Micro and Nanoscale Energy Transport
Dr. Arvind Pattamatta
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 32
Single Phase Liquid Flow and Heat Transport in Micro channels Part 1

So, that is 4 by 3 pi a b c divided by 2 pi by l the whole q. And what are; a, b and c many of you are taken a b and c as k x, k y, k z. How can that be because, k x, k y, k z is the respective coordinate in the x y and z axis, where as a b c refers to the maximum lengths, in the corresponding you know, coordinates? So, for that you have to put the given equation right the given dispersion relation in terms of that ellipse equation you understand. So, x square by a square plus y square by b square plus z square by c Square equal to 1 of that form then you will get what is a b and c.

And once you write your number of states is a function of this. So, directly you will have this in terms of energy. So, all you have to do is differentiate this with respective energy. So, that will give you d n by d e. So, it becomes very easy to work out once you get n as a function of e, directly you find d n by d e divided by l cube and you will get your density of states. So, always density of states we as expressed in terms of energy not in terms of k. So, that I think was slightly tricky problem you are I gave you hint also, I think you should have heard about it. So, apart from that most of you could solve the other 3problems, related to the Boltzmann transport equation and quantum, quantum dot those where all fine.

And many of you used in the third problem the atomic weight of the oxygen. So, I have asked you to calculate everything for a molecule. So, many have you took atomic weight as 16 and you have calculate it mean free path and other thing. So, there you got one I think the second problem was the little bit more trickier one. So, the approach is quite different from what we have attempted in class, I wanted to see how many of you can do that this is slightly out of the box thinking, because the other once are more or less you know been talked especially the forth problem right, and you already know the first problem how it should come out the answer should come out. So, many of you have not really attempted to apply the boundary condition. I am sure that mean these are how the quantum numbers turn out to be, but you have any way reduced that this should be the right answer.

So, therefore, you got it n pi by l and l m and n, but where you went wrong you thinking were required was to calculate the degeneracy of the first four energy levels. So, many of you have just used l equal to 0, 0, 1, 1 you know 01, 01, 01 like that. So, how can quantum numbers be 0 in the case, of quantum dot in the case of the confinement? If your quantum number is 0 there is no wave function; that means, there is no particle inside the box, which does not make sense. So, there is definitely a particle we do not know only where it is inside the box, what is the exact location. That also if you calculate based on the wave it turns out to be middle of the box most likely probability, but the degeneracy part was a tricky one, also you are supposed to know what are the values of energy first. So, if you substitute 1, 1, 1, you know that that is the lowest value of energy and then now the next question comes. So, you have 1 to 1, 1, 1, 2, 2, 1, and 1, for this you have the next higher value of energy. So, after this what is the third level 1, 2, 2? I think many of you have gone to 3, by that time by the third or fourth energy level you have already gone to 3 even before going to 3. So, you will get different values of energy which is higher than the previous one.

So, that is what you have to say first 4 energy levels is, first 4 consecutive values of progressive values of increasing energy and the corresponding degeneracy. So, these are the things which require little bit different thinking from, what you have done also in the assignment. So, in the open notes examination, so, generally you are tested for problem which is slightly different from the approach we take in the class, because you do not have to memorize anything for sure. So, therefore, the emphasis on thinking little bit different and make sure that we test your understanding thoroughly.

Anyway, so, now, in the remaining we have about ten classes are. So, we have another three to four weeks right and. So, what we will do is look at the different forms of the micro scale energy transport. So, we have in the last at couple of classes we talked about the nano scale or micro scale gas flows, a gas flows in mini and micro channel. So, in that case we talked about these slip boundary conditions, and then solved few examples for which analytical solution can be determine to get the velocity profiles and similarly temperature profiles so. So, therefore, that in that case you are thinking whether a proper continuum can be used if continuum equations can be used in the bulk transports. So, what about near wall, so, in the near wall we talked about the velocity and temperature jump conditions.

