated with this formula you can look up any book in engineering mathematics that will have this derivation, but you have to be careful in sort of using the engineering mathematics terminologies here because usually at times in certain books you will find the Fourier series written in terms of sin and cos. We will not use the sin cos invention, we will rather use the complex exponential convention and this is the form of Fourier series that we will be using it. So, what this tells us is that any periodic signal can be decomposed into some of different phasors which are harmonics of each other. So, each of these components are having the associated rotational speed as n times 2 pi by t. So, the fundamental is 2 pi by t does the fundamental speed.

The next value for n equals to 2 you will get the phasor to be rotate e at twice the rotating speed of the fundamental, phasor for n equals to 3 it will rotate at thrice the rotating speed. So, all the frequencies though different frequencies are present there are related their interrelated in the form of integer multiple. There is a fundamental frequency fundamental angular frequency which is 2 pi by capital T, capital T being the periodicity of the signal that you are looking at, and the all the other components are having an angular frequency which is integer multiple of this fundamental frequency.

This is the very important attribute of periodic signal which is not there in at periodic signal. So, periodic signal therefore, have discrete frequency there is nothing like band of frequencies in periodic signal. It can only have discrete frequency, each of this components are what we discussed as phasor. So, each components of this Fourier series summation is a phasor. In practice we do not really employ this integration formula there are better signal processing algorithms to capture these Fourier coefficients both they are efficient as well as accurate and they are known as fast Fourier transform algorithms or FFT algorithms. And the best part of these FFT analysis is that it can be implemented on a real time as your acquiring the signal it can be implemented on a digital computer it works on the discretize data, and you get the frequency related information associated with your signal.

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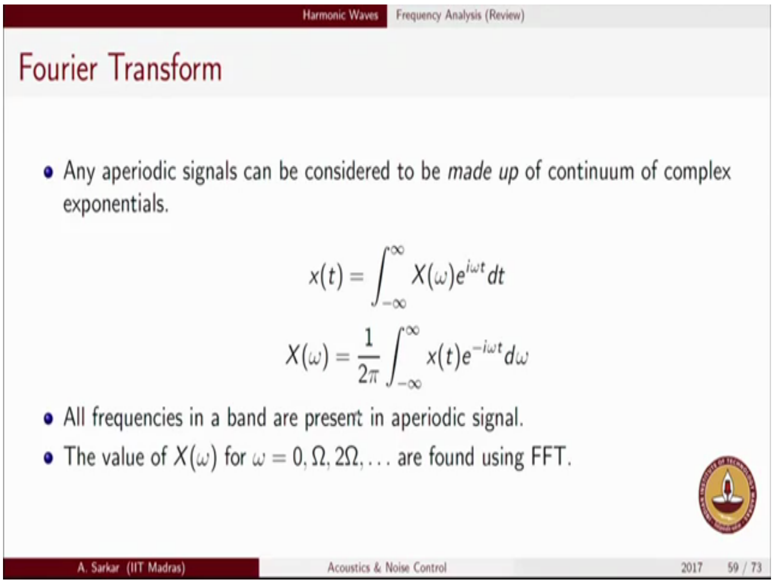

So, these aspects are more like a signal processing expect they are true not only for acoustic signals, but any kind of dynamic signals you will have the same issues getting repeated.

So, we will not do well into those aspects any further, but none the same we will talk about Fourier transform just to give you the opposite picture. So, Fourier series as we understood is applicable for periodic signal, but if you have an aperiodic signal of signal which does not repeat ever right. One example of it could be Gaussian signal, signal which is of the form of e to the power minus t square it never repeats right. So, therefore, in such cases you could not contemplate that there is a single periodicity right. The periodicity basically goes towards infinity and as such there are continuum of frequencies that are present please contrast this situation with the that of a periodic signal in periodic signal you have discrete frequencies. This summation is infinite, but the fact there it is a summation and not an integral suggest that there are only contributions associated with these values of frequencies.

For the other values of frequencies, the contribution is just not there; in signal processing books another helpful way to sort of depict these 2 situations is this.

