Micro and Nanoscale Energy Transport
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Lecture – 39
Nanofluid Heat transfer Part 1

So, very good morning, today we will look at another important subtopic which is on Nanofluids. Now when it comes to Nanofluids; I mean what is very specifically nano scale about Nanofluids? It is actually the big question mark and many people are trying to understand is there any deeply buried nano scale phenomena that can be studied and explaining the novel applications which describe the anomalous increase in thermal conductivity and viscosity and also related to convective heat transfer. So, how does a nano scale phenomenon play a role in explaining all these to Nanofluids.

Therefore, let us take this topic because you know Nanofluids are increasingly becoming a quite popular, to replace conventional coolants for lot of cooling applications because of their favorable thermophysical properties and also because of favorable heat transfer characteristics. But also it is to be understood that whether there is specific nano scale phenomena that plays a very important role here because when people treat Nanofluids mostly they treat it like a bulk macro scale bulk fluid continuum fluid this all the navier stokes equations. So, it is very difficult to understand what is the nano scale aspect of Nanofluid.

It should not stop with the physical synthesis of Nanofluid where you add nanoparticle; that is not the real nano scale phenomena right. So, from the materials point of view you make nanoparticles. So, from the material scientist point of view nano technology synthesizing nanoparticles, but from the heat transfer point of view this is not really nano scale. You mix two components and you have a Nanofluid. So, from our heat transfer point of view this now looks like a bulk fluid with a different set of properties. So, from heat transfer point of view what is really nano scale about this?

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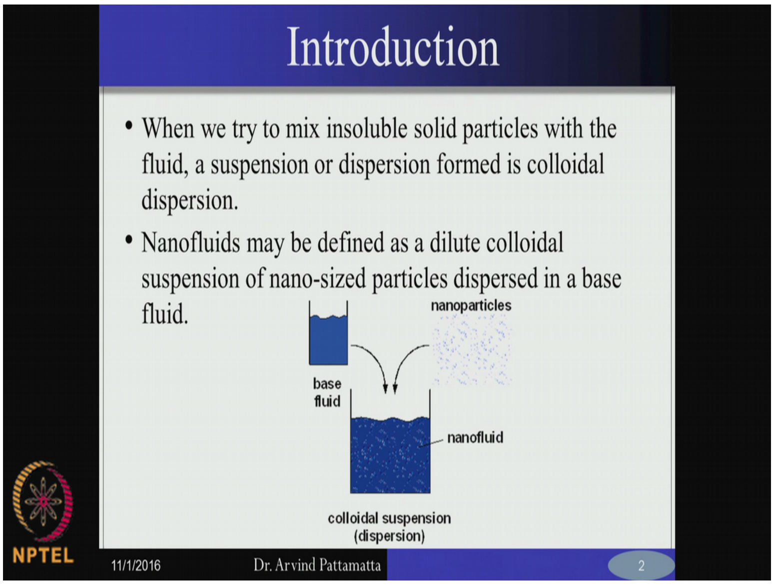

I think this is what we are going to see in the next couple of lectures and as many of you are probably have some idea and those of you who do not have it is a very simple process where you synthesize nanoparticles and you mix the nanoparticle with a base fluid. So, base fluid could be any conventional fluid or coolant it could be a plain water or it could be coolant like ethylene glycol which is used for example, automotive cooling applications. So, you mix this in the base fluid at very very small dilute concentrations; that means, you are talking about volume fractions that is the volume of nanoparticle to the total volume of the entire system. So, that should be less than about 2 percent or 3 percent maximum. From the volume fraction point of view this is really really tiny and this forms a colloidal a suspension.

One of the important challenge is in preparing synthesizing Nanofluid is how do you maintain stable suspensions of these nanoparticles. Because once you mix nanoparticle which are usually having a higher density than the base fluid because this could be metallic particles or this could be some oxides. They could have density which is definitely higher than that of the base fluid and gravities acting and all these nanoparticles.

If you do not do any special treatment and you simply add them, stir them and then you wait for few minutes you will see immediately all the particles are you know getting sedimented and you know there is no suspension actually formed. Therefore, one of the challenge is in preparing Nanofluids is to prepare a stable solution that is a suspension of nanoparticles. Therefore, there are different techniques to do this now we can also say argue that I can do this with even millimeter size particles. What is so special about adding nano sized or nano meter sized particles and preparing Nanofluid? With millimeter size particles have been there for a long time you know we call them colloids colloidal dispersion or colloidal suspensions have been there for you know may be seventy, eighty or hundred years. The only problem with that is when you are using it in place of a standard coolant your pressure drop will increase because of the large size particles number one, number two you have problem with Clogging, corrosion because these are really big particles; this can cause damage to the coolant tubes.

They are little bit more intrusive. Therefore, if you reduce the size of these particles; very important from the mechanical hydrodynamic point of view you are pumping power is not compromised. There is no big difference between the base fluid pumping power and the Nanofluid pumping power and also the other problems with respect to Erosion or corrosion; these are minimized with Nanofluids and also it is easier to suspend Nanofluids more than bigger (Refer Time: 06:21) nanoparticles bigger than comparatively larger size particles because of higher surface area to volume ratio.

Higher surface area to volume ratio means the surface tension forces are more stronger and we can also use certain means to therefore, the make the particle stable because one advantage is you have higher surface tension, but also this could lead to aggregation of particles. So, higher surface tension means you know it will try to push pull particles together and therefore, there may form you know Clogging or agglomeration of particles which is also not favorable. You need suspensions, but not agglomeration suspensions. You need independent particles suspended freely. This is the idea of synthesizing Nanofluids.

