Electron Diffraction and Imaging

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Lecture - 23
Coherence

Welcome you all to today's class on Electron Diffraction and Imaging. Today we will discuss about a Coherence. What is the importance of coherence? In this particular course, we know that diffraction is a phenomenon for which we have to invoke the concept of wave nature of the probe and that too for diffraction to take place, we assume that the beam is coherent and the scattering is elastic. So, what do we mean by coherence and this is what we should understand.

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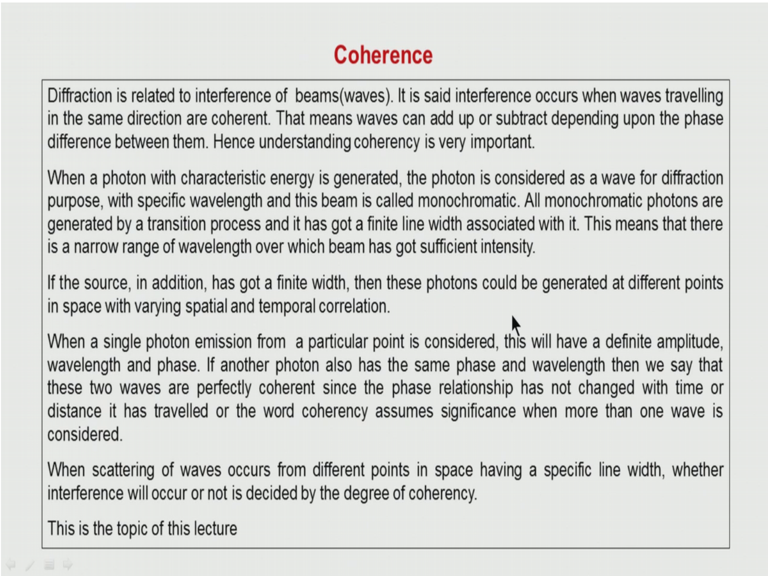

Once we understand this we can tell; what is the condition under which the diffraction will take place or we can say that; what is the condition under which interference will take place? What we mean by coherence is that if a wave has got the same frequency as well as the same phase relationship or there is a definite lag or lead of phase, then we say that the beam is coherent. In such cases, the amplitude of the waves can be added up to find out the net amplitude of the resultant wave. The details of it are given in this which we can go through and read and understand but I am just illustrating what is coherence with the help of an example.

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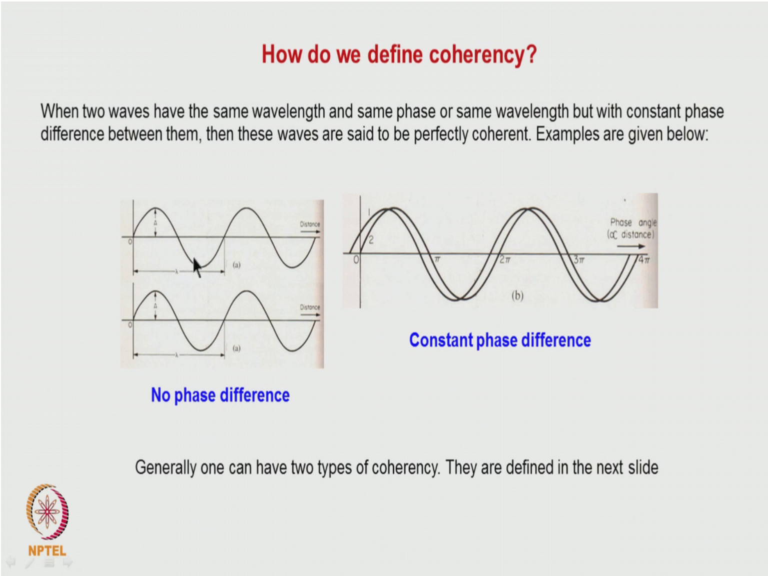

Here what we have, what we are considering is a wave which has a particular amplitude a and a particular wavelength which is associated with it. Another wave is also emanating from the source or that wave also has got the same amplitude and the phase, the same amplitude and the same wavelength and if you look at the phase at the initial point, both of them has got that same phase. So, everywhere if we look at it, here that has a maximum phase. If we look at it at every point of this wave, both of them will have the same phase relationship or we can say that there is no difference in phase between these two waves.

This is one condition and the other condition essentially is that this wave if we consider the amplitude remains same, the wavelength is exactly that same. Only thing is that with respect to two, one is lagging a little bit. So, between these two if we look at it, the phase difference is always has got a constant value. So, both the cases we can say that the beam is perfectly coherent in these two cases. If we wanted to find out what is going to be the resultant and amplitude of the resultant wave, then we can just add up the amplitude at every distance in x in these. Along the x axis whatever the distance we can add, what is the amplitude in both the cases, add together and that gives the resultant amplitude.

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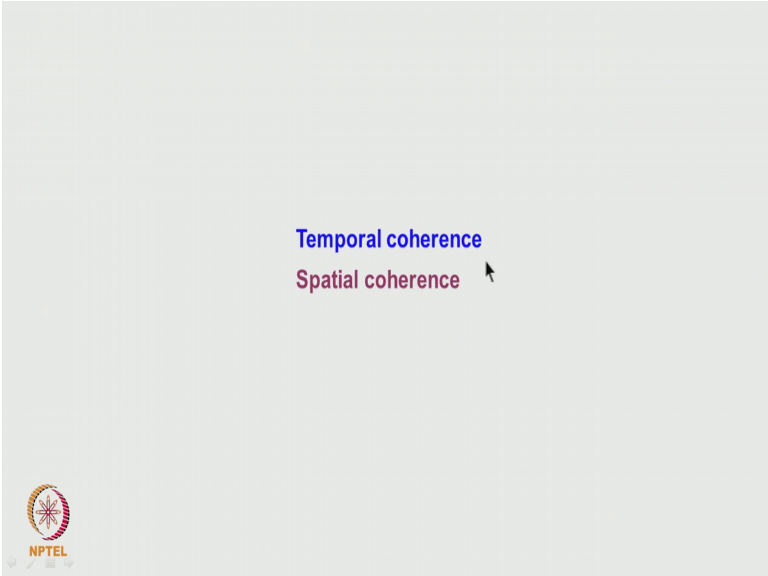

This is what will be shown in the next few slides and it will be illustrated how we get the resultant amplitude, but two coherences which we have to consider, one is called as a temporal coherence and another is called as the spatial coherence. We will discuss what is the difference between these two coherences, what is the importance or significance of these in electron microscopy.