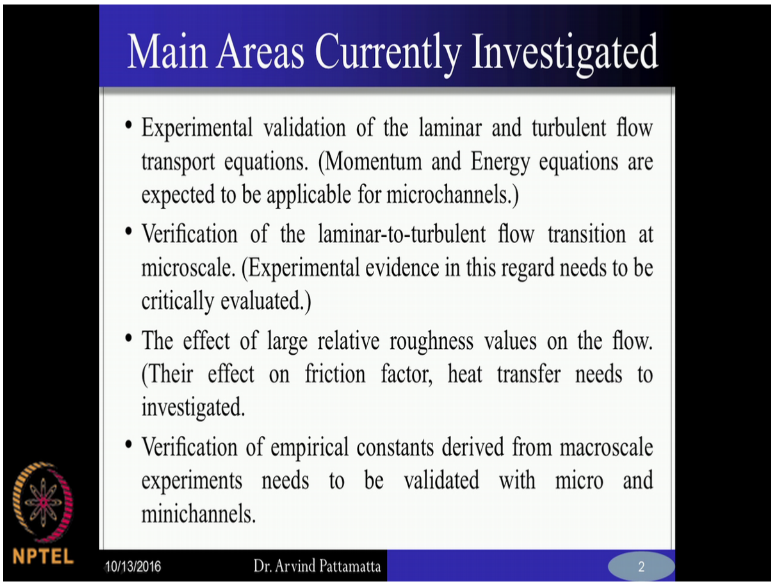
So, this was given by the Maxwell velocity profile, as well as the Smoluchowski temperature jump condition. And where as if you talking about Knudsen numbers higher than 0.1, so, in that case, even the continuum equations were question and then we saw how the Burnett equations can be used. Which are obtained as the higher order approximations of the Boltzmann's transport equation? So, you have Knudsen number power 0, Knudsen number power 1, and Knudsen number square and so on. So, Knudsen number 0 terms will give you the oiler equation, Knudsen number to the power 1. First 2 terms now will give you the Navier stokes, and if you include the third term it will give you the Burnett equation and so on.

So, therefore, in all this cases the viscous stress tensor and heat flux vector. So, all these become increasingly more and more complicated correct. So, the computational effort also becomes difficult. So, therefore, higher Knudsen numbers for gas flows involve solving therefore, different from the Navier stokes equation Burnett equations and again for the more practical cases we try to retain continuum equations with the modifications in the slip. So, the basic slips are the Maxwell's and Smoluchowski slip, but you can also extend that to second order slip to in order to retain the continuum equation.

So, most of the practical studies have been using this kind of an approach. The next thing now we will do is go on to the liquid phase the liquid flows; by liquid flows you have now different kinds of problems. So, you can talk about single phase. So, this is the first introductory part of the liquid flows so; that means, you are talking about only one phase which is pure liquid and again when you talk about liquid flows the equations are not now that interesting or that difficult compare to the gas flows. So, you will be still talking about the continuum equations and the boundary conditions also are not a problem because most of the time your nodes and numbers are much lesser than 0.01.

So, therefore, there is no problem with applying the no slip boundary condition. And only the interesting issue will be the different kinds of the flow physics that come out at mini and micro scales. Which are different from the macro scale problem? So, what we will do is we will, any way know the conventional macro scale liquid flow transport equation we will try to apply this to micro scale and identify what are the important physics that come out in the micro scale and give emphasis only on that.

(Refer Slide Time: 10:13)

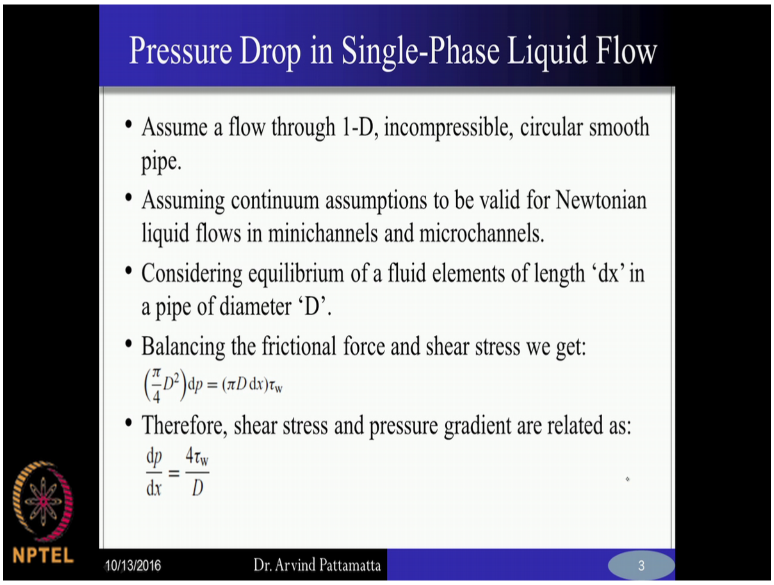


So, you know that, you know most of the momentum and energy equations, that are generally applicable to the macro; macro scale that is a Navier stock can be apply to the micro channels with liquid flow. So, that is probably the easier thing, but what are probably more interesting is two things, one is the effect of relative roughness.

