

**Micro and Nanoscale Energy Transport**  
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**Lecture – 4**  
**Continuum Heat Transfer and its limitations**

Good morning. And welcome back to the second week of this introductory lecture on Micro Nanoscale Energy Transport.

We will try to quickly conclude the generic introduction that have you been making so far. I would like conclude by once again emphasizing where the macroscopic laws we have seen that the different regimes interpreted in different ways using mean free path, characteristic length, nodes and number. So now, I want to again recap because this is very very important when you are looking a transport processes where you can use a macroscopic or continuum laws.

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**Limitations of macroscopic laws**

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Macroscopic Laws break down when:

- The length of the system is comparable to the mean free path of the carrier
- The time scale of the physical system is smaller than the relaxation time of the heat carriers in solids. (phonons and electrons)

Experimentation at submicron scale poses great challenges

Numerical simulations have become critical for basic understanding and design purposes

$\Lambda \ll L$  (a)

$\Lambda \gg L$  (b)

Two regimes, (a) Fourier's law is valid, (b) Fourier's law is invalid,  $\Lambda$  is the mean free path of the heat carrier (phonons or electrons)

Heat Transfer & Thermal Power Laboratory, IIT Madras

So, you can say that generally the macroscopic laws breakdown under two conditions. One is you compare your characteristics length scale with the mean free path of the energy carrier. So, that is the first criteria to decide whether you can use the continuum equations or not. Typically, if you are looking at the illustration here the figure shows your classical continuums approximation where the mean free path is much, much smaller than your characteristic length scale between say two plates. And here you find

that these energy carriers are so many numbers that densely part and they have sufficient collisions with each other and also collisions with the boundary of the system.

So, in this case you have sufficient statistical unsample of particles to define macro scale property, such as internal energy temperature, density and so on. Whereas, if you look at case b; so this is where your mean free path is much larger than your separation between the plates. And therefore, the chances of the inter carrier collision will be much lesser compare to the collision between the energy carrier on the boundaries of the system. In this case you will find new phenomena happening because of the more predominant collision of the energy carriers with the boundaries compare to the system a.

So, this is what we attribute a sub continuum phenomena; the kind of phenomena that you observed when you are much below the regular continuum approximation length scales. The other is your time scale. So the time scale, in this case also you have to be careful if you take your physical system and then you have a time scale of this physical system, and then you have the time scale of the collision of these energy carriers, this is call the relaxation time scale. So, it is call relaxation time scale because this is the average time require for energy carriers to colloid and come back to an equilibrium state to relax basically to an equilibrium state. So, this is called a relaxation time scale.

So, you have basically a physical time scale number; one for example, this could be like laser pulse which you are me radiating on a particular material, so that is your physical time scale. So, this is pulse width could be say nano seconds, picosecond and femtoseconds. The other is your time for the energy carriers to colloid and come to equilibrium state, and that is your relaxation time scale. That is typically if you take the example of say electrons in a metal it could of the order of picoseconds  $10^{-12}$  seconds.

Therefore, if you are now eradiating with the femtosecond laser  $10^{-15}$  seconds pulse width. So, this is much smaller than your relaxation time. And therefore, again there is a chance that you are basically leading to a local non equilibrium and therefore your continuum laws cannot be used. So, not only the length scales are important, but also the times scales are also equally important. There are cases where you

have perfectly large dimensions, but you are talking about using very very small time scales and this can also lead to sub continuum phenomena.

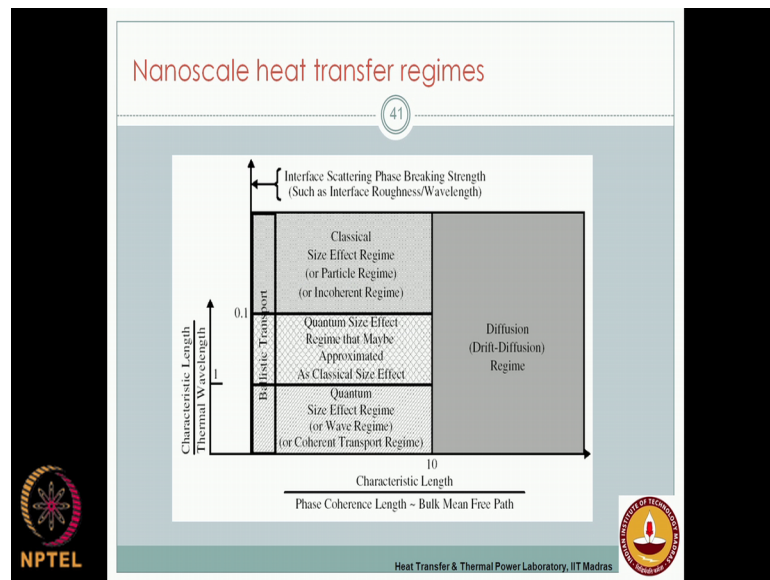
Therefore, you should also understand that anything dealing with sub continuum whether it is in the time scale or length scale they are very difficult to observe with our sensors the way we do it with our conventional measurement. And even the sensors that we are using you cannot use them in the conventional way. For example, temperature can be defined only in the continuum sense. So, we cannot put a thermocouple when you are talking about a hotspot that I was explaining in the previous lecture, in silicon on insulated device.