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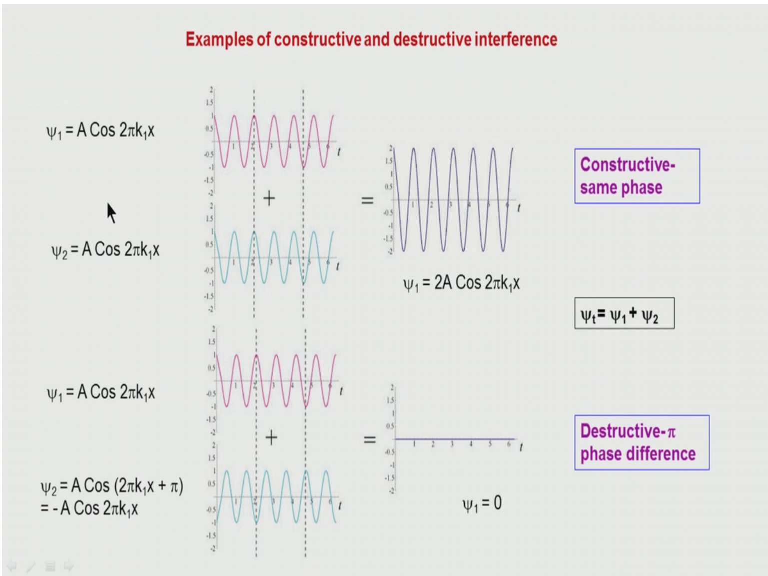

We are just taking an example of constructive and destructive interference here. We are considering a wave which has got a particular with respect to time. Now, we are trying to draw that and this wave is represented the wave function psi 1 equals a into cos 2 pi k 1 into x, and the other wave is represented also a psi 2 equals a into cos 2 pi k 1 x. So, that is the phase difference between these two waves is 0. The resultant wave if we look at it is essentially has got amplitude which has doubled because how we can write it is psi 1 plus psi 2. When we can write it that will become just 2 a cos pi into k 1 into x and the amplitude has just doubled.

So, if we look at the intensity, that intensity will be the square of this wave. We find that compared to the particular wave, the amplitudes have increased and what we should notice here is that here we are first adding the amplitudes and then, finding out the intensity and this is how we define for a coherent beam. For a coherent beam, amplitudes are added and then, the resultant intensity is found out.

In the case of an incoherent wave as we have mentioned earlier, intensity of each of the wave is found out and then, they are added together. Then, we are considering another case where that wave is that same as that first case, but the second case we have introduced a phase shift of plus pi. When this phase shift has been introduced to this wave and now, if we look at the amplitude psi 2, it becomes minus a into cos 2 pi k 1 x and these two waves when we add together, the net amplitude if we look at it, it turns out to be 0. That means in this case, there is a complete destructive interference has taken place that wave has just vanished and there is no fluctuations, ok.

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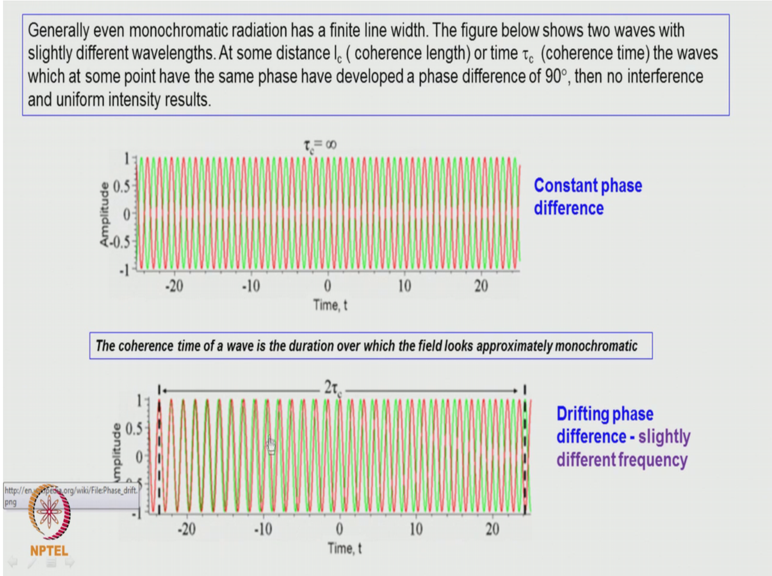

So, this we call it as a destructive interference. Normally most of the cases when we talk of the two cases which we have considered, we assumed that this wave has got a wavelength lambda or lambda 1. Essentially this is the wave length of the radiation which is an ideal case, but normally most of the radiation has got the line width associated with it.

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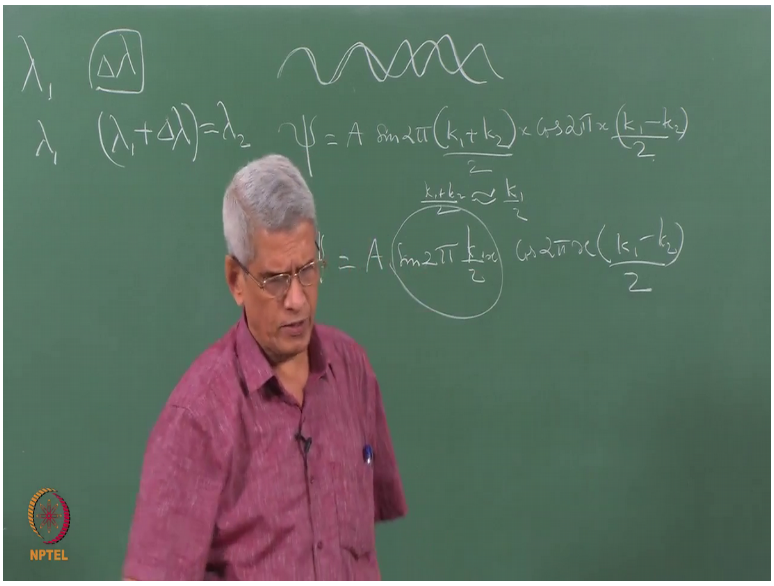

The line width essentially tells; how much is the spread in the wavelength. So, this could be some value delta lambda which we can take it, that is what the reality of the situation is. So, though we call that the beam is monochromatic, but when it has a slight wavelength associated with it, then it is equivalent to considering at an extreme case one with wavelength lambda 1 and another wavelength with lambda one plus delta lambda this is equal to lambda 2; as if two waves with these two wavelength and assume that both are emanating from the same point, originating from that same point and that is what it is being shown here, but as it propagates, we can see that since the wavelength is different, gradually we can see that there is a small phase shift which is occurring after they have traveled a particular distance. We can find out that phase shift has become pi and in this case, the waves completely disappear and a destructive interference is taking place.

In one definition this distance is called as half of the coherence length and that this time is called as, no this distance is called as twice the coherence length, and this distance is called as twice the coherence time.

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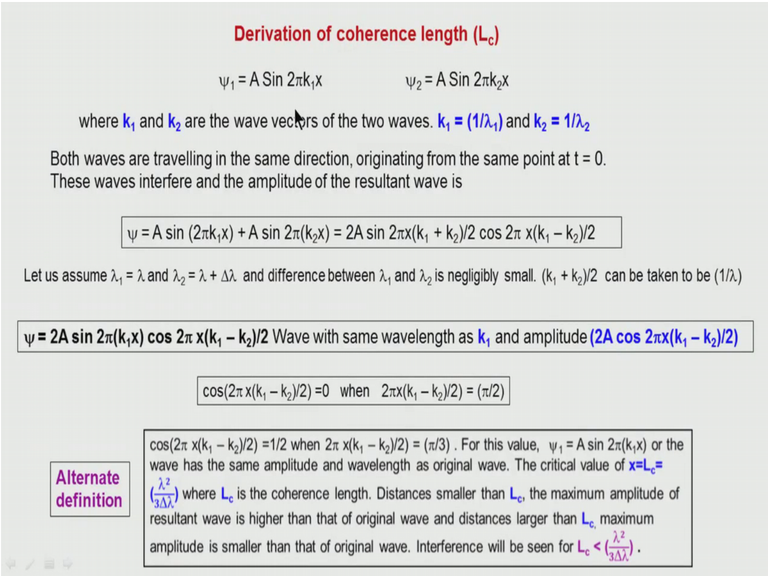

We can just find out an expression for this coherence length, the coherence time and that is what we are trying to do for which what we are assuming that there are two waves which is 1 psi 1 equals a sin 2 pi k 1 x that psi 2 with the sin 2 pi. Only thing is that k 2 is that k vector. So, k 1 and k 2 are two different wave vectors and this is corresponding to wavelength lambda 1 and lambda 2 and we assume that the differences between the wavelengths are extremely small.