So, when you have talked about micro channel, which is of the size of few 100 microns. So, there now the effect of the roughness of the walls which can be few tens of microns can become significant. So, this can lead to different issues. So, one is the transition from laminar to turbulent. So, the relative roughness effect not only affects the basic friction factor and heat transfer coefficient, but also the transition is quite different in the micro scale, micro channels, because of the relative the effect of the relative roughness at the walls. So, therefore, what is more challenging is, and what is interesting is also that you have certain empirical correlations for friction, factor nusselt number for the macro channels, you want to verify whether this can be applied at the micro scale directly. So, if not, what is the addition modification that has to be brought in?

So, if everything is similar from macro scale there is nothing challenging at micro scale. So, the same correlation can be applied. So, the same physics so, there is nothing interesting so, but definitely there are things which are distinct that micro scale.

(Refer Slide Time: 12:16)

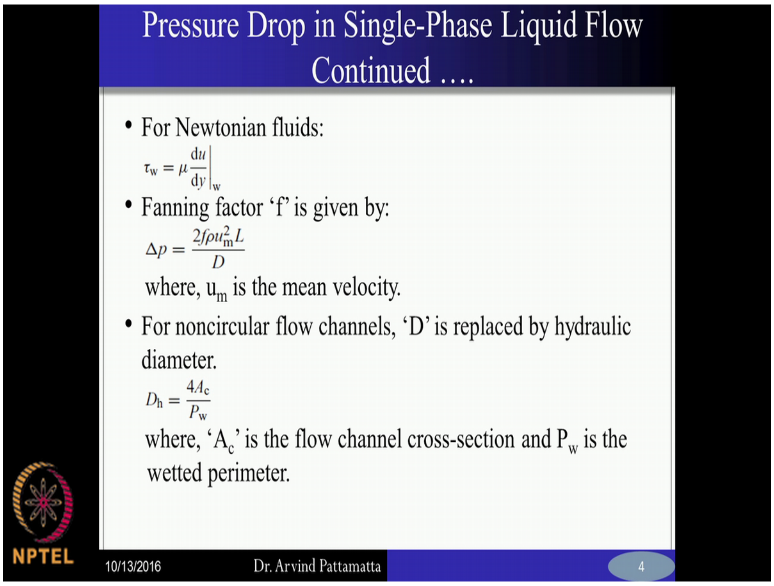


And we have to investigate therefore, whether this correlation can be modified and applied suitably for the liquid flow in micron mille channels. So, most of the cases you know 99 percent of the cases, you do not have any problem with facing higher Knudsen numbers. So, you have a perfect assumption of continuum and Newtonian flow and also no problem with jumps for velocity and temperature at the walls.

So, therefore, you can assume if you for example, take the case of flow through smooth circular tube without any roughness effects. Whether, it is macro scale or micro scale now it is not going to make any difference. A perfectly smooth wall will have the same value of friction factor and nusselt number irrespective of whether this is channel of few centimeters diameter or few microns diameter. So, already you know that for the case of single phase flows in macro channels. So, you will talk about the fully developed flows in either tubes or channels. So, you simply apply the balance of forces. So, one is the pressure gradient along the x direction, if you take x as the axial direction.

The other is the walls shear stress right. So, even if you write down the Navier stokes equation and simplify that you end up with only the viscous diffusion in the vertical direction; the other is your pressure gradient along the axial direction right. So, these two will be balancing each other and from that you can integrate and calculate what are the velocity profiles, right and from which you can calculate the relation between the center line velocity profile the mean velocity profile, and you can calculate additional quantities integral quantities like friction factor.