So, measure the temperature of the hotspot the level of nano meters you cannot use you know a thermo couple, because that the definition of temperature itself is not valid at such small scales. And therefore, I mean the thermocouple will not sense the right non equilibrium phenomenon that is happening. The temperature can be some kind of an average energy density, that is indicator of an average energy density but it is not your equilibrium thermo dynamic equilibrium definition of temperature.

Such kind of measurements will be very very difficult, therefore to do what such nano scales. And therefore usually measurements are very hard and very difficult and also very very sophisticated involving electron microscopy. You need to combine several things together it is not just putting a thermo couple and you know just observing it.

Therefore, we are also relying on increasingly on the theoretical and numerical models by solving this sub continuum transport equations based on Boltzmann equation and so on doing molecular dynamics, Monte Carlo. These are the different ways sub continuum levels of numerical transport that can be used and they can actually give a lot of physical understanding before we can you know do this experiment.

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So, this is another very interesting picture that I got from a review paper some time back and this review paper is from Professor Ganichand's group of MIT. This is very very clear the way they have classified the different levels or regimes of transport on two scales. So, on the x axis what you have is basically the inverse of your nodes and number, so we define nodes and number as a ratio of your mean free path to the length scale and what is plotted here is just the inverse of this. And therefore, the one that is going towards the right hand of the x axis will be the continuum.

So, the node and number 10 to go to very small values, so this particular quantity will be very large and therefore the one portion that is towards the right hand of the horizontal axis will be the continuum transport regime. And that is why it is called diffusion regime. Diffusion meaning that you have plenty of energy carriers which collide with each other and therefore the majority of the transport is by the diffusion or collision of the energy carriers.

And on the vertical axis you have another non dimensional ratio. This is basically the ratio of characteristic length to the wave length. This is something also really interesting. If you are looking at sub continuum, how do you decide whether you are in a continuum or sub continuum range? So, you look at the x axis you define the nodes and number and accordingly you say for large nodes and numbers you are looking at sub continuum and so on.

Now with in this sub continuum regime it is not just one whole sub continuum regime there. Again there is a regime where you have to be careful what is the length scale compare to the wave length of the energy carriers. So, if you are talking about length scale of the order of a strum, now you find there is another regime coming out within the sub continuum range where the wave effect becomes very important. As I already told you that you know we have a wave particle duality and you can always solve either of them. You can use either the wave approach or the particle approach, but the particle approach has a limitation that as long as your device dimension is much larger than your wave length then the particle approach can predict, you know very well.

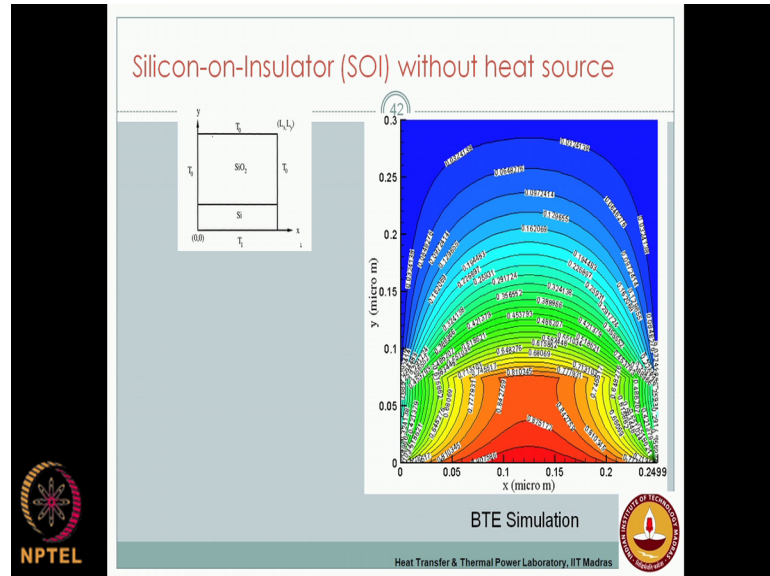
But once the dimension falls below the wave length then what happens the wave phenomena will become more important and particle phenomena, particle methods cannot predict the wave nature. You have to therefore go in to quantum mechanics which describes all these energy carriers in the form of waves. The wave nature has to be explicitly understood and therefore solved in order to describe basically the sub continuum transport when your length scales fall below the wave length of your energy carriers.

Therefore, we have the two excess here the primary excess horizontal excess your Nusselt number, the secondary excess is your the ratio of characteristic length to the wave length. And when your characteristic length falls below the wave length, so this is where your wave regime becomes important you solve your Suring this wave equations which we will do in a classes, introduce that and see how the wave nature can be explicitly understood, and how it becomes important in sub continuum scales. Whereas, if you are talking about the order of several hundreds of nano meter, and comparable to the wave length wave length could be you know 1 nano meter 2 nano meter, whereas the device dimension is they order of 100 nano meters.

In that case now you are above the thermal wave length regime and therefore you describe by what is called as a classical sub continuum regime or also particle based regime. So, these are very very interesting regimes. You have to be aware that not only just using a simple sub continuum modal, but also depends on the ratio of the device dimension to thermal wave length you may also have to use some time Schrodinger's equations. So, you cannot explain all the phenomena using the particle based method

itself. That means, Boltzmann transport equation itself is not enough for describing all the phenomena. So, most fundamental would be going to the shooting as wave equations.