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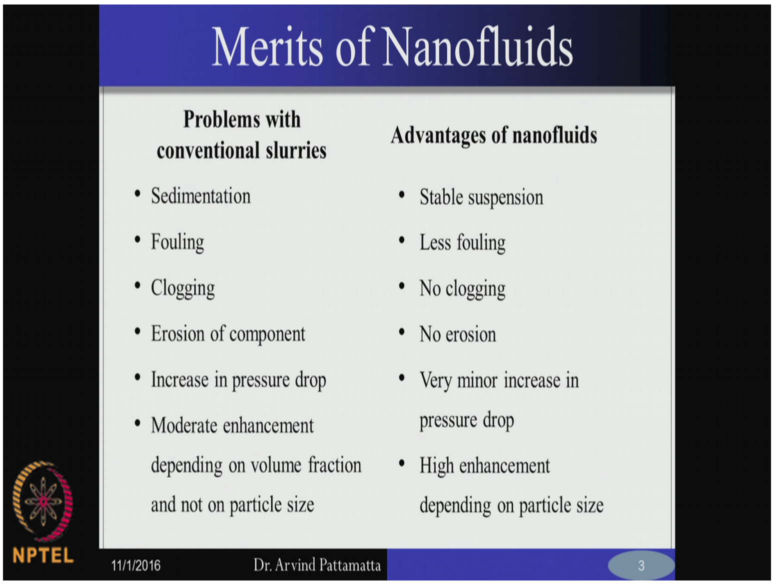

Therefore what is the difference between Nanofluid and conventional macro scale colloids; slurries? There are different terms chemical engineers refer to refer them as colloids and some people call them as slurries. One of the major disadvantage is Sedimentation; because if you the larger the size of the particle more prone to Sedimentation and when you use them in heat exchanger applications; you have problem with Fouling; that means, these particles can go and get deposited on the walls and over a period of time that will spoil the heat transfer rates between the fluid and the walls. It will change the profile of the wall itself and they can also lead to Clogging whenever used it (Refer Time: 08:04) filters. So, these filters can be of the size of few microns and if you use large scale particles they were simply go and block the filter passages.

Also very important point is increase in pressure drop. If you have very large particles you have to increase the pumping power to you know pump the same flow rate. Therefore, your pressure drop is going up and therefore, you can get probably higher thermophysical properties, but you have associated problems like this which make them practically difficult to implement with colloids, but still colloids are popular in certain sense people use them chemical engineering; but what is so attractive about Nanofluid? One is that it is easier to forms stable suspensions because of higher surface area to volume ratios. Therefore certain phenomena which occur at small scales can be made use of to create stable suspensions.

One of them is a concept of E D L; Electric Double Layer. Once you have higher surface area to volume ratio the E D L formation becomes very dominant and now you have the electrostatic forces which come into picture which can be used to keep the particles stable and also prevent the agglomeration of particles. From other point of view you can also say that the Fouling is less, there is no Clogging, no Erosion and almost hardly any change in pressure drop compare to the base fluid.

Therefore there are so many advantages in favor of Nanofluids. That is why people increasingly you want to use Nanofluids instead of the base fluid as a coolant and where are the applications when you say coolants? Practically anywhere where you have a conventional cooling fluid we can replace that with Nanofluids because of that favorable thermo physical and heat transfer characteristics.

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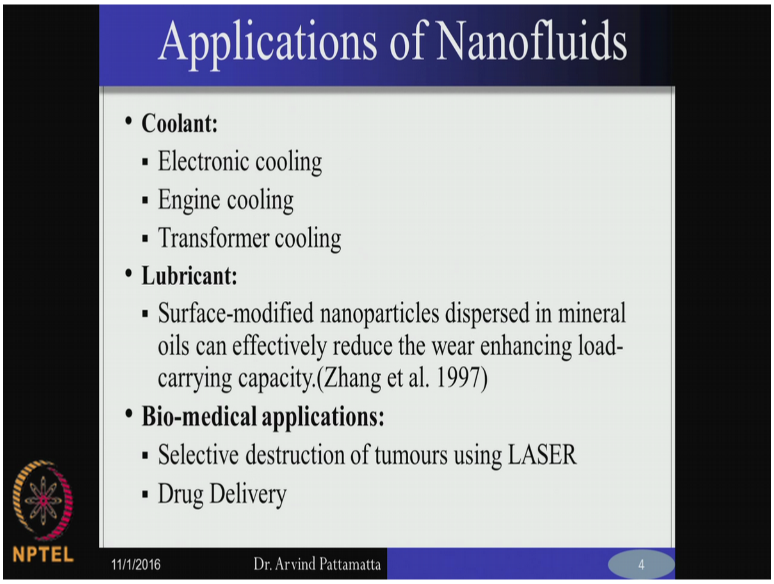

For example, you can talk about Electronic cooling, Engine cooling, Transformer cooling. These are three diverse areas. One is electrical the other is electronic, the other is mechanic. You can also talk about lubrication also is some people have been trying all these are kind of in the research stage; you do not have practical system available in the market where you can buy engine coolant with nanoparticles. Still we have not got that because one thing is synthesizing nanoparticles on such a large scale and creating stable suspensions for several months or years is also still a challenge.

On a laboratory scale you can synthesize Nanofluids and try to maintain stability for days or weeks or may be a month. But on a large commercial scale it is still yet to take off. We also have if you just refer to nanoparticles you have lot of biomedical applications.

One is selective destruction of tumours using lasers. What you do is you send the nanoparticle in to the body; these nanoparticle can be coated with some chemical which can actually go and detect the where the tumour location is it can actually go and target. This is also called targeted drug delivery. Usually what we do in chemotherapy? You burn off lot of healthy cells along with the cancerous cells that actually sometimes becomes worse than having cancer itself. So, the most important therapy is targeted drug delivery; that means, you target your drug exactly to the point where the cancerous cells are present or the tumour is present. So, nanoparticles are a very good way because the size of these nanoparticles is comparable to the size of the cells. So, therefore, you can coat these nanoparticles with appropriate reactants or chemicals and also drug which can go and detect the tumour, get struck to the tumour and then the drug is released.