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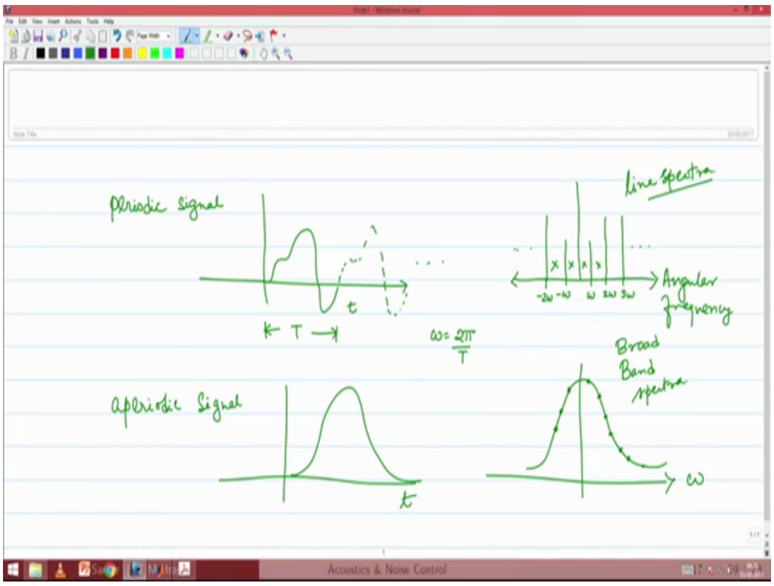

So, if you have periodic signal it will repeat. So, the time signal will repeat afters certain duration. So, let say this is the fundamental form and it keeps on repeating right. So, they associated frequency the fundamental frequency associated with this is going to be associated with this capital T time period. So, when you say omega equals to 2 pi by capital T that is the fundamental, and then you have frequencies. So, this is the frequency axis now both in the positive and in the negative direction. So, you will have components associated with omega 2 omega 3 omega and so on right. And on the other side also you will have components associated with minus omega minus 2 omega and so on.

So, this this minus omega simply means that it rotates in the other direction it does not rotate in counter clockwise direction, but it rotates in the clockwise direction right. So, you will have contributions associated with each of these frequency. So, we call this a line spectra right there are no contributions associated with any value of omega this is better called as omega axis or the angular frequency axis. So, the picture in time relates to this picture in frequency domain, wherein you will have only contributions associated with discrete values of omega. No contributions are expected from any value of omega which lies in between omega and 2 omega 2 omega and 3 omega and so on right.

Whereas if you have an aperiodic signal, in the case of an aperiodic signal such as a Gaussian function you will have a frequency content. So, this is time axis and this is angular frequency axis you will have a frequency content, which is actually spread all through. So, here I am not going to say that the frequencies are limited at omega 2 omega 3 omega all the omegas are present right. So, this is called a line spectra to make things very clear right whereas, this is having broadband spectra right. So, aperiodic signals will have a broad band spectrum and there are ways to determine these do this spectrum estimation, but we will not get into all that right now, but suffices to say that FFT can be employed to determine both the value of so what can be done using FFT is that you even for an aperiodic signal. If you are interested to find what is the value at certain discrete points on this spectrum you can do that using FFT there is way in which you can employ FFT to determine both the approximation to Fourier series as well as Fourier transform right.

But typically we will be interested in the line spectrum more than the broadband spectra. Random noise is again another example where you do not expect a line spectra because if there is a line spectra; that means, it will repeat after a certain time interval capital t, but as per the very definition if it is random it cannot repeat right. So, therefore, for all noise which are generated from some random processes let say turbulence induce noise for example, you will not expect a line spectra you will expand expect a broadband which is of the form that we have shown it to you. So, using various signal processing algorithms both line spectra and broadband spectra can be estimated those estimation techniques are beyond the scope of the current course.

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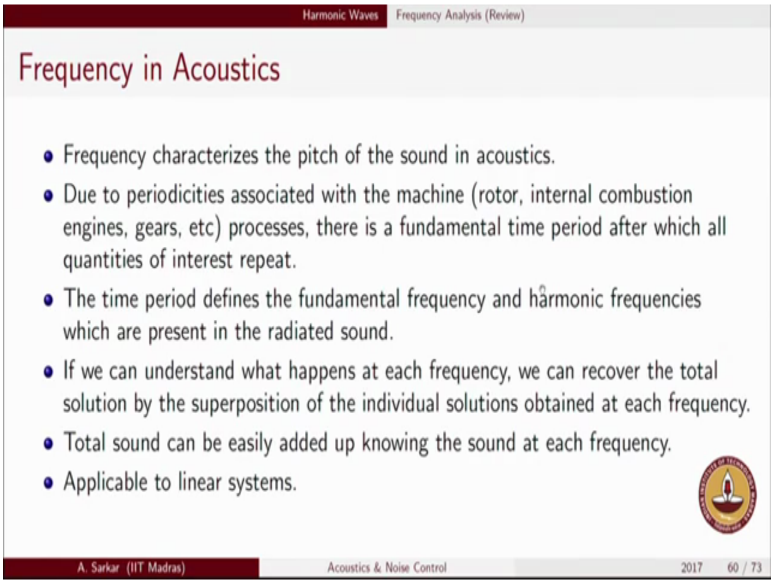