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Well, now again with this let us look at one example where we have a semi conducted device. So, this was one of the simulations headed few years back by solving the Boltzmann transport equation. So, we have a system where you have silicon phylum which is deposited, on top of this you have the silicon dioxide which is a dielectric. So, usually this is a substrate. Typical semi conducted device what I call as SOI, silicon on insulator. So, the S I O 2 here is the insulating sub striate over which you have silicon which has a very good thermal conductivity, whereas S I O 2 has a very poor thermal conductivity here.

And we just try to solve a simple heat diffusion problem in this system. The order of magnitude of silicon thickness will be the range of few tends of nano meters, and silicon oxide substrate might be may be 5 times the size of the thickness of the silicon phylum here. And you have homogenous temperature defined at three boundaries, and at the bottom boundary you have a higher temperature.

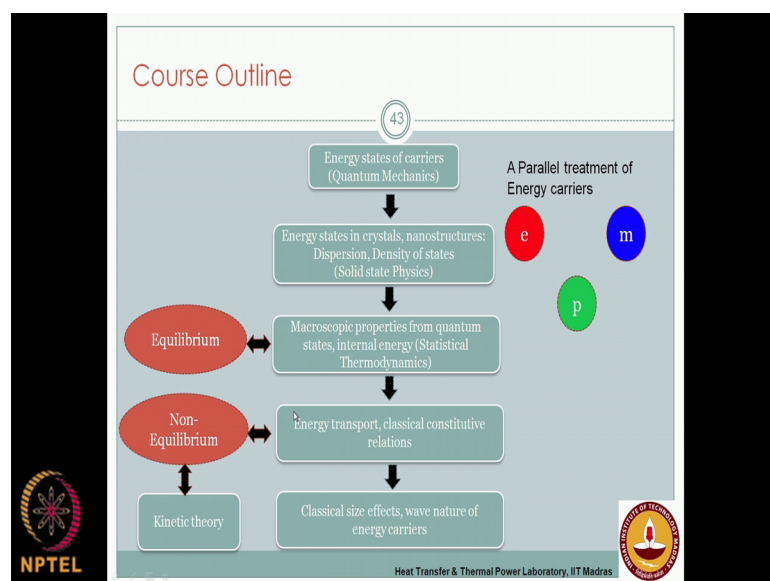
With this if you look at your solution of heat diffusion equation where you apply your Fourier's approximation and then you solve it, you will be seeing a set of temperature contours and also at the same time you will not be seeing any kind of discontinuity in temperature between silicon and S I O 2. That is no interface thermal resistance that we

are modeling here. According to the macroscopic laws your temperature has to be continuous at the interface, and your heat flux also has to be. But now when your solving this using a sub continuum modal you feel see that now you the major differences that you see in the silicon dioxide layer where it is a slightly tending towards the continuum level you see your classical Fourier kind of equation pattern you get these elliptical contours.

So, the nature of the contours that you observe here typically are the once that are predicted by solving the Fourier's equations. If you remember your solution of heat conduction equation, so if you have a high temperature and then low temperature on the other three sides this is how your temperature contours look. But you see with in the silicon layer where it is a sub continuum phenomena it completely non-linear. So, we have a linear pattern of temperature outside the silicon layer and within the silicon layer you have high non-linear temperature contours. And there is also a temperature discontinuity which is happening at the interface, there is also sloop discontinue.

So, you see that this high non equilibrium process, if you use the Fourier's equation; obviously, they will completely failing to predict the values of temperature magnitude and also the qualitative variation of temperature with in this particular silicon phylum. So, this is why we have to go to the modals based on for example, Boltzmann transport equation to understand this particular non-linearity and sub continuum scales.

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Therefore, just too finally conclude the wave that we will be looking at this course on just the nano scale energy transport. I mean this consists only the nano scale part the micro scale part I will come back to that later on. But first when you focus on the nano scale part we start with the defining the energy states of carriers. That is the most fundamental level so were we apply the quantum mechanics and then we define these energy states of carriers.

And from there we have to go to the macro scale level were we define properties and therefore we have to also understand how do we apply these quantum mechanics to certain system. So, we will be applying it to systems which are semi conductor, crystals, nano structures, and so on. We will see how this energy states get modified in this particular systems. You have fundamental energy states which are for a single particle in a box or free electron or whatever.

These are the theoretical ones, but when you apply it to a system where it is confined by certain dimensions and certain structure of the system so you have the energies levels or the fundamental energy states get modified. So, we will see how that gets modify this is called dispersion of energy states. So, we look at the dispersion and then we will go to the macro scale by using statistical thermo dynamics, we have to now collectively take an average of all the energy occupied by the carriers at different states. So, that will be done by using this statistical thermo dynamics.

And then, this will be still an equilibrium value, equilibrium distribution from this statistical thermo for different energy carriers. They have different equilibrium distribution function. Then how do we look at non equilibrium that is the transport of heat for example. For that we will look at energy transport trough non equilibrium phenomena by solving the particle nature of energy carriers say Boltzmann transport equation. And parallely we will also look at some hand waving approach by using kinetic theory. And also we will show that using the particle based sub continuum Boltzmann transport equation we can take moments and therefore we can derive basically the macro scale equations.