Other assumptions which we make is that both the waves are travelling in the same direction and then, if we try to find out instant in the resultant amplitude or the wave at it that formula turns out to be the sin equals a sin 2 pi k 1 x plus, then using this trigonometric identities, we can find out the expression for the net wave. The resultant wave amplitude turns out to be 2 pi k 1 plus cos 2 pi k 1 k 2 by 2. This is what it turns out to be the amplitude of the resultant wave

Since k1 and k2 are very close by, we can assume to be equal to k1 plus k2 by 2 to be equal to k by 2. If we take that, then this becomes a into sin or k 1 by 2 pi k 1 by 2 k 1 minus k 2 by 2. This expression is quiet similar to the amplitude of the one of the wave and then, we have a factor two comes in the picture here and then, this term cos 2 pi x into k 1 minus k 2 by 2, its value is going to decide how as a function of distance x, the amplitude is going to change there is if we take this value to turns out to be 0, ok.

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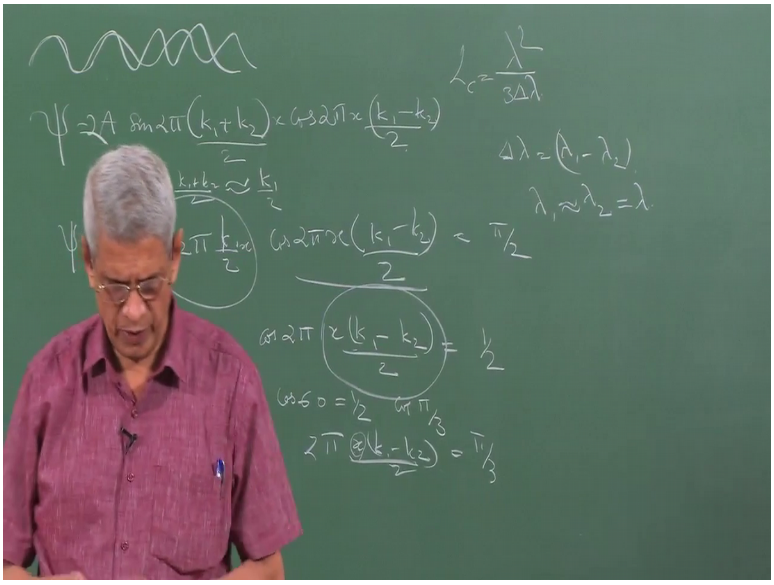

For this cos value to be turn out to be 0, then this term 2 pi x into this one, this should be equal to pi by 2. There is one condition when we can take it when that becomes 0. Then, the net amplitude essentially turns out to be just equal to 0. This is one way in which the definition like the way we have defined it earlier.

The other definition is that if the value of cos 2 pi x into k 1 minus k 2 by 2 if this value turns out to be half, then what will happen then in this case, the psi will turn out to be a into sin 2 pi k 1 x by 2. So, this is nothing but equivalent to that one of the waves. So, that means there is neither increased nor decrease in the same amplitude and it remains the same as the original one. So, for this condition to be satisfied, then this term has to turn out to be equal to pi by 3, that is we know that cos 60 equals half that is cos pi by 3. That means 2 pi into x into k 1 minus k 2 by 2 should be equal to pi by 3. So, corresponding to this value since we know k1 and k2, this will have 1 by lambda 1 and 1 by 2 lambda 2.

Now, we can find out that value for this distance x, this x corresponding to this particular value turns out to be the coherence length distance up to which when two waves with slightly different wave lengths, but having the same frequency, but having the same phase at what distance the amplitude of the wave becomes the same as that of that incident wave, that distance is called as the coherence length. This distance will turn out to be nothing but lambda square by 3 delta lambda. This delta lambda will be nothing but lambda 1 minus lambda 2 and since lambda 1 is nearly equal to lambda 2, this is written as equal to lambda. That is how we derive this expression l c equals lambda square by 3 lambda.

So, this is another way in which the coherent length is defined. This we call it as the temporal coherence length. This is the distance over which when the waves travel, the interference vanishes.

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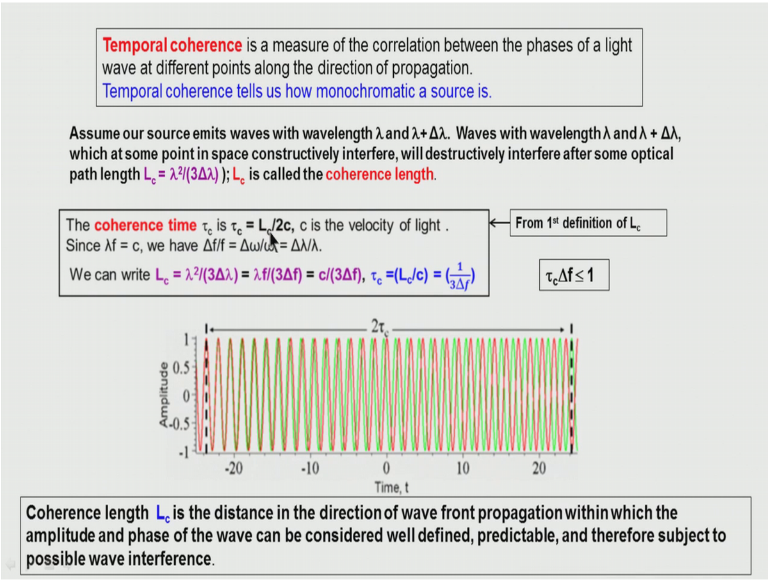

This is exactly that a same derivation which has been shown from this, the coherence time which we can write it tau c equals the coherence length divided by this naught 2 divided by just c, that is equals l c by c. This will turn out to be roughly 1 by 3 delta f and this one can just do this simple derivation which is shown here, ok.

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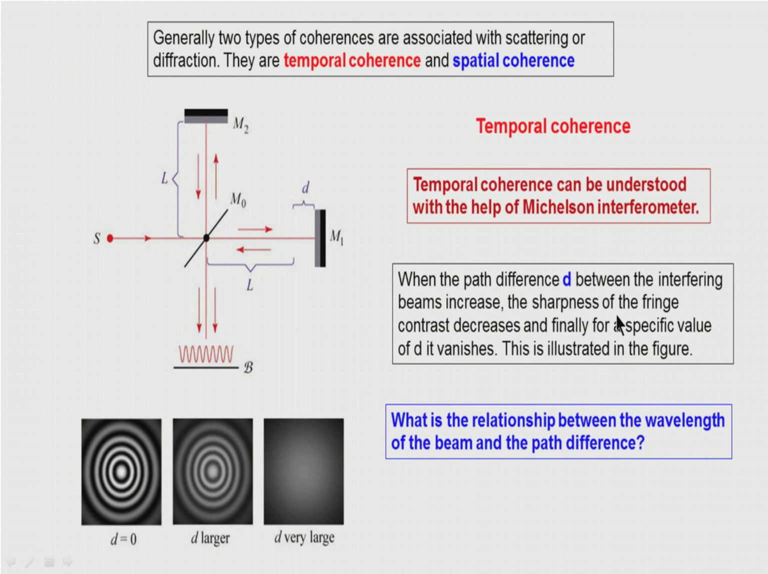

So, tau c becomes delta f tau c into delta f is almost of the order of 1 or slightly less than 1. What is the experiment which we can do to find out this coherency? One of the experiment is essentially the Michelson interferometer which has been used to find out this coherent length. What is essentially done here is that we have a monochromatic source of light and light is travelling in this direction. We have kept a partially reflecting mirror here to the ray pass through this and part of the ray because this has kept at 45 degree part of the ray is reflected.