So, let us once more recapitulate therefore, what is the role of frequency in acoustics as by now you would have been convinced that frequency is characterizing what is (Refer Time: 18:58) called as the pitch of the sound in acoustics right. And if you observed when we did the demonstration for 500 hertz and 1000 hertz the higher pitch refers to higher frequency. Or typically the other way of remembering is that women have a higher pitched voice right. Their frequency content is higher male voice which is deeper and resome more resonant probably will have a lower frequency content will have a pitch which is low. The high pitch sound the shrill sound is high frequency sound right. Now as I said that machine noise mostly leads to a certain inherent frequency which is their due to the machine operation.

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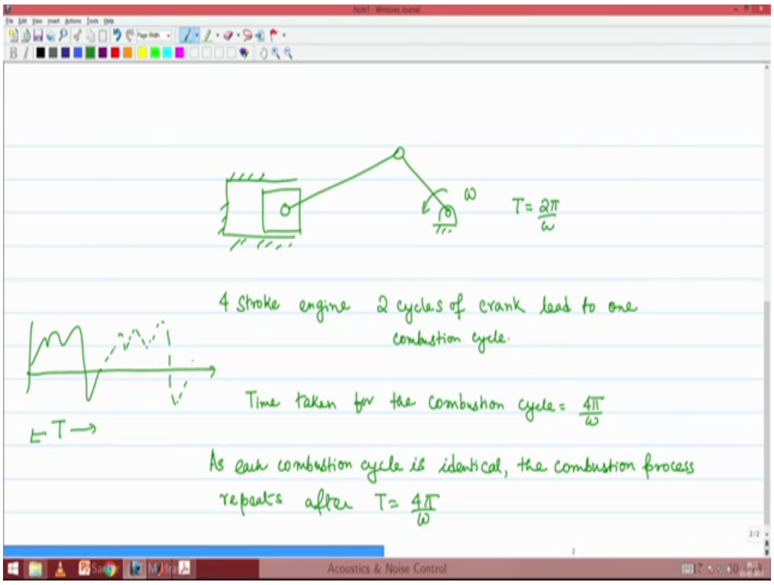

So, let me just elaborate what I meant by this statement. So, will suppose we are talking about an IC engine mechanism. So, this is a good hold slider crank mechanism for the IC engine and let say the crank rotation one way or the other is omega. So, therefore, the time period is 2 pi by omega after which the crank comes back to it is original position. So, the time required for the crank to come back after one cycle is 2 pi by omega right; however, the engine cycle if it is a 4 stroke engine which is what which is predominantly used, forced for 4 stroke engine 2 cycles of crank lead to one combustion cycle right. That is well known that is the combustion cycle will repeat after 2 revolutions of the crank right. So, what is the time taken for completing 2 revolutions of the crank?

So, the time taken for the combustion cycle will be 4 pi by omega right, in second see if omega is an ra radians per second then this time is in 4 pi by omega is in seconds. So, the point here is if you are not changing let say any parameters associated with the operation of the engine, then every combustion cycle is supposed to be identical, unless some pathological failure suddenly happens in the engine which is let say ruled out in the present discussion. So, you would expect all the combustion cycles to be identical right. So, inherent with the operation of the engine there is a fundamental periodicity that your getting. So, whatever is the noise whatever is the vibration associated with one cycle of the combustion process, the same noise and same vibration would be perceived even in the next cycle. Because the combustion cycle in terms of it is thermodynamics in terms of it is dynamic forcing nature all of it is just identical.

So, therefore, as each combustion cycle is identical, the combustion process repeats after t equals to 4 pi by omega. So, anything which is cost due to the engine combustion, whether it is vibration whether it is sound whether it is anything if you would like to whether it is exhaust anything which is fundamentally dependent on the combustion process of the engine will repeat after this value of time. So, any vibration data that you take at any point will repeat after this capital T time. The next instant will be just identical I should have made it identical. So, it will have exactly the same features. So, I need not worry about the second cycle, whatever happens in the first cycle will keep happening in the first cycle.