Essentially, what I mean is that if you will look at this particular picture I still solve the Boltzmann transport equation here and here. But you see that the Boltzmann transport equation tends to the continuum equation. Therefore, it is automatically becoming a



Fourier kind of approximation in the macro scale limit. So, this is how you can show that you can derive all the macro scale equation from this fundamental equation. And we will also have a parallel treatment of energy carriers.

That means, when I say energy carriers this could be for example metals it could be electrons; electrons are the predominant energy carriers so of heat or charge electric current, and if you are looking at semi conductor, dielectrics, the predominant way of energy transport is by the vibration of the lattice structure, so that we will assign our virtual particle called phonons. And then if you look at liquids and gases this molecules, as I said the nano scale phenomena generally not observed in liquids its primarily gases the rarify gases, therefore these molecules we will mostly focus only on the gas flows, and that is why the kinetic theory is also apply.

With this I will like to stop the power point and then what we will do now is start making some derivations. Before we directly going to the nano scale energy transport we will start with the macro scale, because that is already known to you. We will derive the heat conduction or heat conduction convection equation first the macro scale. Then you understand what are the properties that are already should be known to you in order to solve this particular equation. And that is the starting point for your continuum solution. So, you assume that you already know certain properties and how do this properties come from. They come from the micro scale.

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Macroscale Continuum heat transfer

open system

Heat transfer

(ignore w)

Net efflux generated by  $dV = w(T_e - T_i) = w\delta T$

unsteady  $\int_{cv} \rho \dot{w} \delta V$  :-

$$\frac{dU}{dt} = \dot{q}$$

$$\dot{q} = -k \nabla T$$

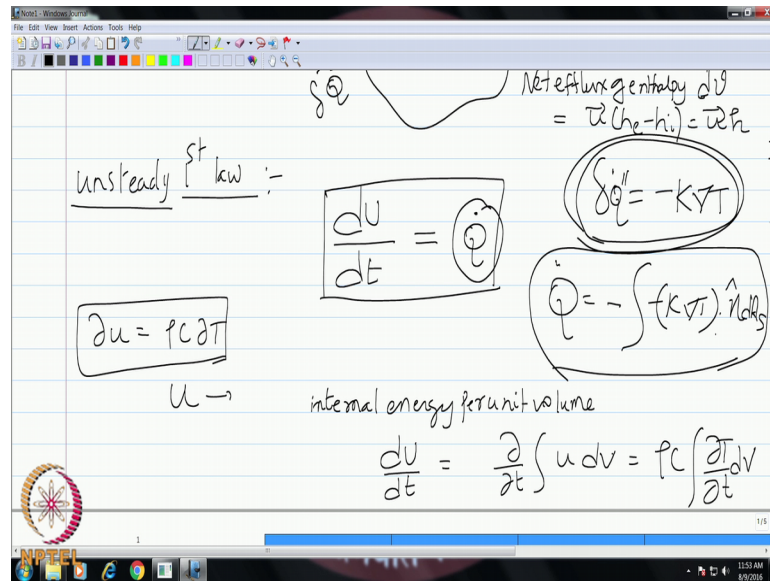
In order to make you little bit more aware about this we will first take the case of closed system. So, I hope all of you are able to see the projection, is that ok? So, we take a closed system some arbitrary system which is a not a control volume, because it is close to mass transport and then you take a particular differential control element or a control volume. And let the surface area of this differential control volume be  $dA_s$ , and the differential volume is  $dv$ .

Now the normal vector of this particular element is always pointed outward normally from the surface. This is your normal vector. And, in order to look at only heat transport, let us if you say energy transfer in general you can have both heat and work transfer. So, you can say that there is a particular quantity of heat which is flowing in to the system. So, this is positive and it is going in to the system. And certain quantity of work which is done by the system, so work done by the system is positive here.

But when you are looking at only pure heat transfer, let us ignore the work interaction. So, we will simply ignore work transfer that means there that is always; of course, you derive the energy equation very rigorously the influence of work on heat is definitely there. It could be in the form of viscous dissipate or pressure work. All these are coming from the work which basically increases the internal energy of the system. But in this case we now ignore those terms and derive only a pure heat equation without the effect of work.

Therefore, what will be the first law? First law of thermo dynamics, all this is now you know macro scale you can say this is your macroscopic or macro scale, continuum heat transfer. So, can you write down the first law for this system and then; so all of you should be able to write down the first law. Write down the unsteady first law not the steady state form.

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First law; that means, you have a time derivative walls. For example, ignore the macro scale kinetic and potential energy of the system, so then what do you have? Change in internal energy will call  $du$  by  $dt$   $u$  is the total internal energy of the system, so we are ignoring the macro scale, kinetic and potential energies. So, this should be equal to  $q \dot{}$ . So, this is for the entire control volume that we have represented, but the representation that we have  $q \dot{}$  here we can say this is  $d q \dot{}$  for example, for this differential element.