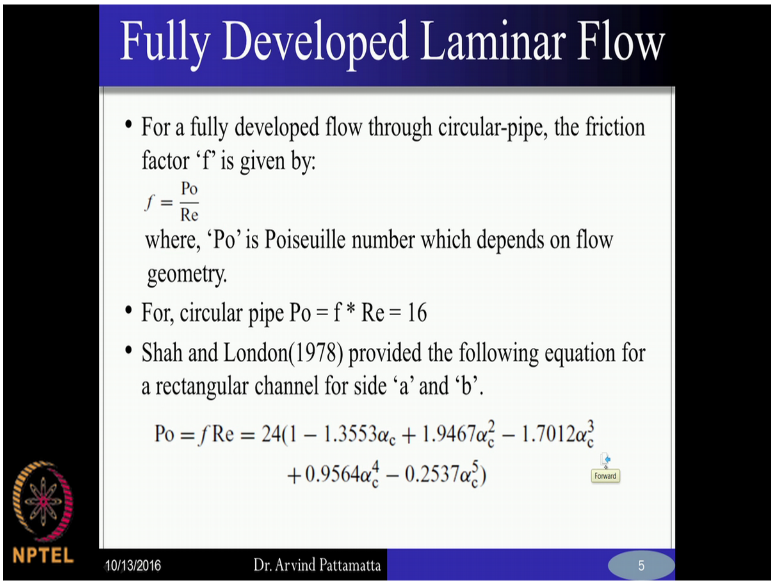
(Refer Slide Time: 14:20)



So, all these are known you know. So, these are coming from simply macro scale fluid mechanics is nothing different here, except you should know that we have two different friction factors right. So, one is defined based on the wall shear stress and that is call, the fanning friction factor. So, that is tau wall divided by half rho u square right. So, that is you are fanning friction factor, the other is your Darcy friction factor. So, we defined Darcy friction factor in terms of pressure gradient if you say d p by d x. So, d p by d x times the diameter divided by half rho u m square. So, that is your Darcy friction factor. So, and your fanning and Darcy friction factor are related such a way that your Darcy friction factor is four times the fanning friction factor. So, generally if you therefore, derive a relation for the fanning friction factor for a circular cross section what is the relation you get 16 by r e therefore, the Darcy friction factor will be 64 by r e.

So, this is a something that is already known to you, but I want to again highlight the difference, because most of the time in these correlations that I will be talking about we will be mostly using the fanning friction factor. So, apart from that you also know that if you have a non circular cross section, then instead of using the regular diameter of the channel you replace this with the hydraulic diameter which is defined as four times cross sectional area by perimeter.

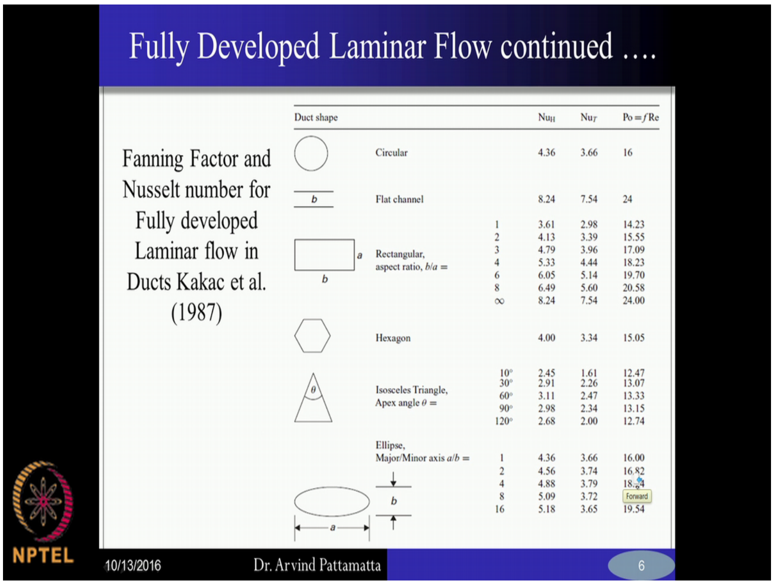
(Refer Slide Time: 16:16)



These are the things which you already know, and then you go ahead and you calculate your expression for friction factor now depending on again, whether it is a fanning or Darcy this Poiseuille number will be difference. So, this Poiseuille number is nothing, but product of the friction factor times Reynolds number. So, which is are constant for most of the fully developed duct flows, and depending on the duct cross sectional area, you have different values of Poiseuille number. And this Poiseuille number also depends on the aspect ratio if it is a rectangular duct.