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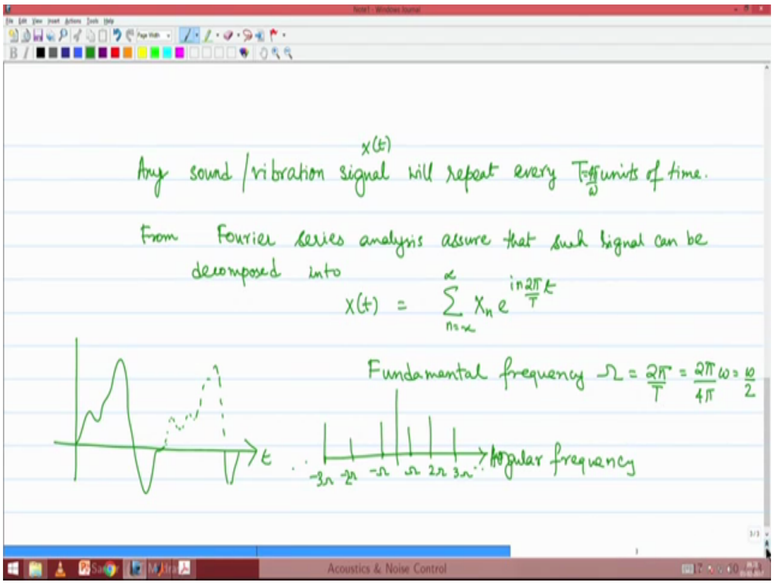

So, this leads me to believe that any sound or vibration signal will repeat every t units of time where t is 4 pi by omega. So, therefore, what are the frequencies associated with the sound and vibration that is generated by a 4 stroke engine. The frequencies associated will have a fundamental frequency of 4 pi by sorry with the fundamental frequency will be 2 pi by capital T, and then there would be other frequency is also because of the integer multiple nature. As we have come to know from Fourier series analysis and why is Fourier series analysis applicable remember Fourier series is applicable only for the case of periodic signal. We have already argued that the noise and vibration signal has got to be periodic because it is being generated by a combustion process which is having a periodicity of t equals to 4 pi by omega.

So, everything has to repeat itself at that capital T. So, therefore, Fourier series analysis assures that such signal can be decomposed into. So, if you have this signal to be x of t, then this x of t could be written as x n e to the power I n 2 pi by capital T into t right. So, the fundamental period is going to be omega equals to I should I call this capital omega. So, therefore, the fundamental period or the fundamental frequency capital omega is equals to 2 pi by capital T.

You will also have harmonics of this because Fourier series tells you that this is got to be a summation from n equals to minus infinity to plus infinity right. There is an infinite series summation which is possible, but in reality this infinite series is actually could be truncated to a finite number of terms. So, the point is this that whatever x of t that you have recorded in time domain which hopefully repeats, whatever x of t that you have repeat you have generated in time domain in frequency domain angular frequency I will call it, it would lead 2 components which are at capital omega minus omega 2 omega minus 2 omega 3 omega minus 3 omega and so on right.

So, there will be components associated with each of these frequencies and therefore, you get a line spectra right. So, line spectra will be obtained for any noise and vibration which is fundamentally originating because of the combustion process of the engine, but you must add a pinch of salt also to this analysis, associated with the combustion process there are some random effects also let say, there are some turbulences which occur in the flow processes that are happening. So, if it is because of some turbulence related issues which is truly a random process.

Then those frequencies will spill over to the other frequencies not necessarily confined to omega or it is integer multiples, but at present we are saying that there are if, if it is a completely deterministic process and if it definitely repeats after time capital T there are no random components involved then you should expect only a line spectra. And that is a very important analysis when we come to engine noise and vibration analysis. This is essentially what is called as order analysis. Because you will see that this capital omega is basically 2 pi by t and if you put back the values of capital T which was 4 pi by omega this is basically omega by 2 right.