Therefore, we can integrate this for the entire control volume from the small differential amount of volume that to that we can integrate to the entire control volume. And we can also use the Fourier's law to describe the relation between  $q$  and temperature. In generic case how do you write this? Without fixed coordinate system, coordinates free representation.  $\text{Del } q \dot{}$  should be minus  $k$  into  $\text{del } t$ , so gradient of temperature. Obviously, you know why the minus sign comes here. So, this temperature gradient heat flows from higher temperature to lower temperature it will be negative. Since have convention is heat should flow from high to low, so we put a minus sign to make this quantity of positive quantity.

Therefore, now for the entire system or control volume all we have to do is integrate this over the surface area. This gives you the total heat transfer, and not only that it is a little bit you have to be careful with the normal that you are using so we dotted it with the

normal times  $dA \cdot s$ . So therefore, as you can see that the direction of your heat flow on the normal or opposite to each other, therefore in order to make this heat transfer positive in to the system we have to put another minus sign out. Because the normal is pointing opposite direction to the heat transfer, so we have to put a minus sign so that this quantity is positive because now minus  $k \Delta t$  will be positive, but dotted with normal will be negative so put in a negative sign in outside. So that that the heat flows in to the system is positive. So, this is on the right hand side. Now, therefore on the left hand side we will introduce internal energy per unit volume.

Therefore, how do you write this in terms of specific internal energy? So, let us call this  $u$  as your specific internal energy. This is internal energy per unit mass, therefore for the entire volume how do you write this capital  $U$  or let us say  $d$  capital  $U$  by  $d$   $t$ . Therefore, for the entire volume can be written as  $d$  by  $d$   $t$  of integral  $u$   $d$   $v$ , so here I am going to define slightly again I call this as internal energy per unit volume. Therefore, this can be written as  $d$  by  $d$   $t$  of; in terms of temperature we can use the relation between  $d$   $u$  will be  $\rho c_p$  or  $\rho c_v$   $d$   $t$  which we can use here to write in terms of temperature. I will just use  $\rho c$  and write this in terms of  $\rho c$   $d$   $t$  by  $d$   $t$ , and this is over the; so I am assuming  $\rho c$  is any way constant properties, so I can take it out said the integral and this is now integrated over the entire control volume.

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The image shows a whiteboard with handwritten mathematical equations. At the top, there are some faint symbols:  $\partial t$ ,  $\partial s$ , and  $\partial t$ . The main equation is:

$$\rho c \int \frac{\partial T}{\partial t} dV = \int k \nabla T dA_s$$

The right-hand side integral is circled. An arrow points from the circle to the text "Gauss divergence theorem". Below this, the equation is rewritten as:

$$\int \rho c \frac{\partial T}{\partial t} dV = \int \nabla \cdot (k \nabla T) dV$$

The whiteboard also features a logo in the bottom left corner and a taskbar at the bottom with the time 11:58 AM and date 8/6/2016.

Now if I put them together I have  $\rho c \int dT$  by  $dV$  is equal to I have  $\int \nabla \cdot (k \nabla T) dV$ , so I will just write this as  $dV$ . So, I can now use the Gauss divergence theorem to convert this surface integral to a volume integral. And therefore, this will be divergence of  $k \nabla T$  over the entire volume using Gauss divergence theorem. I am converting the surface integral to volume integral by introducing the divergence operator. Therefore, now this can be written as  $\int \rho c \frac{\partial T}{\partial t} dV$  is equal to this is also a volume integral, now this is also a volume integral  $\int \nabla \cdot (k \nabla T) dV$ .

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$$\int_V \rho c \frac{\partial T}{\partial t} dV = \int_V \nabla \cdot (k \nabla T) dV$$

$$\int_V \left( \rho c \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) \right) dV = 0$$

$$\Rightarrow \boxed{\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)}$$

Therefore, if you take this integral volume integral common you have  $\rho c \frac{\partial T}{\partial t} dV$  minus  $\nabla \cdot (k \nabla T) dV$  is equal to 0. And therefore, in this case in order to satisfy this particular condition your  $dV$  is not 0. You have finite differential volume here, so therefore the integrand has to be 0.

Therefore, this will go lead us from the integral formulation to the differential formulation. The differential representation that for now is  $\rho c \frac{\partial T}{\partial t} dV$  will be equal to  $\nabla \cdot (k \nabla T) dV$ . This is your the generic heat transfer or heat conduction equation for a closed system. This is a coordinate free representation, and you can use whatever coordinate system that you are working with, you can use the corresponding a gradient operator and divergence operator and that should be view the

particular the corresponding set of equations for that coordinate system. So, this is a generic coordinate free representation derives for an arbitrary control volume.

So now, this is a conduction equation because we have not considered any mass transfer here. Same thing can be extended to an open system there you also have a mass flux, in that case you are also write the continuity equation to balance what is the incoming mass and what is the outgoing mass. And there is also what we call as enthalpy  $p$ , which is advection by this mass. In the case of open system if you want to do the same apart from the; so we will just extend this in the case of an open system, because if you want to derive the convection equation so you have to do this for the case of open system and also you have to consider the mass transport and the corresponding transport of enthalpy due to mass. So, let us say you are the corresponding velocity entering of the fluid entering this particular control element boundary, this  $u$  vector and they corresponding enthalpy that is entering this particular control volume is  $h_i$  and you have a corresponding enthalpy of the fluid which is leaving this control volume let us say that is  $u$  vector  $h_e$ .