We have kept another fully mirrored, fully reflecting mirror is kept here and another fully reflecting mirror is kept here and both these distances are kept initially the same and then, we find that these two waves which has been split from here, they travel the same distance and come back and they join here. Once they come here and we should be at if we keep a screen here, we should be able to see an interference pattern and here we will see that the pattern which we see what we can do is, this mirror, the distance we can just gradually go on changing it. That means we can change the path difference continuously.

So, as we change the path difference because when the phase difference is that same, there is an amplitude increase. So, we will be getting an interference pattern and when the distance we go on increasing it for a particular distance. As the distance increases, gradually we can see that d 0, this distance and this gap as it is being increased, we can see that the intensity of the constructive and destructive interference will come down.

That we can make out from here as the x changes for a constant value, this amplitude factor.

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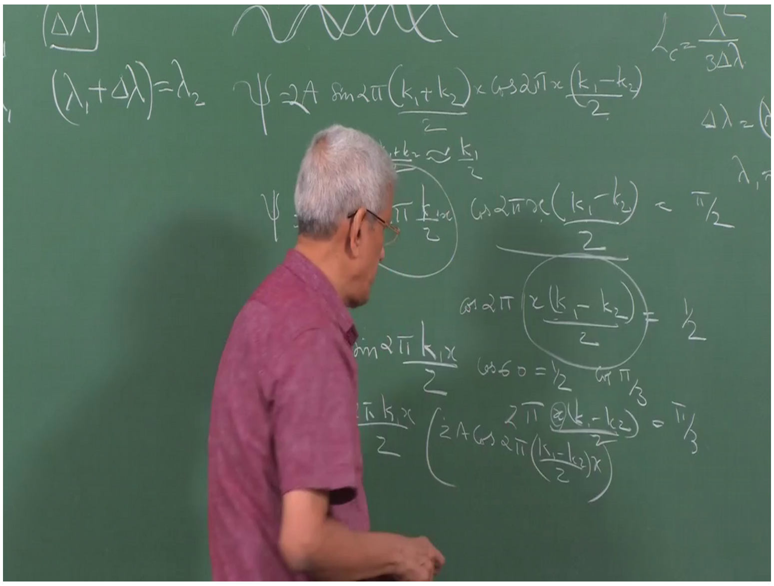

Now, this can be written as sin 2 pi k 1 x by 2 into 2 a cos 2 pi k 1 minus k 2 by 2 into x. This is the amplitude term depending upon that value of x or the distance d which we use it, this term will be changing and as the value of d becomes very large for at a critical value, we find that interference pattern completely disappears. This distance d a is related to the coherence length.

This way we can find out what is the temporal coherence length of the beam. That is what is the path difference over which even when we say that the beam is monochromatic because in an ideal case if we say that the beam is perfectly monochromatic at any distances, constructive and destructive interferences should take place, but we find that in reality, there is a line width which is associated with it that puts a limit on what is the distances over which this constructive and destructive interferences could be seen beyond which we will might be seeing the difference pattern. That is what information we can get it from that Michelson interferometer experiment.

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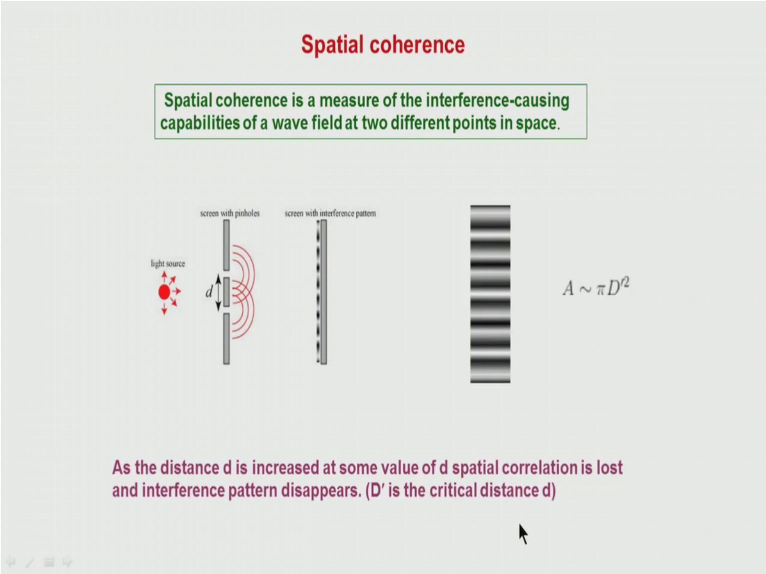

Other important aspect which is especially important in the case of electron microscopy is spatial coherence. What do we mean by spatial coherence? That is if light is emitted from one particular point and a light is emitted from another particular point on that source, will the photons which are emitted from various points all of them will be in phase or not; normally need not be.

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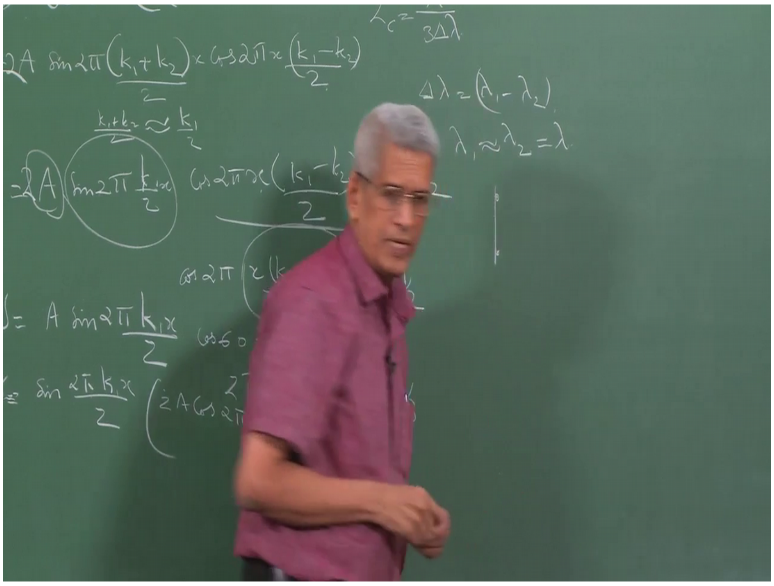

What is the distances over which the spatial coherence could be maintained? This information can be obtained using Young’s double slit experiment is one example, one experiment which we can use to get information about it. What is being done here is a point source from which the light is being emitted and the two slits are there. So, when the light reaches here, we can assume that the spherical waves are being emanated from each of this. So, at these two points if we look at it, both of them will have the same phase relationship and then, this waves interfere and because from here to here at different points when the waves come from both the slits, there is a path difference which is introduced. So, it gives rise to an interference pattern.