So, exactly you can do that or there are also other ways like you send the nanoparticle and you only detect the tumour and from outside you know where the nanoparticle is then you put the laser exactly at the at spot; the nanoparticle will be a good volumetric observer of radiation. It observes and locally it releases the heat and destroys the local cancerous cells. Therefore, either it can be an active agent; it can deliver and it can react or it can deliver and you can have an external laser to irradiate and then you can (Refer Time: 13:14) the local tumour. There are different ways of doing this, but increasingly in medical applications, in cancer research nanoparticles have become very attractive.

So, much more than the application in coolant; they have found very attractive applications in cancer research. So, how do you prepare Nanofluids?

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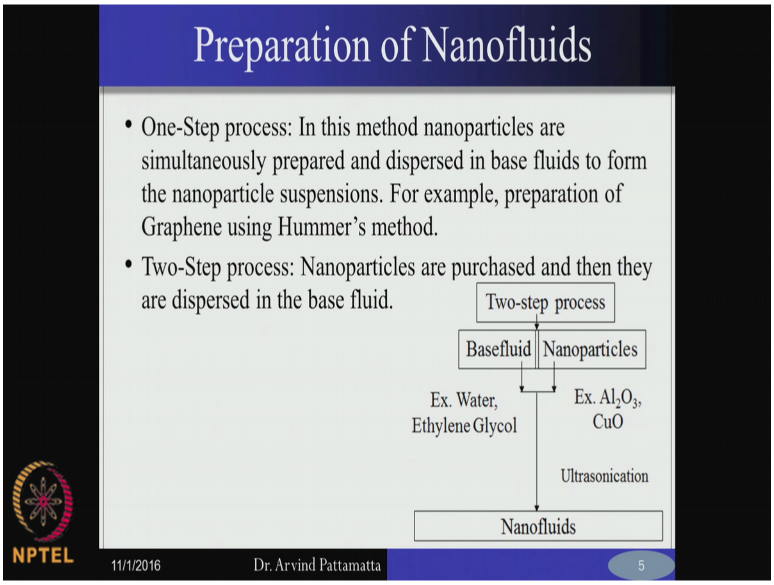

There are basically two ways. One is called a One-Step method; in which you directly synthesize the nanoparticle within the base fluid and then you prepare the Nanofluid in one step; that means, you synthesize and form Nanofluid together simultaneously. One example is how do we prepare prepare Graphene Nanofluid. So, Graphene Nanofluid is prepared by chemical process this chemical method is called Hummer's method or there are also other techniques. So, in a Hummer's method you prepare Graphene oxide in whatever in as a suspension in base fluid could be ethylene glycol or Water and from Graphene oxide it actually reduces to Graphene using a chemical a treatment or a reduction process.

All this happens within the base fluid and finally, you have Nanofluid prepared along with the synthesis process. So, this is a completely One-Step process. You mix your synthesis and Nanofluid preparation together. This is a very simple approach and usually this is done if your Nanofluid is prepared by chemical synthesis, but if you are doing it by mechanical or physical process like chemical vapour deposition or you actually break down bigger structure in to smaller particles using mechanical process and prepare nanoparticles like metallic nanoparticles for example. In that case; the nanoparticles is separately prepared you buy it from the market and then you mix it with your base fluid and then you prepare the stable suspensions. Then they become a Two-Step process.

Therefore, the first step is synthesis of nanoparticles, second step is you mix nanoparticle with a fluid and then you do what is called as a sonication; that means, you basically stir it at very very high frequencies; ultra sonic frequencies for considerable amount of time so that as much as possible your suspensions become very stable and they do not sediment. There are also chemical ways of making it stable like modifying the Ph. The other attractive way is looking at the potential of an E D L. Therefore, how do you understand whether the Nanofluid created is stable or not?

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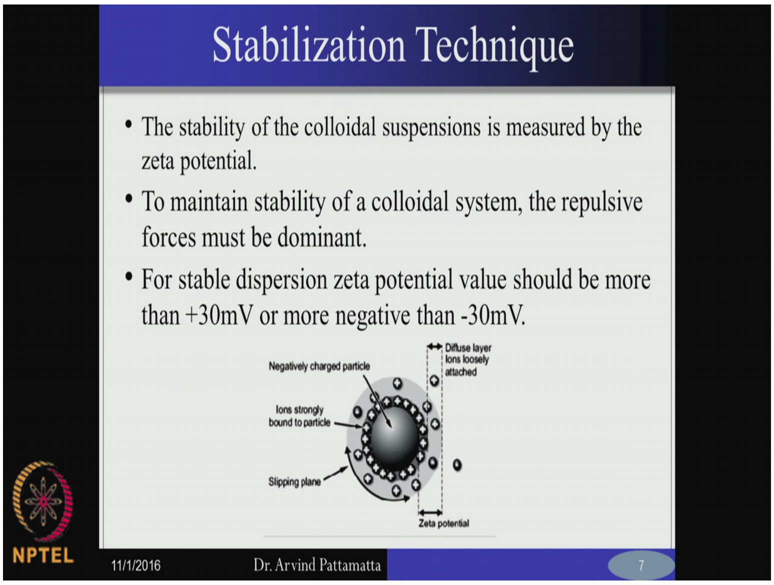

One of the simplest ways is to measure the zeta potential. Since now we are talking about nanoparticle. Now you can imagine your electro osmosis and electrophoresis process. Now, this particular particle will have a particular charge at the surface. Suppose if this has a positive charge; this will attract all the negative ions from the base fluid and this will form a stern layer and diffuse layer and how do you define zeta potential? How do you in general case in any electro kinetic process how do you first define zeta potential? It is the excess potential which is there at the edge of the stern layer and that is excess potential.