Therefore, we can introduce net efflux of enthalpy which is basically leaving this particular differential control volume will be  $u$  vector times  $h_e$  minus  $h_i$ , we will call this as  $u$  vector times  $h$ . So this is  $h_{net}$ , this is your net efflux of enthalpy which is basically leaving this particular differential control volume. Therefore, for the entire control volume what is the net efflux of enthalpy, it is the integral of this over the entire control surface. Because this is a flux is a surface related phenomenon.

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For the entire control surface the net efflux of enthalpy

$$\text{enthalpy} = \int_A (\vec{u}h) dA_s$$

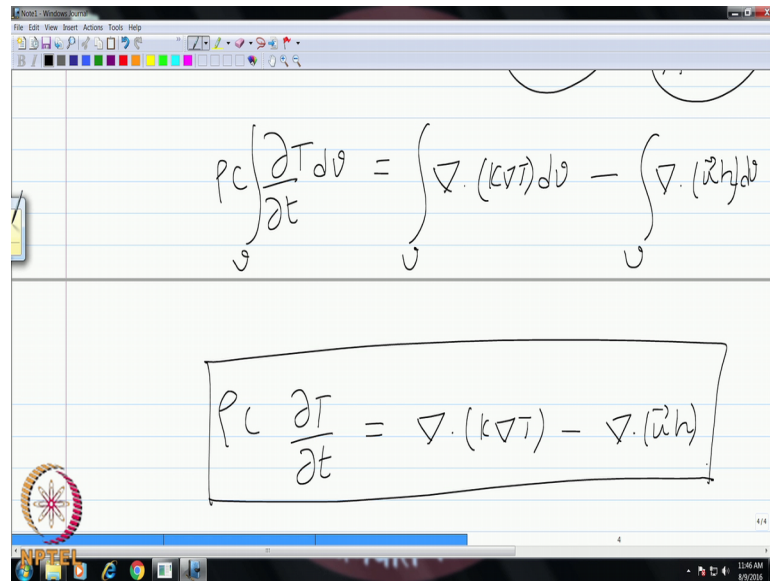
1st law :-  $\frac{\partial}{\partial t} \int u dV = \int_A k \nabla T dA_s - \int_A (\vec{u}h) dA_s$

So therefore, we will just extend this two the entire differential control surface so for the entire control surface. So, the net efflux of enthalpy will be; you have integral, you have  $u h$  and integrated over the differential surface area and over integrate it over the entire area, so that will give you the net efflux of enthalpy or with entire surface.

Now the first law will now get modify. The first law for the open system; if you want to write it down should now also include apart from the terms. So we had  $d$  by  $d t$  of integral  $u d v$ , and you also have heat transfer the conduction. So, we had integral over the entire surface area  $k \delta t$  differential surface area  $d A s$ . Apart from this now you also have the heat transport by the enthalpy transfer of enthalpy, so net efflux of enthalpy is also resulting in change in internal energy.

So that should come, because now the net efflux is actually leaving the system that should come on the right hand side with a minus sign. So, you have this integral  $u h$  in to  $d A s$ , is that ok. So, this particular term here is your net efflux which is leaving the entire control surface. Therefore, we have negative sign. This is purely the heat transport due to conduction. This is your conduction heat transfer and this is your advection heat transport.

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The image shows a digital whiteboard with a blue grid background. At the top, there is a menu bar with options like 'File', 'Edit', 'View', 'Insert', 'Actions', 'Tools', and 'Help'. Below the menu bar is a toolbar with various drawing tools. The main content of the whiteboard is a handwritten equation in black ink:

$$\rho c \int_V \frac{\partial T}{\partial t} dV = \int_V \nabla \cdot (k \nabla T) dV - \int_V \nabla \cdot (\vec{u} h) dV$$

Below this equation, the same equation is enclosed in a hand-drawn rectangular box:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - \nabla \cdot (\vec{u} h)$$

In the bottom left corner of the whiteboard, there is a circular logo with a star-like pattern and the text 'NIPTE' below it. The bottom right corner shows a system tray with the time '11:56 AM' and the date '8/27/16'.

Once again if you apply with Gauss divergence theorem to the right hand side and convert the surface integral to the volume integrals and then write this in terms of rho c d t d v. So, everything is in terms of volume integrals, so what do you have on the first term on the right hand side will become a volume integral and then becomes divergence of k delta t d v; minus you have again volume integral of divergence of u h d v, is that.

Now I can again club this together equate this to 0 and therefore the integrand has to be 0. So this will give me the differential equation to be rho c d t by d t is equal to del dot k delta t minus del dot u h. This is my conservation of energy in the presence of flow that is for an open system. Now I can also introduce the relationship between h and the enthalpy and also temperature.



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Incompressible (liquid)  $C_p = C_v = C$

$$dh = C dT$$

$$\rho C \frac{\partial T}{\partial t} + \nabla \cdot (\vec{u} \rho C T) = \nabla \cdot (k \nabla T)$$

Heat transfer  $\leftarrow \frac{\partial T}{\partial t}$  Advection  $\leftarrow \nabla \cdot (\vec{u} T)$  Thermal conduction  $\leftarrow \nabla^2 T$

$$\frac{\partial T}{\partial t} + \nabla \cdot (\vec{u} T) = \alpha \nabla^2 T$$

So, for incompressible substance now we make an approximation that the substance is incompressible. So it could be for example, a liquid or you can use a compressible substance like gas but under the incompressible flow approximation, that is your Mach numbers are extremely small in that case the substance can behave as an; although its compressible fluid practically the regime that your observing makes it an incompressible fluid.