The question which arises is that this is some distance d which we have maintained. If we go on increasing this distance, d will we be seeing that same type of an interference pattern one and will the amplitude of the constructive and destructive interference will remain the same or not or for some particular distance d will this pattern itself will vanish. This experiment show that d is being increased, the intensity of this constructive and destructive one, they gradually start decreasing at some particular separation.

They completely vanish. This decides the spatial coherence, ok and that area corresponding if that psi d dash square, this is the critical distance d at which the spatial coherence is lost.

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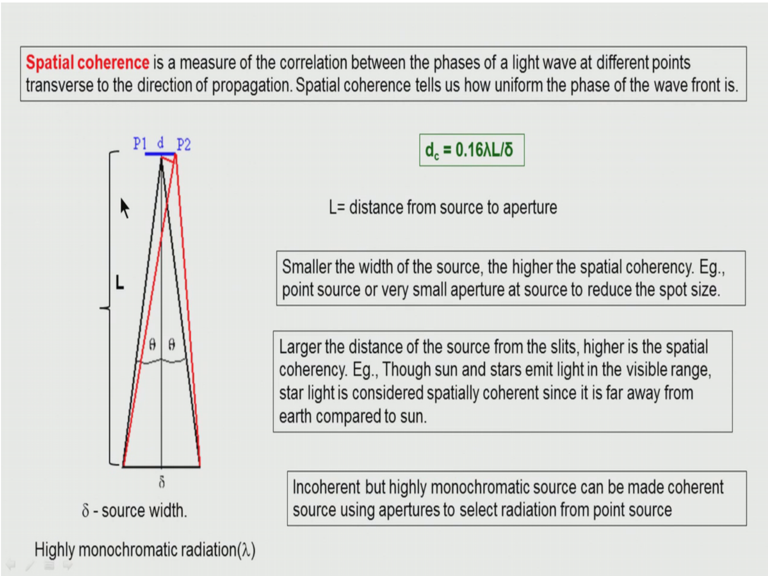

This is given by this formula. We will just look at the derivation little bit later. Essentially the spatial coherence or that separation between the two slits is given by the formula 0.16 lambda is a wavelength of the radiation, l is the distance from the source and delta is the source width and here we assume that the beam is a perfectly monochromatic beam, but its a wide source. So, as the size of the source increase, we can make out that d c decreases. That means the spatial coherence becomes poor and poor and as the wavelength increase, the d c increases. As the separation of the slit from the source is increased, then also the spatial coherence d c is increasing.

This is one of the reason when we find that though the sunlight when we comes from the sun and similarly, the light comes, the stars we say that light from the stars are coherent and not from the sun because the sun is much closer to us whereas, the stars are very far away. Though the size of the star is much bigger, still using deform we find that because the distance is very large, the d c turns out to be very high. So, the spatial coherence is quiet high.

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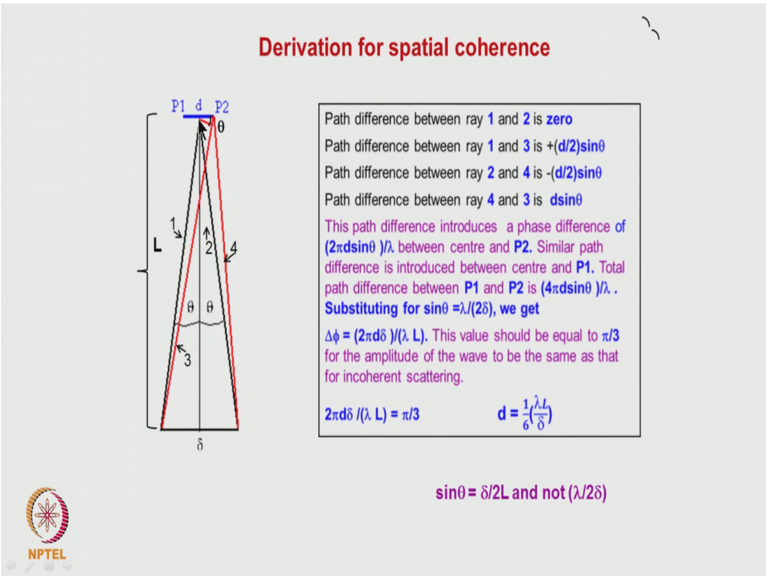

We will just look at this derivation. This is the size of that source beam which starts from both the ends of the source extreme and if we consider it when they come and meet at the middle of the slits are there, the beam can come and here if we consider it at this particular point, the path difference turns out to be 0, but from here compared to a ray which starts from here and reaches here that introduces a small path difference and that path difference will turn out to be this distance d by 2 into sin theta.

Similarly, the ray which comes from here from the center to this one, this introduces another path difference minus d by 2 sin theta. So, essentially if we look at the path difference between the rays, this one which emanates from here and one which comes from here and meet here, the path difference turns out to be d sin theta and similarly, we can find out the path. So, the total path difference between this and this finally turns out to be 4 pi d sin theta by lambda. This is what the total path difference which turns out to be corresponding to this, we can find out the phase difference. When this phase difference turns out to be as we have looked at it expression, this phase difference turns out to be equal to pi by 3 and that value corresponding to that if we calculate this is the d value turns out to be 1 by 6 into lambda l by d l by delta.

This is what essentially we have seen. So, from this what we can make out is that if the beam is being made as the source width is reduced, the spatial coherence length even if the separation between the waves spatially is quiet large, still an interference can take place. That is very important or if the source is at a very long distance, then also the spatial coherence is enhanced or the width of the source is like points source. If we take it, then also the spatial coherence is enhanced. This is what we can infer from this transparency.

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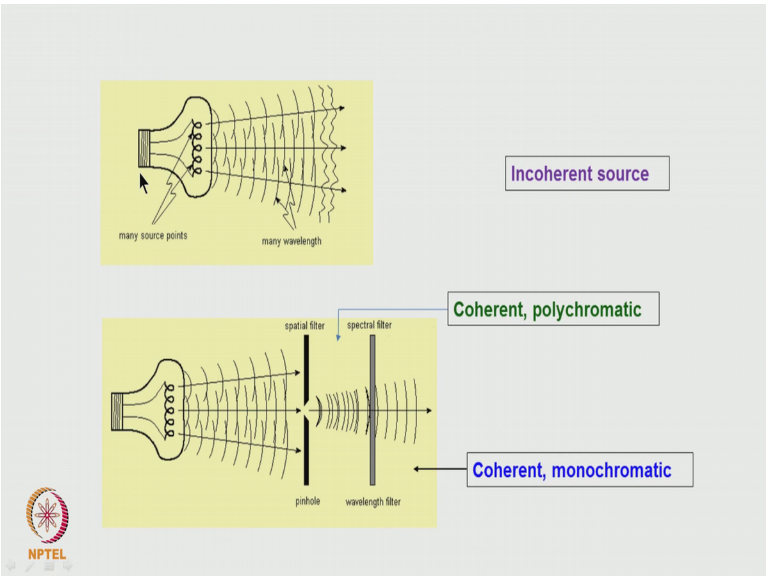

Here one example is that tone, this is a light source. It is an inherent bulb which is emitting tungsten bulb which is emitting photons at different points. Photons are being generated with different wave lengths and different frequencies. If we put a spatial filter, a spatial filter is nothing but a aperture with a very small size whole in it. When we put it like a point source, the light is coming from here. Though the wavelength is different, this source become highly spatially a coherent source and to this, but the beam is a polychromatic beam which if you put a filter, then we can get a monochromatic beam which also exhibits high spatial coherence.