Far away from the particle; in the base fluid it will be excess potential will be zero. It is essentially the excess potential at the edge of this stern layer. There is again a potential which is existing at the wall due to the stern layer that will be even higher, but we are only measuring at the interface between the stern layer and the diffuse layer. That will be the zeta potential. If you measure this zeta potential; zeta potential should be considerably higher; that means, if the wall is positively charged then this value can be of the order of you know plus 30 milli volts are larger or if it is negatively charged you can have minus 30 milli volts are larger or smaller if you look at from negative sign point of view.

Therefore, it should be considerably large enough so that this zeta potential will be a repulsive potential right because if you have a considerably large zeta potential of one particle; the other particle will cannot simply come and form agglomerate. This large zeta electrostatic potential will repel these two particles because they have like charges. Therefore, many very important criteria when you first look at stability of Nanofluid system is measure the zeta potential. You take some nanoparticles, measure the zeta potential and if these values are greater than either plus 30 milli volts or minus 30 milli volts and larger magnitude that means, these are quite stable suspensions. They either lightly to form stable sustentions, if this is between minus 30 and plus 30 milli volts; then you have trouble. You cannot find stable suspensions. This is either minus 10 milli volt or plus 10 milli volt or less may be 5 milli volts plus or minus; that means, you cannot really whatever you do you cannot form stable suspensions because they can come together and agglomerate, so this one example.

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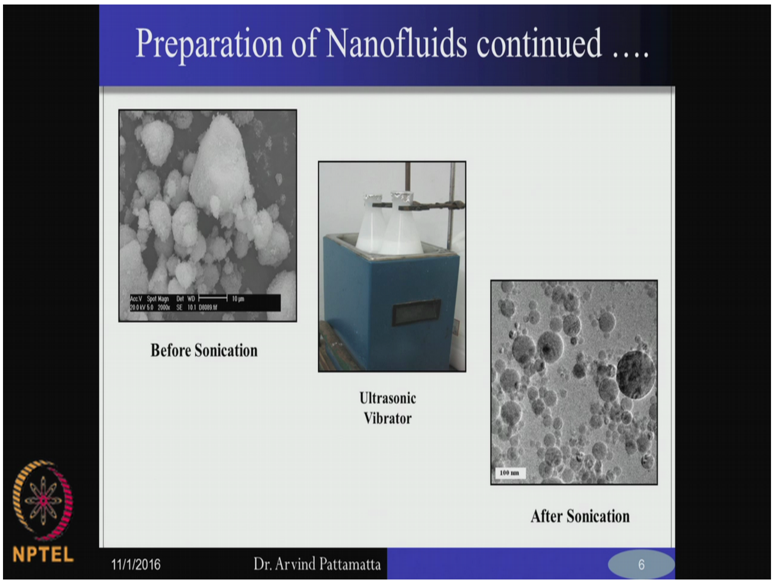

If you just disperse the nanoparticle in base fluid more likely because they will all come together and form agglomerates as you can see on this figure on the left side. These are big agglomerates of nanoparticles. Typically their values of zeta potential are low. Now, when you put this under the Ultrasonic Vibrator this process is called Sonication; where you induce ultrasonic frequencies and you cause stirring or vibration of this nanoparticles; what you indirectly do is basically try to increase the zeta potential.

One thing is all these big clusters get broken off number 1, number 2 you are aiding in the formation of a very good diffuse layer; due to which the zeta potential value also considerably goes up and then finally, after Sonication that if you sonicate for several hours then you look under the microscope. These microscopes are very higher resolution microscope. They can be either scanning electron microscope or transmission electron microscope because they have to resolve of the order of few nanometers. You cannot put it under conventional optical microscope and see them. The field of resolution is of the order of few nanometers. You have to use either scanning electron microscope or transmission electron microscope. These are such images; you are sonicate them and then you see. then you see then you find the suspensions have broken down in to smaller size and they are kind of distributed fairly uniformly; they will not the perfectly uniform because there is a size distribution.

You have may be sizes of nanoparticles from few nanometers to few 100 nanometers. Therefore, when you buy nanoparticles the supplier will always give you what is the average diameter of the size of the nanoparticles and what are the range about which you can fit a Gaussian curve minimum maximum and the mean. You can never find all the nanoparticles to be of the same size and shape. There will be some distribution; size distribution and also shape distribution. But if you look at typically metallic nanoparticles and oxides they are more or less of spherical nature.

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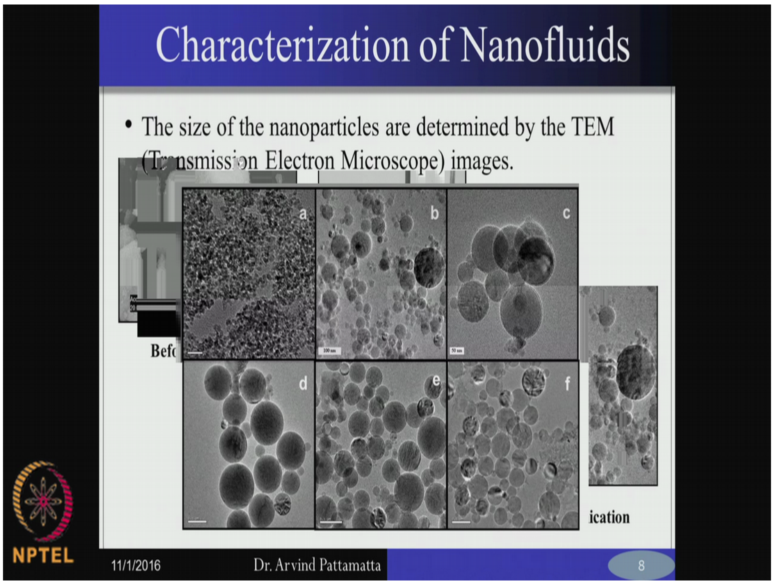

This is also showing some more images if you put this under Transmission Electron Microscope; how the different spherical all these are spherical nanoparticles how they look depending on the kind of sizes. For example, the first image a shows this resolution of the order of 20 nanometers. You can see that these are really small nanoparticles smaller than 20 nanometers where as there are particles which could be of the order of 100 nanometers which is there, image b.