So, therefore, you know. So, you can simply write down expression for Poiseuille number and depending on whether you use fanning or Darcy friction factor again this can be different for example, since we are talking about fanning friction factor here, for the circular pipe the Poiseuille number will be 16.

(Refer Slide Time: 17:26)

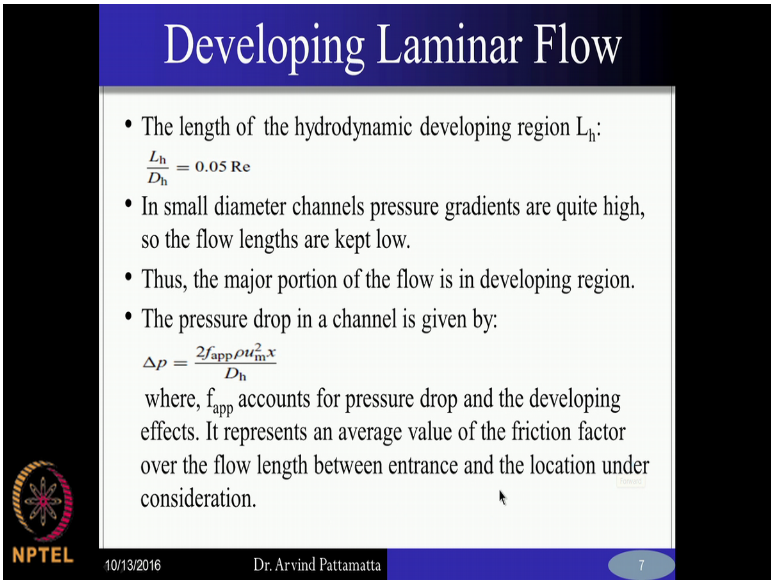


So, now if have a rectangular channel with sides a, and b. So, if you for example, consider rectangular channel as shown in this particular figure, where you have the shorter side as a, and the longer side as b. So, you can define the ratio of a by b. So, your aspect ratio is b by a, but you can define parameter alpha c, which is the ratio of a by b, the shorter side length divided by the longer side length. So, if you calculate a by b there is one popular correlation given by shah and London. So, from which we can determine the Poiseuille number for any expectation. So, for any rectangular cross sectional area, for different values of a by b we can calculate the corresponding Poiseuille number.

So, as you can see that for different cross sectional shapes duct shapes. So, you have different values of Poiseuille number, for the rectangular case you can use this particular shah and Londons correlation, to get this value and they have been tabulated here. So, out of this you can see by varying the aspect ratio you know. So, if you increase the aspect ratio you see that a Poiseuille number increases and at the same time your heat transfer coefficient also increases. So, the nusselt number h and t, so, that mean; that means, that nusselt number for constant heat flux boundary condition nusselt number for constant wall temperature boundary condition. So, the nusselt number for the constant heat flux boundary condition is usually higher than the constant wall temperature condition, and with for a case of a rectangular channel with increasing aspect ratio you see that the values of nusselt number increases and. So, does the value of Poiseuille number and again for the other shapes such as the hexagon triangle ellipse also they have been calculated although they are not very common shapes.

So, especially shapes like hexagon or triangle is not. So, commonly used in heat exchanges right. So, all this is the knowledge coming from macro channel.

(Refer Slide Time: 19:58)