So, the frequency is associated with the noise and vibration value that you are getting from the engine process is going to be dictated by the crank speed of the engine. Omega is the small omega is the crank speed. So, if you have a crank speed of 1000 rpm then associatedly you will have the fundamental frequency at 500 rpm, but you will also have 1000 rpm, 1500 rpm, 200 rpm and so on. So, all integer multiples associated with the fundamental will be there, but the fundamental frequency will be half the frequency at which the crank is rotating. So, this was about engine vibration. So, engine vibrations and engine noise definitely leads to this sort of a frequency analysis.

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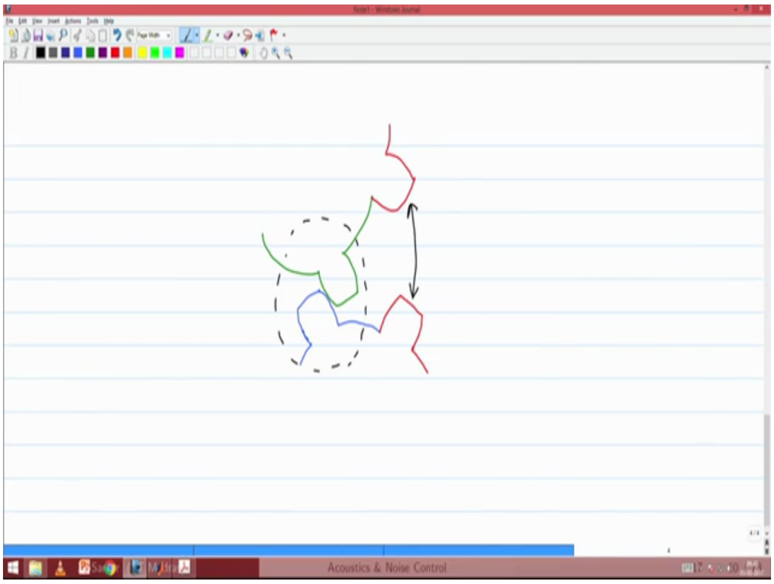

If you look at gears something similar is happening. So, I will just draw a pair of gear tooth which is in mesh. So, this is one of the gear and this is the other. So, maybe I will dry one more.

So, at this instant what is shown is one pair of gear tooth which is in engagement which is what is shown here right. So, associated with this pair of engagement there are certain force getting transmitted there are certain vibrations that is happening which is eventually convey to the gearbox and that leads to vibration of the gearbox which eventually leads to the acoustic emissions from the gearbox, but the point is whatever is this force transmission and vibration transmission that is occurring between this pair of gear tooth, after a certain instant of time you will expect these to red pairs of gear tooth to be in contact right. So, this entire process will repeat itself in this instant whatever is happening between the green tooth then the bluetooth is not sacrosanct between the green tooth and the bluetooth, that entire process will again repeat after a duration of time when the next pair of tooth which is indicated in rate will come in contact.

So, again you will see that there is inherent frequency associated with the machine operation. So, gears and engines being be to very obvious examples, where you can see that there is a repetitive process. So, any rotating or reciprocating part you should be able to identify what are the fundamental frequencies associated with that with the operation of that machine element. So, due to periodicity is associated with the machine which could be a router, which could be an internal combustion engine which could be a gear which could be a compressor any unit will have this fundamental frequency which is dictated by the basically this speed of operation of that machine.

So, the time period defines the fundamental frequency and harmonic frequencies which are present in the radiated sound right both for the sound as well as for the vibration same thing will happen. So, the point is now that we understand that our machine elements will emit sound at discrete frequencies, and we also know for sure that the equation of acoustics the governing equation of acoustics is a linear partial differential equation. So, we will adopt divide and rule policy. Instead of analyzing all the frequencies in one go we will analyze each frequency one at a time and then superpose the solution.

Superposition is valid because we are talking about a linear partial differential equation, because we are talking about a linear partial differential equation and because we have the power of Fourier series behind us and because we have all as seen that various different kinds of practical machines do lead to steady state noise which in effect means that there are a summation possible of these frequency components. Instead of trying to analyze the entire summation in one go, what we will try and do is we will try to analyze each frequency at a time if we are able to get the solution for one frequency then the analysis was down to the fact of just superposition of these results right.

So, that is the process that is adopted from here on where in we will steady harmonic acoustic waves right. So, this is the important statement that I want you to be convinced about. So, if we can understand what happens at each frequency, we can recover the total solution as the superposition of individual solutions obtained at each frequency. No need to understand the entire solution in one go right we understand that the solution will be made up in different frequency components, instead of an understanding the total solution we will understand parts of the solution we will try to find parts of the solution and hopefully if we succeed in that objective, we can recover the total solution simply by adding these individual parts which are associated with different frequency. So, therefore, total sound can be easily added up knowing the sound at each frequency. And all this is applicable because it is a linear system that we are dealing with why is a linear system because governing equations are linear partial differential equation.