In that case you can say that your specific heat capacity due at constant pressure in constant volume will be same. And therefore, your  $dh$  which can be written as  $c_p dt$  can also be written as just  $c dt$ . So, you use your relation this as  $c$  in to  $dh$ . So therefore, you can rewrite above equation purely in terms of temperature you get equation for temperature which will be of the form. So, we will take this just a moment, we take the enthalpy towards the left hand side, so makes this is an advection term. Therefore, plus you have  $\text{div } \vec{u} \rho c T$ . On this side you have  $\text{div } k \nabla T$ . If your properties are constant that is your  $\rho c$  is constant you can divide throughout by  $\rho c$  and you get your energy equation  $\text{div } \vec{u} T = \alpha \nabla^2 T$ ; so you can write this as  $\alpha$  which is your thermal diffusivity which is  $k$  by  $\rho c$  in to  $\text{div }^2 T$ .

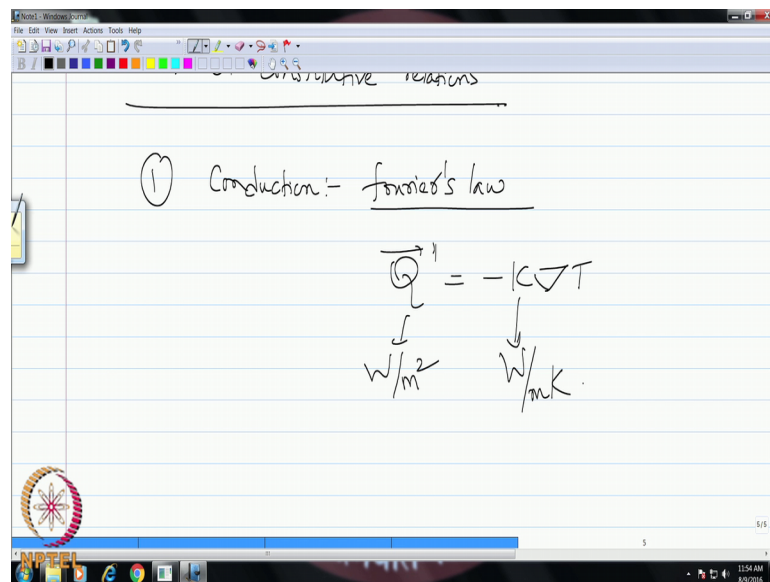
This is your heat conduction or this is your heat transfer equation. This is your advection term, and the right hand is your diffusion term. Once again this is in a coordinate free representation, so you can use it in any coordinate system that you want. So, the reason

that we have derived these equations which are already familiar which to make you understand that we have made certain assumptions here, for example; we made the assumption that your enthalpy  $p$  is related to temperature through the heat capacity, and also we have heat capacities coming here the internal energy formulation.

And therefore, you should understand that how do we basically fundamentally compute this property called heat capacity. How do we know this value? Where does it come from? Now in our equilibrium thermo dynamics we assume that this measured a known for most of the macroscopic substances, but then these are actually having a micro scale basis. So, you derive the equations to estimate the heat capacities from statistical thermo dynamics, and that that is what we will do slowly. But you should understand that even in the continuum equations the parameters or properties that you use have a micro scale basis and they come from, therefore understanding the micro scale heat transfer.

So, this is why we are starting with what we already know. All although you should understand that fundamentally we should start with micro scale heat transfer, know things like heat capacity before we derive the continuum equations.

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Now the next step that we will be doing this to quickly write down some of the continuum classical constitute of relations. So, we call this as Classical constitutive relation. So, why do we call this as constitute of laws, because in the above equations that we have written and we derived the conservation of heat. For example, originally

had the equation in terms of the heat transfer rate or the heat flux, so now we wanted to close the relationship between heat transfer and the temperature through this particular relationship here, and therefore we could express your  $q$  dot like this.

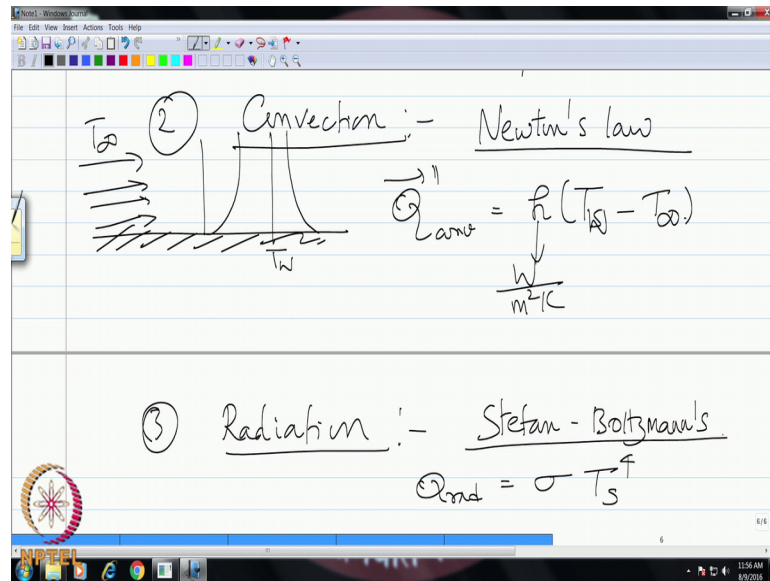
In fact, they should be at  $\delta q$  double prime this is your heat flux, and this is your heat transfer rate we multiplied by the differential amount of surface area. So, this relationship to close is required otherwise we will not be able to solve the energy equation. So, we need to express  $q$  in terms of temperature, now therefore the resulting equation becomes scalar transport equation in terms of temperature. So, this is called a constitutional of relationship. And although this has been known without very it has been originally proposed by Fourier, without any rigorous background we can also show from the micro scale laws these classical constitutive laws can be derived.