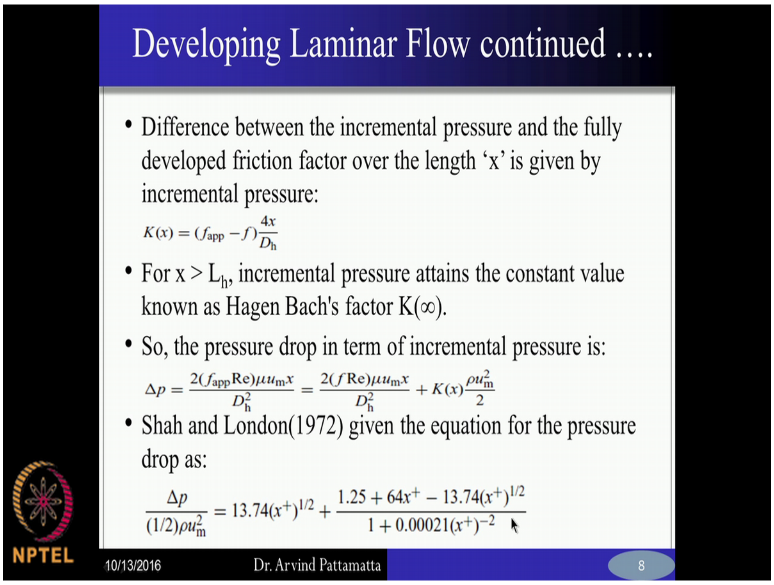
And the same thing can be apply, if you have a smooth micro channel also. The other thing that is important is what is calling the developing length, hydro dynamic developing length. And this hydro dynamic developing length can be estimated for macro channels as 0.05 times Reynolds number. So, for example, if your Reynolds number is equal to 100, so, this value will be 5. So, this is a non dimensional length at which the boundary layers will merge and then after which you do not have any change in the elastic profile.

So, therefore, you said d u by d x is 0 beyond this particular length. So, less than this you have all the gradients in the axial direction also they are very important we cannot neglect them, and that becomes a very complex region you have to solve the Navier stokes equation, considering the inertial terms also you cannot neglect the initial terms in the developing region. So, now, what is now different about the micro channel case for example, So, usually as you see that one of the effects of keeping the channel diameters small, for the same value of Poiseuille number so; that means, you have a friction factor which is constant say if you are operating at Reynolds number of 100, and you reduce your channel diameter from say 1 m m 2, 1 microns. So, three orders of magnitude, so, what happens to the pressure drop it increases by three orders of magnitude?

So, keeping your friction factor the same you see that your pressure drop increases in a several orders of magnitude when you go from macro to macro channel. So, therefore, since your d p by therefore, pressure drop is large you do not want to have long channels, when you have micro channel, because when you have to put lot of pumping power to overcome this pressure drop. So, therefore, typically all the micro channels have relatively much smaller lengths compare to the macro channels. And therefore, most of the times you will have construable length of this channel to be developing. So, in the case, of large long channels you can always neglect this developing length to be very small for example, if your Reynolds number is 100 if your l by d is 5, and if your diameter is of the order of 100 microns, and your length is of the order of several centimeters, then you can safely ignore the developing length, but if your length is of the order of few mille meters then this becomes very important a contribution.

So, therefore, you can say that what is different now in the micro channel case, one possible thing is that the effect of the developing region will become more important, in the micro channel compare to the macro channel. And again when you are therefore, talking about the pressure drop it could be micro or macro, but where the developing effects are significant. We have to now modify the earlier formulation what we use as f with what is the called as f apparent. So, why apparent means we are now not just talking about fully developed region, but also region which is developing and therefore, the total pressure drops will accounts for both these regions apart of which it is developing and part which is developed. For the developed case, we know what is the Poiseuille number? Its constant value for a given cross sectional area where as for the developing region. So, you have to write down some empirical correlations to determine this and therefore, in general for case where you consider developing and the developed region you replace this f with what is called as f apparent this has two components, one accounts for pressure drop which is for a fully developed flow the other is a developing effects.

(Refer Slide Time: 24:38)