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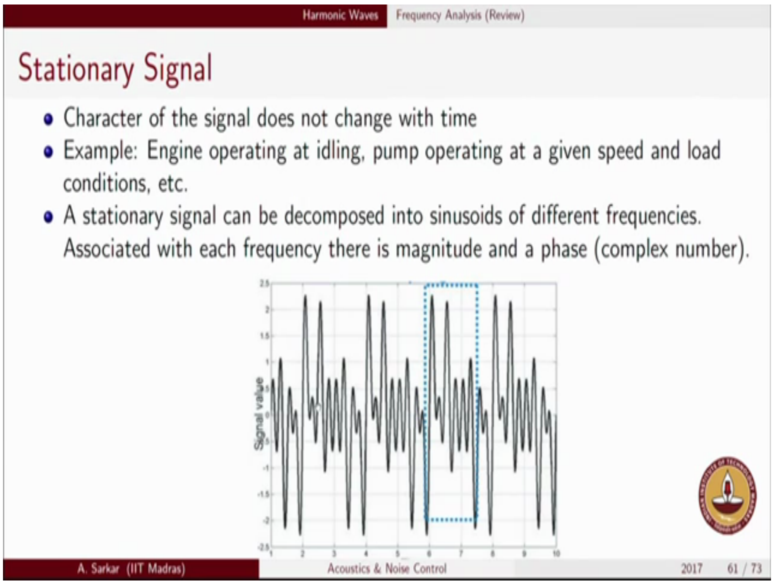

So, with that background will get into the harmonic acoustic wave in a moment. Just before we depart to that part, as I said stationary signal is defined as a signal where in the characteristics does not change with time. And examples are more practical examples of ways in which stationery signals can be generated are engine operating at any condition, at idling at full load and half load whatever you think is at.

Similarly, a pump operating at given speed unload conditions these are all stationary signals. Only if there is a certain change in the operating parameters of the machine you will encounter a non stationary type of sink. And because it is a stationary signal it can be decomposed into sinusoids of different frequencies. And associated with each frequency there will be a magnitude and phase by the way magnitude and phase is precisely this number x n. This x n happens to be a complex number in the way in which it is written here. Because x t is real and your multiplying a real number with a complex number and then integrating.

So, x n will be a complex number this is exactly the complex amplitude of the phasor that we have talked about. So, each fellow sitting here is a phasor and instead of analyzing the sum of all phasors, we will analyze each phasor at a time, and then we will superpose the solution it is a very efficient method of dividing and ruling right. So, that is the approach that we were going to follow. So, this is a simple illustration of a stationary signal. So, something which is not like a purely academic exercise. You can see here this pattern this blue box which is covered in this blue box is repeating right.

So, this signal; obviously, comes without any noise. In practice when you recover signals there will be cable noise which as I said is a random component. So, spilled over this actual signal there will be some randomness it will not look that clean, but none the same if you can approximate that a certain pattern is repeating you should be able to convince yourself that it is a stationary signal. And once you are dealing with stationary signal you are very sure that frequency analysis techniques are applicable and from there on you can shift from time to frequency domain analysis technique.

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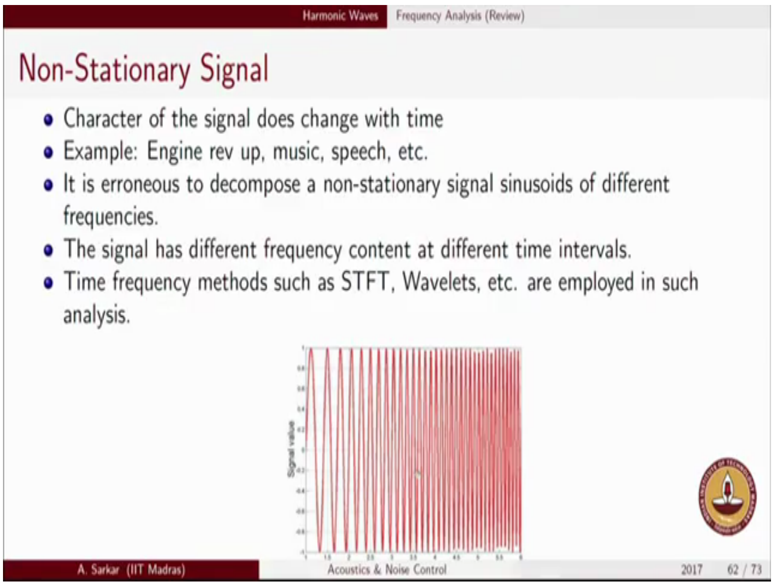