If the line width of the radiation turns out to be very small, then this beam can exhibit very high temporal coherence as well. So, both spatial and temporal coherence can be obtained even from sources which are incoherent by putting an aperture and using different types of spectral filters.

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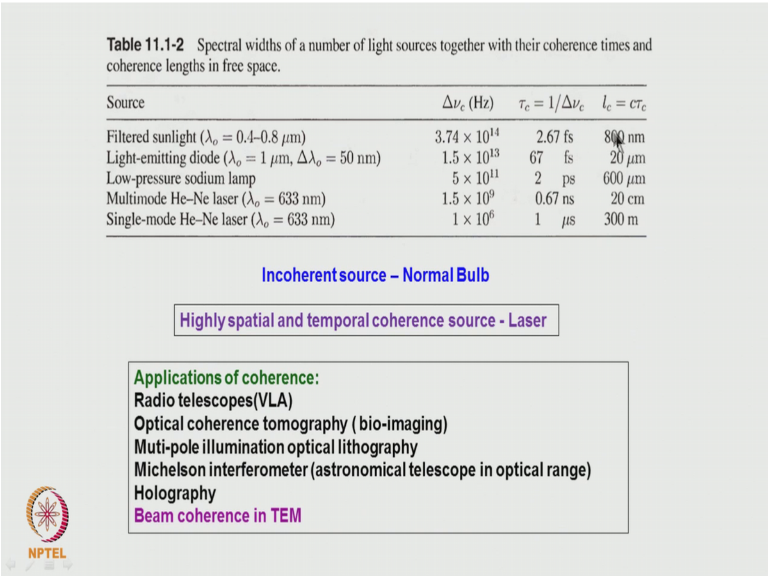

In this table, I am just showing some examples of sources and what is the coherency which they exhibit. If we take sunlight which is unfiltered, the wavelength of the sunlight is from 0.4 to 0.8 micron. This corresponds to a frequency of 3.74 into and tau c. The coherence length corresponds to this operation turns out to be 2.61 femtosecond and this gives the coherence length is only 800 nanometers. So, we wanted to get any interference pattern, this is the distance by the time light has travelled.

This distance from the source already the coherency has been totally lost. This is the distance over which the coherency is being maintained. If we take light emitting diode, this is lambda 0 equals 1 micron delta lambda, then it is about 20 micron is the coherent length and if we take the sodium vapor lamp, we can get a coherence length of about 600 micron. If we take a multimode neon laser, then the coherence length turns out to be 20 centimeters. If we take a single mode helium neon laser, then the coherence length turns out to be 300 meters. That means, whatever be the path difference which are being introduced, still the interference patterns could be obtained using this beam, ok.

What are the applications of this coherence because suppose we wanted to collect signals from which are coming from different stars, from space, then many radio telescopes are used to collect the signals to enhance that intensity. Then, there are very large array of telescopes are kept some particular distances, so that phenomenon of coherence could be used to add up all the signals which are coming from each of them. Then, in optical coherence tomography is one which is used in bio medical imaging and then, in lithography it is being used Michelson interferometer which I had explained.

Then, holography is another technique where coherent beam is very much necessary, especially if wanted to look at the magnetic structures or ferroelectric materials in an electron microscopes. We can use this holography technique to create interference pattern and get information about it for which the spatial coherency of the beam is very much important. Having said that so far what we talked about coherency in terms with respect to an electromagnetic radiation, ok.

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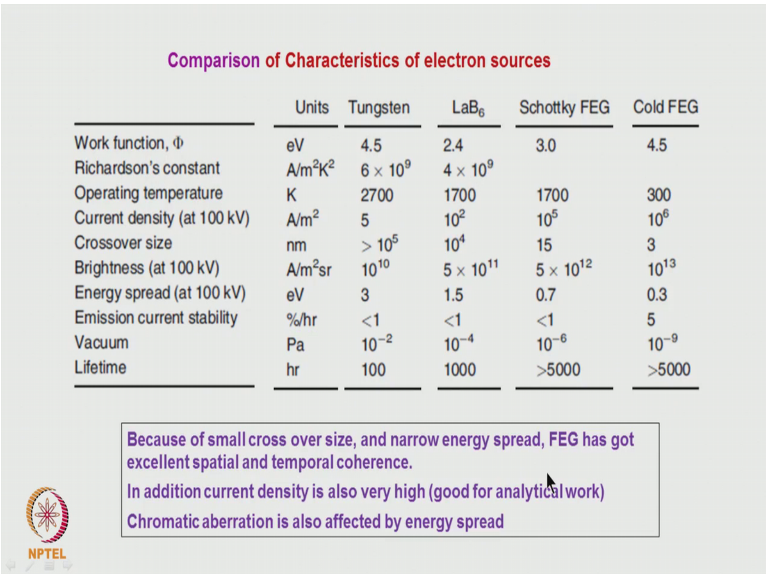

Comparison of Characteristics of electron sources

	Units	Tungsten	LaB_6	Schottky FEG	Cold FEG
Work function, Φ	eV	4.5	2.4	3.0	4.5
Richardson's constant	A/m^2K^2	6×10^9	4×10^9		
Operating temperature	K	2700	1700	1700	300
Current density (at 100 kV)	A/m^2	5	10^2	10^5	10^6
Crossover size	nm	$> 10^5$	10^4	15	3
Brightness (at 100 kV)	A/m^2sr	10^{10}	5×10^{11}	5×10^{12}	10^{13}
Energy spread (at 100 kV)	eV	3	1.5	0.7	0.3
Emission current stability	%/hr	<1	<1	<1	5
Vacuum	Pa	10^{-2}	10^{-4}	10^{-6}	10^{-9}
Lifetime	hr	100	1000	>5000	>5000

Because of small cross over size, and narrow energy spread, FEG has got excellent spatial and temporal coherence.

In addition current density is also very high (good for analytical work)

Chromatic aberration is also affected by energy spread

Now, let us talk about coherency in electron microscope, where exactly it is necessary because the types of source of electrons or either tungsten filament or LaB 6 filament lanthanum hexaboride filament or Schottky FEG or cold FEG, these are all the filaments which are being used. The work function corresponding to each of them are given here what is going to be the Richardson's constant and the operating temperatures at which the filaments are operated tungsten is operated 2700 lanthanum hexaboride, 1700 Schottky FEG as 1700 Kelvin and the cold emission, cold field emission gun is operated at 300 KV can make out that the tungsten, the current density is very poor where as in the case of cold Schottky cold emission, the current density is almost around 10 to the power of 5 to 10 to the power of 6 times that of a tungsten filament brightness.

Also, similarly you can see that increases, but what is essentially important is that when we talk about monochromaticity of a beam because the monochromaticity of the beam determines the temporal coherence that in the case of tungsten filament, it is about 3 electron volt, and whereas in LaB 6, it is around 1.5 Schottky. If it is 0.7 and cold emission, this is 10.3 from this, we can immediately make out that the field emission guns the beams exhibit higher temporal coherence. Other than that what is one more thing which is essentially important is that- what is the area from which these electrons are emitting.