There are distribution which is between few nanometers and 100 nanometers you can see in the image b; there are particles which are of the order of 100 nanometers some which are much smaller of the order of few nanometers. Usually you have this size distribution and you will be mostly working with the mean value of the nanoparticles. Now, moving on to the thermophysical properties once you achieve a stable suspension by Sonication and you stabilized based on the zeta potential.

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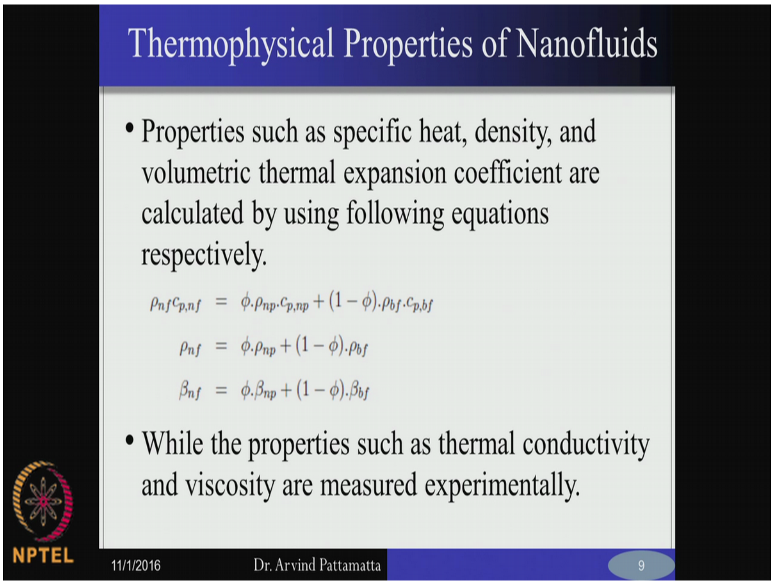

Now you go ahead and you treat it like one fluid and you want to now measure the effective properties. You have actually two components in this; the nanoparticle and you have base fluid, but for all practical purposes you are now going to deal with one homogenous fluid with effective properties which is taking the values of properties of both the nanoparticle and the base fluid together. We defined effective properties of Nanofluid and how do we estimate this effective properties.

If you take properties such as specific heat, density and volumetric thermal expansion they are simply volume averaged. That means, you take the volume fraction you measure you know how much volume fraction of nanoparticle it is multiplied by a corresponding to property plus the volume fraction of the base fluid multiplied by a corresponding property. It is a simple volume averaging which will give you the effective property of the Nanofluid. These three thermophysical properties are fairly straight forward to estimate; hat is heat capacity, density and thermal expansion coefficient, but the two other properties; one hydrodynamic property which is viscosity the other property of heat diffusion which is conductivity.

These two are more difficult; you cannot simply do a volume average and say the effective thermal conductivity will be volume averaged or effective viscosity will be volume averaged.

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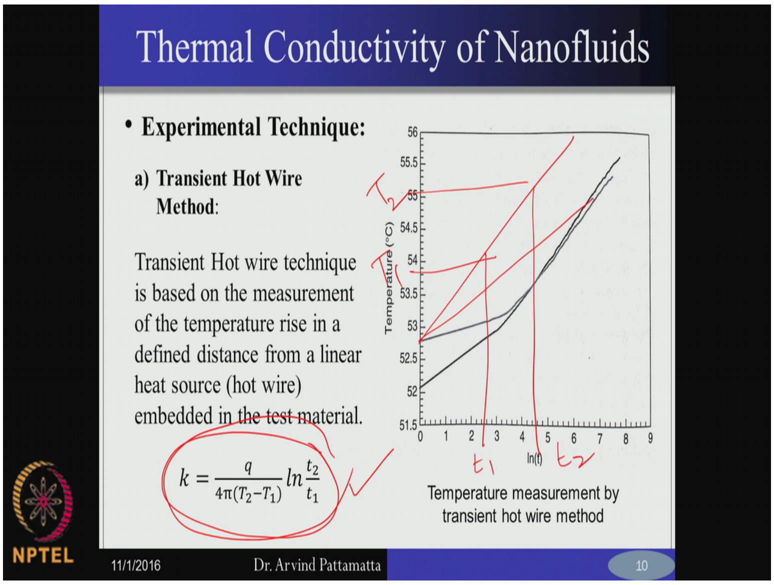

These two are more rigorous and we use specific techniques to determine the thermal conductivity and viscosity. When it comes to thermal conductivity one of the most popular methods to measure the thermal conductivity of any fluid it can need not be even Nanofluid it can be any fluid is the transient hot wire. The Transient Hot Wire technique is nothing but it is actually a line heat source if you.