As a counter example and non stationary signal is shown here. This is exactly the chirp signal. So, if you look here carefully what happened in this early part of the time window you see the frequency looks like less here or the time period is more. Whereas, in the later part of the time period of this time window you see that the frequency is high and the time period is less. The time period associated with these repetitions have got condensed, so this is exactly what I demonstrated is the chirp signal in the case of MATLAB demonstration few minutes back I demonstrated this to you.

So, this is a non stationary signal. So, this is something that we will not get into in this course, but at the same time I wish that you appreciate what are the limitations of the current assumption. You will not be able to analyze this signal, using any FFT techniques or frequency domain techniques try doing Fourier transform of this and then inverting that Fourier transform you will not be able to recover this signal. So, this is a very important concept that not all signals can be Fourier transformed or should be Fourier transformed. I mean there is nothing wrong if you try to do a Fourier transform of this, but the problem is the inverse Fourier transform of the Fourier transform signal will not give you this back right.

So, you would have lost the information for ever. So, signature of this type of signal is in the fact that the character of the signal changes with time. And this happens when the engine is revving up in the case of my voice; obviously, I do not since I am sounding different syllabus and if my phonetics is changing across the different words that I am speech speaking. It is obviously, not the same whereas, if I whistle it is a stationary signal right. Music speech these are all non stationary signals and we are not going to get into these components in this course.

So, as I said it is erroneous to decompose a non stationary signals into sinusoids of different frequency do not use Fourier analysis techniques to analyze a non stationary signal because you cannot inverse Fourier transform that and recovered the signal back. So, that is why it is not a good idea to use without applying much of a thought if you simply use these FFT techniques you will end up into big problem. And there are specialized methods which are called short term Fourier transforms wavelet us which are used for handling these kinds of signals, but as I said we are not going to deal with this any further.

So, the point here is that the signal has difference frequency content at different intervals. For the chirpy signal that I have demonstrated here and also you have hard through the MATLAB demonstration that essentially shows that the frequency is changing linearly with time. So, 2 pi f t square is essentially it meant that can I took the signal sin of 2 pi t square it mean sin of 2 pi t into t right. So, f basically was 2 pi t right. So, therefore, f was changing linearly with time that is why it was non stationary.

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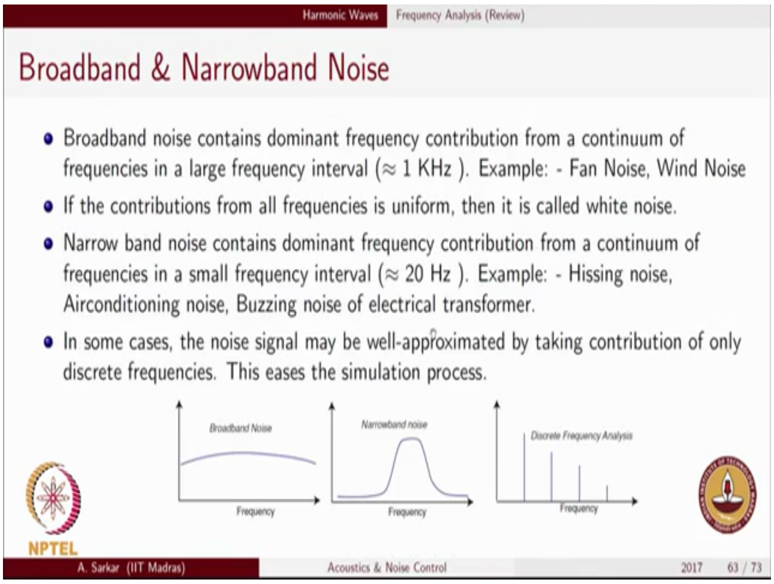

So, we all not going to deal with non stationary signal any more. In the next class, I think we can start with our more formal harmonic wave analysis.

Thank you.