So, these are not some something which is been simply proposed without very strong fundamental understanding. Although, these were intuitively proposed Fourier understood that your heat flux has to be related to your temperature gradient definitely, and he introduce this constant call thermal conductivity. But, this is entire relationship can also be shown from the kinetic theory or also from the Boltzmann transport equation. We will do that you know several classes down the line, but these are called as constitutive laws and they are specifically used only in the continuum regime.

So we will see that number one this is your, if you look at conduction, the most commonly referred one is the Fourier's law. You have also, you should remember modifications to the Fourier's law, and we have what we called as hyperbolic variance. But the most commonly used one which makes the, what is the nature of the heat transfer equation in conduction; the unsteady conduction heat transfer it is parabolic kind of an equation. So, now there are variations to the Fourier equation which makes the c conduction equation hyperbolic. They call this is telegraphic equation. So, hyperbolic is typically a wave equation, so that is why they call this as a telegraphy equation. But let us not bother about that right now.

We have already used the relationship between your heat flux and temperature gradient which is stated in this particular way. And this is your heat flux in watt per meter square. And what is the unit of thermal conductivity here watt per meter Kelvin.

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Now, this is in conduction. What about in convection? Do you know if any other constitutive law that can be used in convection to relate the convective heat transfer to temperature? So, you have Newton's law of cooling. Which again states that you are  $q_{conv}$  are equal to  $h A (T_s - T_\infty)$ ; this is in terms of heat flux we just say  $h$  into  $T_s - T_\infty$ . So, you can imagine case of a flat plate flow over a flat plate, and you have velocity boundary layer and you also have the temperature boundary layer like this. In wall temperature this is your free stream temperature. You can either you  $T_s$  or  $T_w$  whichever. And when you go to radiation, so what is unit of  $h$ ?

Student: (Refer Time: 48:06).

Watt per meter square Kelvin. On finally when you go to radiation, what is that you want to use to Stefan Boltzmann's law. That means, your  $q_{radiative}$  is equal to how do we write it down; sigma into?

Student: (Refer Time: 48:46).

If you take a black body, let us first define it for a black body. It is just the surface temperature from the black body rights to the power 4. If you are talking about transport of heat between a black body and the ambient, then we can say  $\sigma T_s^4 - T_\infty^4$ . This is the net radiative heat transport between a black surface and the corresponded surrounding ambient. If this is not a black surface but a gray

surface you introduce your infinity. And for the black body you have basically a distribution of the energy spectrum  $b$  as a function of wave length.

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$\frac{W}{m^2/C}$

③ Radiation :- Stefan - Boltzmann's

Planck's law :-  $Q_{rad} = \sigma E (T_s^4 - T_0^4)$

$$e_{b, \lambda} = \frac{c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)}$$

So, in variation you have spectral quantity, that means the energy or intensity is a function of wave length and it is also a directional quantity. It is a different direction of this sphere or a hemisphere, the variation and gradation intensity is different, so it is a directional spectral quantity. Usually we average it over the directional hemispherical space, so we call this as total quantity. But still total spectral quantity, so if we look at the black body distribution function as a function of wave length we have the planks distribution function. You know when radiation is start we just state this distribution function but we should understand this is actually originally derived from the micro scale using statistical thermo dynamics.

So, this is related to your wave length and also what else, the temperature of a particular surface. You can plot this for different temperature as the function of wave length that is your planks distribution for black body. And, in fact if you go to nano scales, in the very recent years you can see that the intensity of the spectrum, the intensity of the radiation ammeter by the surfaces which are proximity of nana metes can exceed the blackbody energy density.

As I said these are the new phenomena that emerge at small scale. So, classical theory states that none of the body can exceed the black body distribution intensity, but at nano

scales it is found to exceed. So, there are several phenomena attributed to that call that they attribute this to photon tunneling for example, and even at smaller scale attributed to phonon tunneling and all that. The gaps become too small that tunneling effect start appearing, like in quantum mechanics. So, you should not think that these are all the time valid, so and they fail at very very small scale. These are valid only for the normal macro scale distribution.

So we will stop here, and then tomorrow we will also see look at some other constitutive relations in fluid mechanics mass transfer in electrical engineering. And then we will slowly get in to some physics behind the nano scale transport.

Thank you very much.