From the electrons source size here, if you look at it, it is around 10 to the power of 5 nanometers here. It is around 10 to the power of 4 here, it is about 15 nanometers here, it is about 3 nanometer. That is we can make out that the source is becoming smaller and smaller and with a very high brightness current density and also, if we look at the energy spread is becoming small. So, for field emission guns, we have got very high spatial as well as temporal coherency is there.

Why this coherence is necessary? Because in an electron microscope essentially all the contrast which we look at is essentially some sort of a phase contrast microscopy, where the phase difference is being changed into an amplitude variation. So, in such cases, the coherency of the beam is very important. Then, the other case where the coherency is very important is that when we look at the diffraction pattern is essentially is nothing but interference of coherent beams which are interacting with atoms in crystal structures which are some periodic distances.

Here again the coherency of that beam is very much necessary and in both the counts if we look at it, field emission guns have got high spatial as well as high temporal coherency and apart from the coherency if you look at that resolution of the microscope also which we discussed in some earlier classes, there we had seen that the resolution is also being affected by the energy spread. The energy spread enhances the chromatic aberration of the microscope and in the field emission gun, the chromatic aberration is reduced considerably.

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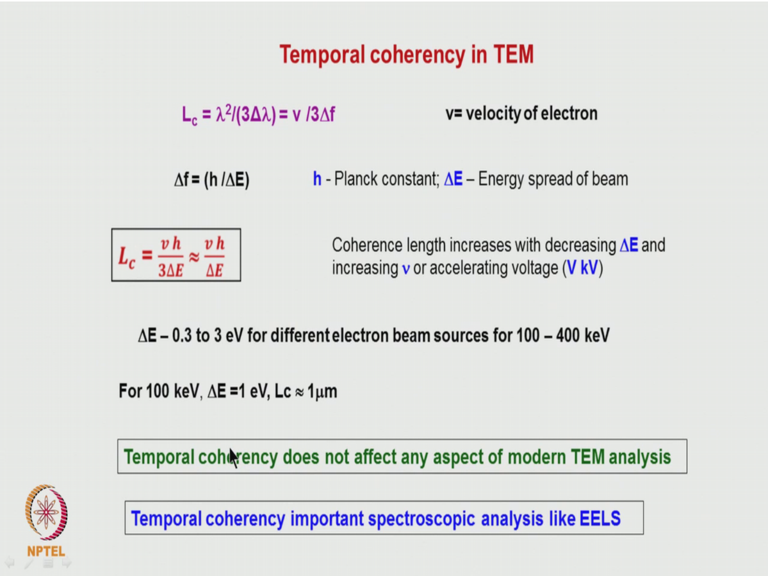

One the next question which comes is that though field emission gun has got a very high spatial as well as a temporal coherency which is the one which is really important in the case of a transmission electron microscope, ok.

Here if the formulas which we have derived that are being given here in the case of a microscope, we can make out that l c is the coherence length temporal. Coherence length delta is 0.3 to 3 EV for different electron beam sources for 100 to 400 KEV. Initial energy of the electron beam for 100 KEV if we consider delta e equals 1 electron volt, l c is of the order of about roughly 1 micron, but the temporal coherency is not that important in the case of a microscope.

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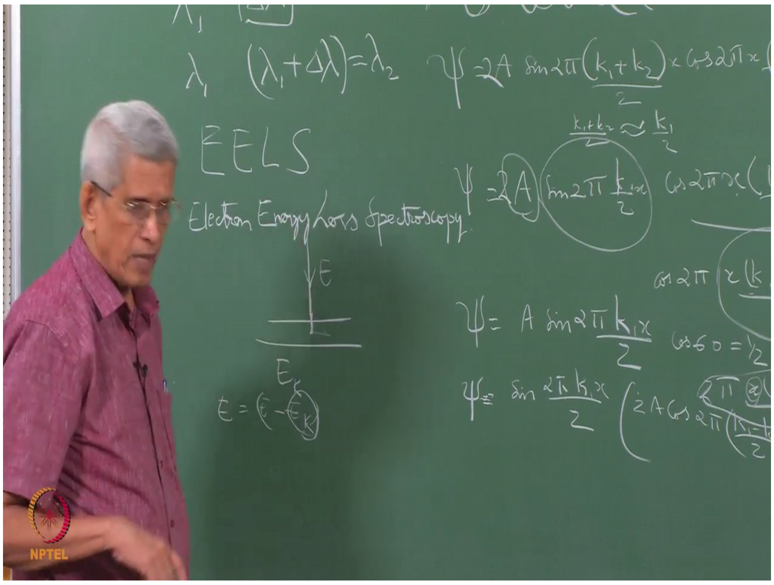

The spatial coherency is very much important, but the temporal coherency is very important in particular type of a microscope which we use it and this is called as the technique which is called as electron energy. This is electron energy loss spectroscopy. About this technique I will be talking about it later, but I will just mention here what it means, so that what is the relevance of this temporal coherency, we will understand it at this juncture, ok.

In electron energy loss spectroscopy, when electron with a specific energy is falling onto the sample, some inelastic scattering can take place. So, electrons from a particular place for an atom electron from a particular level k level can be knocked out. Suppose this is the energy of the electron. So, if that electron is knocked out from an atom sitting here and then, the energy of the electron which comes out will be e minus e k is the energy with which the electron will be coming out.

So, since we know the initial energy since we have measured this particular energy of the electron which is coming out, then we can find out what is going to be e k. This is characteristic for each of the element and this information could be used to identify the various elements which are present on the sample surface. So, if the energy spread is going to be large, the accuracy with which we can measure this e k will turn out to be small. If the energy spread is very small, we can measure this value. Hence, in this case the temporal coherency is very important or what we essentially means that the beam should be as monochromatic as possible. In many microscopes, in addition to the sources we can put a special filter to bring down the spread in the beam.

That is what it has been done. More details about this spectroscopy we will talk in a later class.

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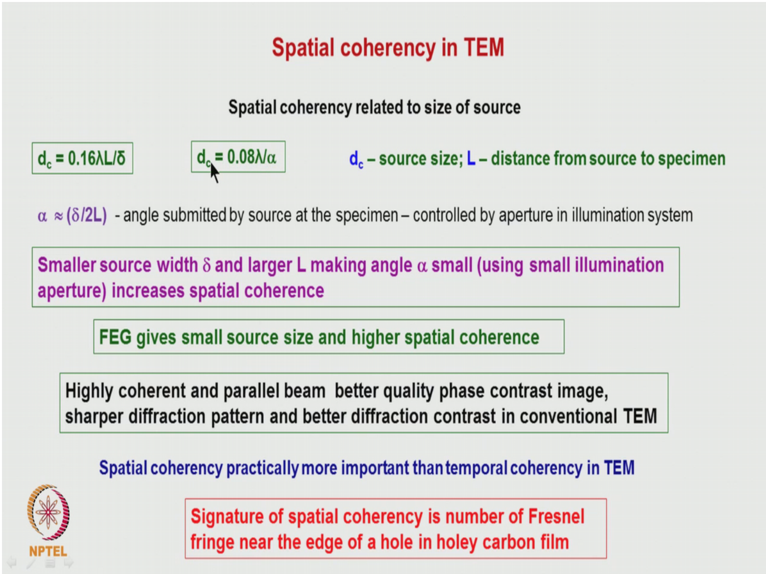

The spatial coherency this is formula which we have derived d c equals 0.16 lambda l by d and this l by d will turn out to be nothing but the angle alpha, that is if we have a source which is here and this is where the width which we look at it.