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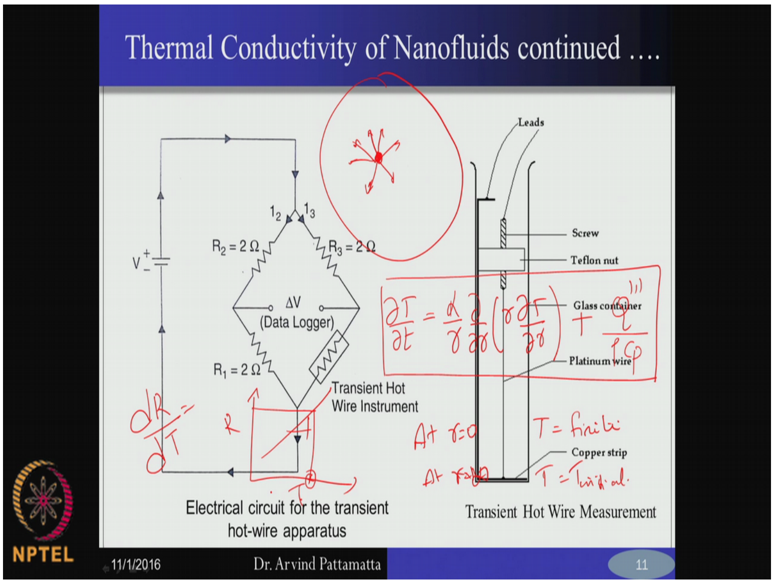

I will just show you the measurement here; you have a container which is filled with the fluid whose thermal conductivity you have to measure and the center of this container you have a platinum wire, you suspend a long platinum wire very thin, so the diameter is very small and the length is substantially large through which you pass current. This will be a local heat source and then you connect this end, this end of the other end of the platinum wire to the one of the as one of the resistances for a Wheatstone Bridge. The Wheatstone Bridge by default is a balanced bridge with 4 arms, 4 resistances. Under balanced conditions there is no net voltage that you measure. One of the arms go unbalance that is due to the change in resistance. You can measure the voltage and we can calculate what is the unbalanced resistance. This is the exactly the principle used in the transient hot wire. So this transient technique; that means, you solved the transient heat conduction problem.

When you measure this you pass current trough the platinum wire and this platinum wire is usually chosen because of the higher temperature coefficient of resistance; that means, the dependence of electrical resistance on temperature is very high. It is quite sensitive to small changes in temperature. When you pass current; this is a point heat source; f you write down the heat conduction equation in cylindrical coordinate system you can consider this as cylinder. This is your point heat source and from which the heat will diffuse radial. You can treat this like a small cylinder with heat source; you can write down the heat conduction equation; transient heat conduction equation which will be d T by d t is equal to 1 by r d by d r r d T by d r. Assuming only one dimensional heat conduction in the radial direction plus you have your heat source you have alpha. This is the equation governing this particular problem.

You can apply boundary conditions that is at r equal to 0 that is at this point in the somewhere on the wire. The value of temperature should be finite and since this is a transient method you do this experiment only for may be 2 to 4 seconds very short time. The heat will not be able to penetrate to very large radius. You can assume that at r equal to R which could be the size of the complete container itself. It is like a semi infinite media so that other end temperature is equal to the initial temperature or r going to infinity; T is still initial temperature.

You apply the boundary conditions. You will be able to find out an expression for temperature and therefore, thermal conductivity. So, that expression turns out to be of the form given here. This is your expression for thermal conductivity as a function of time and temperature. This is coming from solving the transient one dimensional heat conduction in cylindrical coordinates and what it tells you is if you measure the temperature at two different time intervals. The time could be t 1 after certain time t 2 you measure temperature T 2; capital T 2. Based on these values and you know how much of heat is actually supplied to the hot wire the hot wire is the platinum wire here you can find out the thermal conductivity of this.

Therefore, in this case the platinum wire works both as a sensor; that is itself is a thermocouple because from the change in the resistance in the Wheatstone Bridge you can actually calculate what is the actual value of this resistance. You can measure the voltage and the voltage is not zero you know that this resistance is changing because the balanced condition will be the case where there is no liquid you keep the hot wire outside at that temperature you balance the Wheatstone Bridge.

Once you suspend this you pass current and you suspend this in liquid, what happens? The wire starts cooling down; depending on the thermal conductivity of the liquid if the thermal conductivity is high the cooling rate will be higher so that means, the increase of temperature with respect to time will be lower the slope will be smaller. So, depending on that there is a change in the resistance electrical resistance; you measure the voltage and you know what is the value of electrical resistance and since you know the resistance versus temperature dependence because you know d R by d T the coefficient of electrical resistance is a function of temperature from which you can determine what is the corresponding value of temperature. You know d R by d T; you know the value of R. Therefore, if you plot r versus T; suppose this is the linear relationship you have a constant value for the slope which could be something. You know for this value of resistance therefore, corresponding value of temperature. So, like this you plot temperature as a function of time. So, it is nothing, but resistance at the function of time and this resistance is changing because the wire is getting cool down you are continuing to supply heat at the same time the heat is being dissipated to the fluid because of the thermal conductivity.

Therefore, the increasing temperature is determined by the thermal conductivity or the fluid. For a fluid with low thermal conductivity you will find that this will increase quite rapidly; something like this. For a fluid with larger thermal conductivity it will be little bit gradual. So, once you get the temperature versus time; you substitute a two different time intervals you take the corresponding value of temperatures that is t 1 capital T 1 here you have t 2 capital T 2; so substitute into this and find out what is the thermal conductivity.

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That is what. This process itself is a very fast. This measurement will be over in above to 2 to 4 seconds. The time here; the actual time will be maximum 4 seconds by this time there will be no significant natural because natural convection takes long time to get establish the experimental time is much faster than the natural convection time scales. We can safely ignore the effects of natural convection. This is one of the most common methods for measuring the thermal conductivity of fluids any fluid using a transient process there also study state measurements so, but you are going to use that transient measurement for also Nanofluids.