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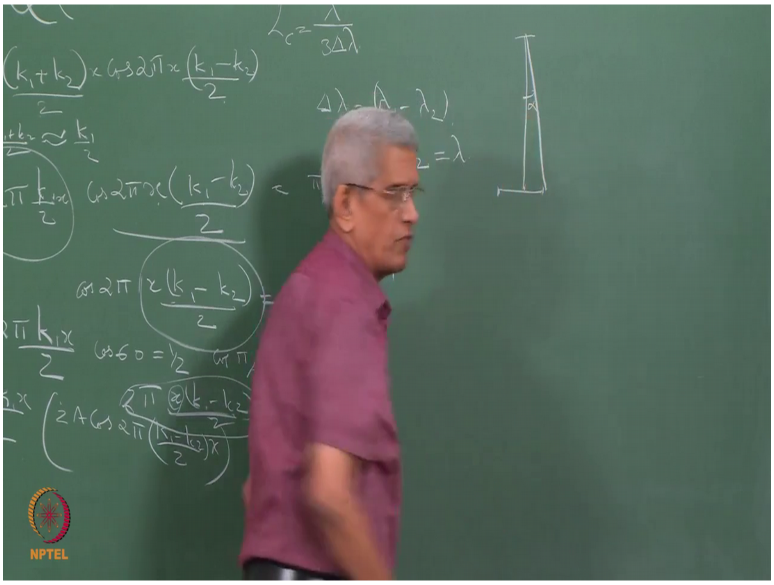

What is the angle this it makes with respect to this angle is what is called as alpha. This depends upon this distance. The source width delta and this distance l that is what essentially is being given and as we could make out that the spatial coherency d c, this distance d c s or d or d c is called as the spatial coherency. This is what we have derived sometime back and since the lambda is fixed, that can be changed or that can be made smaller by using high energy electrons or increasing the energy of the electrons and for field emission gun, the delta becomes very small even for a constant l. We can make out that d c becomes very high.

So, the high spatial coherency can be maintained that consequence of it is that even if the beam is made highly convergent, even then the coherency could be maintained. Generally when the beam is being made parallel, this angle will turn out to be, alpha will turn out to be 0. Then, what d c is going to be very high. This is the condition which we normally use it for diffraction because normally for diffraction what we do is, we use a parallel beam. Why we use a parallel beam is because in that condition, the spatial coherency is turning out to be very high and FEG, what happens is that in spite of even if you use the aperture, the beam can be converged to a point, even then the spatial coherency could be high, spatial coherency could be maintained. That is the advantage of having an field emission tip gun.

A direct example of it: how we can see this spatial coherency in a material. The spatial coherency in a transmission electron microscope strong difference which we can see when we use tungsten filament, ok.

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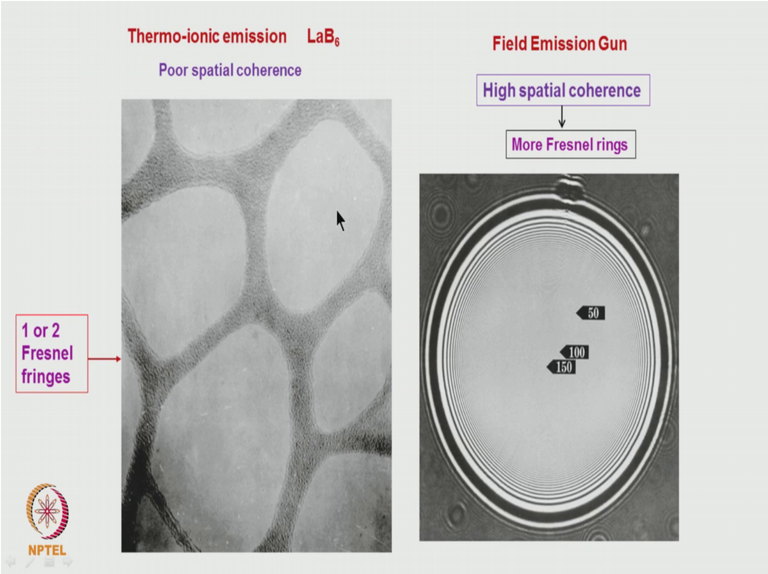

The thermionic emission filament where we can make out that source size is very large, the spatial coherency is very small. In this case, what is being done is a hole film is a sample that is hole film is nothing but a carbon film in which small holes are there. Around these holes when a highly coherent beam which a monochromatic coherent beam which we use it, we will be able to see Fresnel fringes near the edges. In this case, hardly one or two fringes which we could see, where as if we use a field emission gun because the spatial coherency is there, lot of Fresnel fringes could be seen from the edge going inverse.

This itself is an indication of high spatial coherency. The importance of this coherency comes in especially high resolution images as well as in all conventional images also, where essentially the contrast which we get we call it as some sort of a phase contrast. Here I had just tried to explain what is the need for spatial coherency because what will happen is that if we use the thermionic emission gun and try to look at that sample, unless the beam is highly parallel, perfectly parallel, we will not be able to get a high fidelity, high contrast images, especially high resolution images using thermionic emission, but where as in the field emission gun even if the beam is not parallel. Even if the beam is convergent, still we can get very good quality images. We can obtain that is why we should have high spatial coherency in the microscope.

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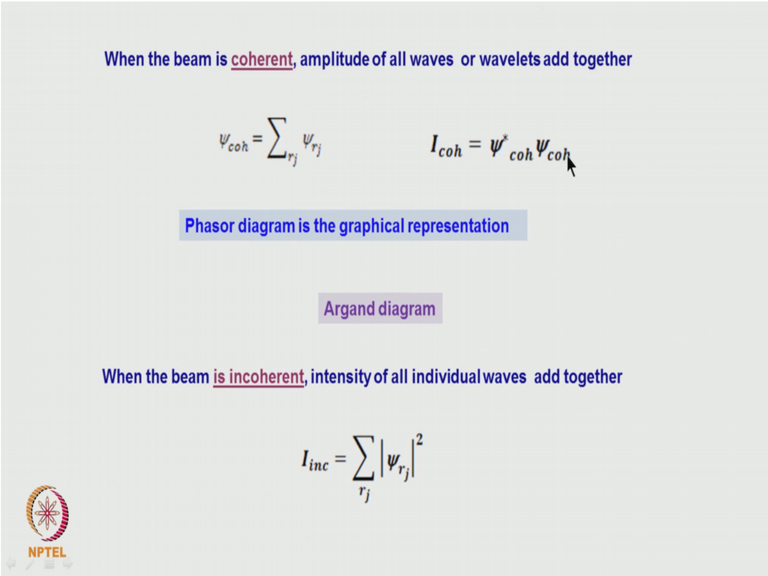

Here just what I had mentioned about how we calculate the intensity of when the beams are coherent beams, in this case essentially amplitudes are added together and then, the psi star is taken to find out the net intensity, where as in coherent beam for each of the wave we separately calculate the intensity and add together.

I hope in this transparency I have made it very clear the importance of having coherent beams. I will stop it here now.

Thank you very much.