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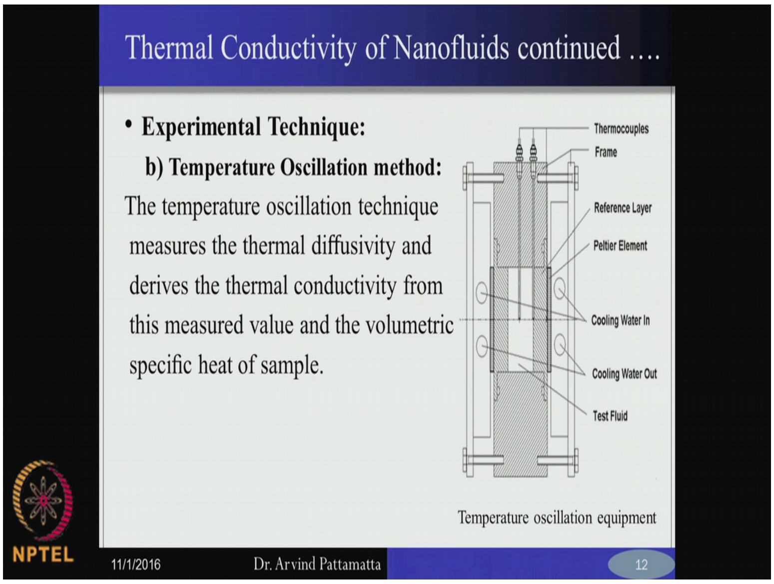

What you have to do is simply replace this fluid with Nanofluid do the same process to estimate the thermal conductivity. Therefore, this is one method the other method makes use of Temperature Oscillation that means, we have heater which will give you a sinusoidal variation in heating and therefore, temperature rise which is correlated to the corresponding change in temperature elsewhere. The heater temperature changes as a function of frequency sinusoidal say for example; that effect is filled by the fluid somewhere could be at the center of the container.

We measure the temperature of the fluid somewhere and we look at the temperature phase change between the heater and the fluid somewhere in the center of the container and this will correlate with the thermal conductivity. This is called Temperature Oscillation method. Now, the difference between these two methods is that the Transient Hot Wire gives you directly value of thermal conductivity which is the straight forward way the other is the Temperature Oscillation method gives you the thermal diffusivity thermal diffusivity is alpha. In order to get thermal conductivity you need to also know what is the heat capacity and density; that can be fortunately obtained from a simple volumetric average, but for fluids again you have to use techniques to measure the density and heat capacity. This is all require to get thermal conductivity therefore, this is a slightly indirect method.

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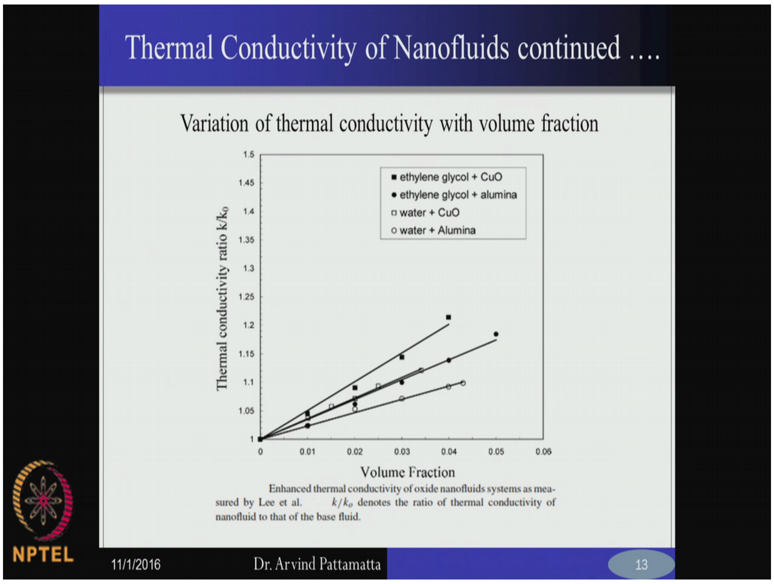

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What is beta? This beta value is used to understand the dependence of density on temperature. Preferably they are use to natural convection. So, if you are dealing with force convection where you do not have much property changes, density changes. Then you do not have to use beta, but in natural convection; the density changes as the function of temperature and so that is nothing but the volumetric expansion coefficient.

Now, after measuring the thermal conductivity if you plot this as a function of the volume fraction, volume fraction is basically the volume of nanoparticles to the total volume. Typically these are very small values you are talking about 0.01, 0.02, and 0.03; these are the typical volume fractions. If you look in terms of percentage this could be something like 1 percent, 2 percent, 3 percent, 4 percent not more than that maximum 5 percent.

Usually we do not go beyond five percent because problems with creating a stable suspension increases and then corrosion you know pressure drop slowly they will also start appearing. Here you see the two different Nanofluids in fact; we have four different depending on the base fluid also. We have copper oxide ethylene glycol; we have alumina ethylene glycol, copper oxide water, alumina water. We have two different base fluids and two different nanoparticles; one is copper oxide the other is alumina.

We can say out of all these ethylene glycol copper oxide gives the highest enhancement in thermal conductivity. For example, at 4 percent; you have thermal conductivity enhancement of 20 percent; whereas for alumina water that gives the lowest; not more than 10 percent. The enhancement totally depends on the basic property of nanoparticle and the base fluid. So, ethylene glycol itself has a higher thermal conductivity than water. Therefore, if you use ethylene glycol and prepare nano suspension the same nanoparticle you put it in ethylene glycol will give you a higher effective conductivity than in water. And in water also you can alter the enhancement by using different kinds of nanoparticles. For example, copper oxide definitely with water this having a higher enhancement compare to alumina. If you go to other nano particles like carbon nano tubes and graphenes;

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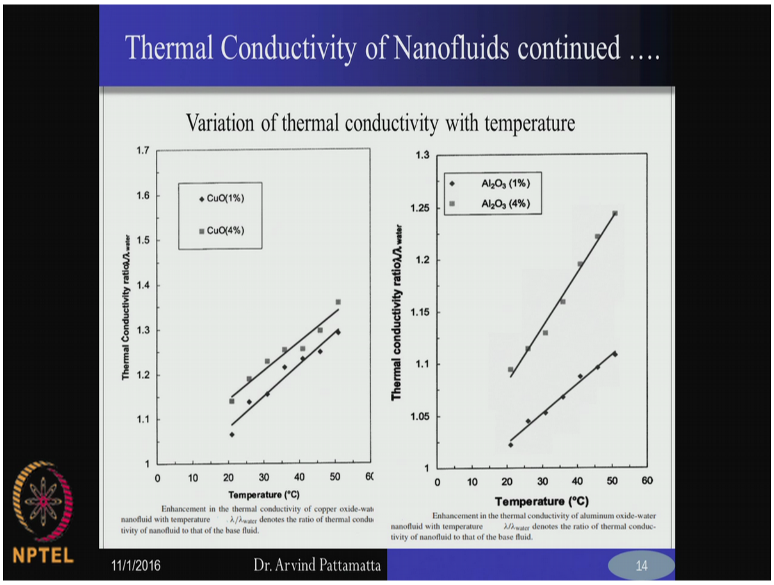

They will give you enhancement even better than these oxides. So, not only the variation with volume fraction, but for a given volume fraction if you measure the variation with temperature you find it is a very strong function of temperature. Both the alumina and copper oxide you see that if we increase the temperature of the Nanofluid from say 20 degrees to 50 degree; you see considerable enhancement from 20 percent to close to 40 percent for copper oxide and for alumina you have enhancement from 10 percent to up to 25 percent. They show very strong dependence on temperature. Why?

This is where the nano effects come in to picture. We can explain the temperature dependence of Nanofluids very strong temperature that much more than what the base fluid exhibits. Because you know that for liquids if we increase the temperature what happens to thermal conductivity.

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Decreases; for gases it is the opposite. That means, for this if this was purely liquid; thermal conductivity should have gone down, but on the other hand we have seen that this is increasing quite considerable. So, completely reverse effect and also acceleration. What is the reason for this? This is where the nano scale effect comes. One is what is called the Brownian Motion. Once you have very small particles suspended as collides; according to what Einstein has proposed; these will exhibit random motion in the base fluid and this motion velocity is a function of temperature and inversely proportional to the diameter of the particle the smaller the diameter higher the temperature more is the Brownian Motion.

Once the Brownian Motion increases what is happening? It is actually kind of stirring the liquid. This will increase the heat transport although from a macro scale point of view the liquid looks static. If you go into the nanoparticle you will actually seen nothing is static it is just always continuously moving around and this increase in the kinetic energy is contributing to thermal conductivity. This explanation people initially could not understand when they simply mix particle and saw the variation with temperature it was quite rapid.

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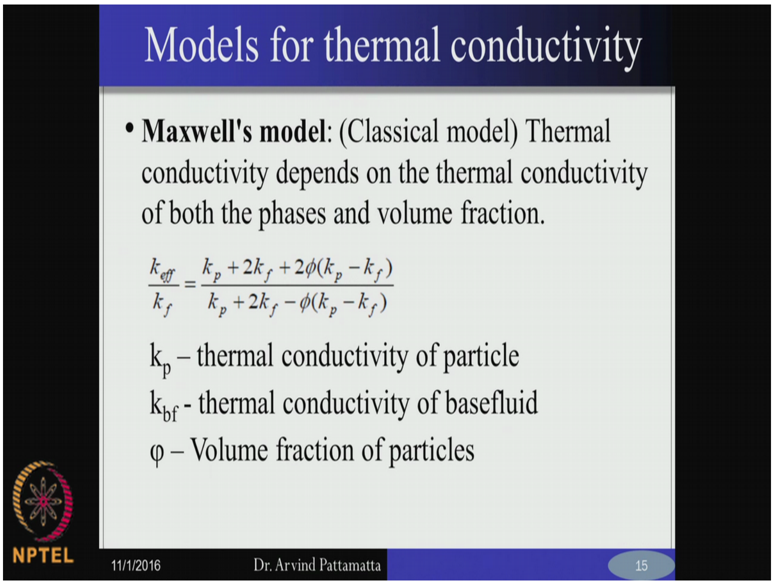

Later on then they related this to the Brownian Motion. Therefore, there are different models which can be use to explain the increase in thermal conductivity as a function of volume fraction. One of the most basic models is call the Maxwell's model. This is also called the Classical model which gives you if suppose you have thermal conductivity of the nanoparticle and the base fluid you can directly estimate what is the effective conductivity of the Nanofluid from this expression; that means, if your phi is equal to 0; your k effective will be equal to k f if phi equal to 1; the k effective will be equal to k p. It is like slightly modified form of the volume average.

We will kind of stop here today; tomorrow we will look at the other models which are used for particles of different shapes because the Maxwell's model cannot be use to explain thermal conductivity enhancement for non spherical particles. In that case there is a modification and also for particles where the Brownian Motion is not very significant for example, then how can it explain dependence on temperature? This does not have any dependence on temperature.

Therefore, the more advanced model should account for dependency on temperature as well as the shape effects. Those things we will look at it tomorrow. So, tomorrow we will complete this and also some aspects of the heat transfer enhancement.