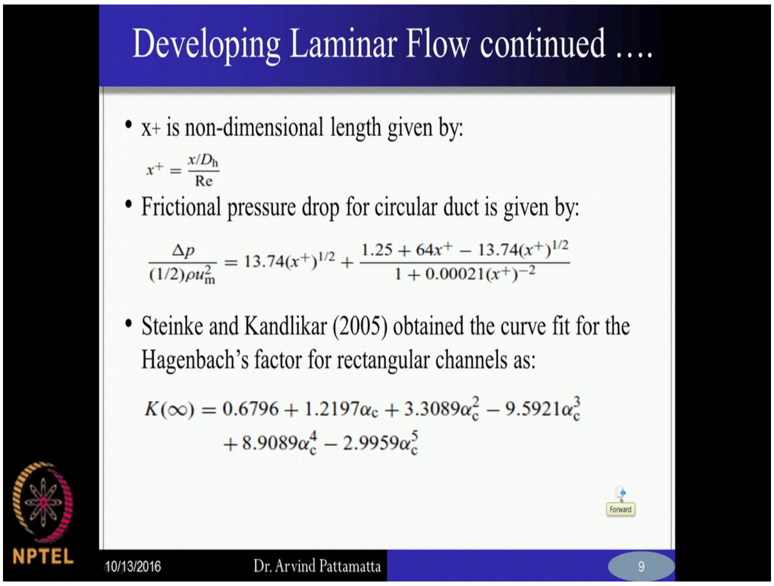


So, therefore, how do we calculate this f apparent is a next question? So, one of the ways to do the common way is to define what is called as an incremental pressure. So, this incremental pressure is denoted by notation k, and you know this is the function of x, because if you are probably within the developing regime itself. This incremental pressure will be a function of x, if you are outside the developing region this incremental pressure become a constant value. So, therefore, k is a function of x is defined as the difference between the apparent friction factor, and the fully developed friction factor. So, this gives you the local variation of the friction factor in the developmental regime you understand. So, difference between the apparent and the fully developed one should give you what is the friction factor in the developed region and this is varying locally. So, that is why your incremental pressure is a function of x.

Now, for x greater than the developing length; So, then this value would become a constant, because beyond this there is no variation in the friction factor with respect to position it is a constant value. So, for this entire region of x is equal to l h your value of k reaches a constant value which is denoted by k infinity, and this is call the Heisenberg factor. So, the Heisenberg factor is a limiting value of the incremental pressure when your x reaches the developing length. So, beyond with this becomes a constant value right. So, most of the cases you know, so, ones you talk about position where which is greater than the developing length you do not have to worry about k of x, but directly k infinity you can substitute and then you can calculate your apparent friction factor. So, therefore, the overall pressure drop which is now defined in terms f apparent.

So, you can re write the previous expression, which is in terms of rho u m square x by d in terms of Reynolds number. So, you can multiply and divide by for example, d and then you can write this in terms of Reynolds number. So, this turns out to be 2 times f app r e into nu u m x by b h square and this f app, now from this definition of incremental pressure you can split it into 2 terms, one which corresponds to the fully developed region the other is your developing region. So, therefore, you write it in terms of 2 terms, and if you are considering length of x which is greater than l h this is k of h will become k infinity which is your Heisenberg factor.

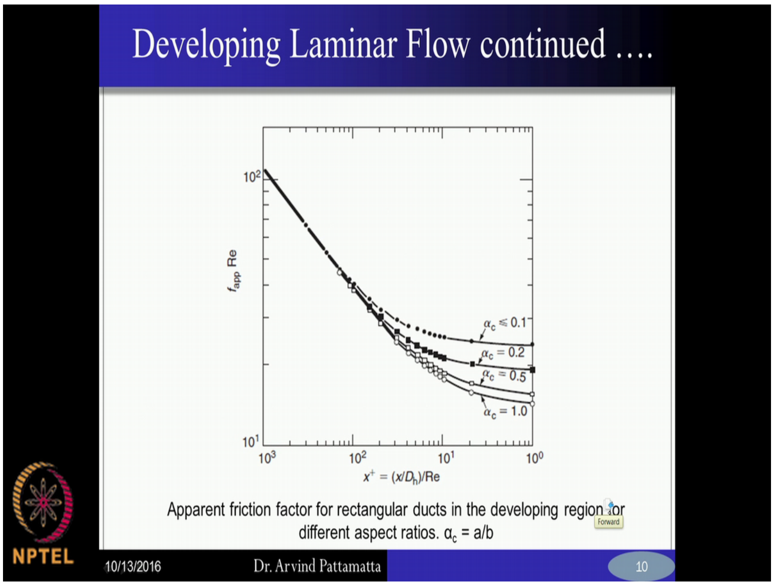
(Refer Slide Time: 28:04)



So, to in order to calculate this particular Heisenberg factor, these common expression which is use to this by Steinke and Kandlikar. So, they have for rectangular channel for example, they have obtain an empirical correlation to calculate k infinity as a function of a by b ratios. So, this is the most commonly used expressions for rectangular channels. So, you can based on this you can calculate k infinity and then you can substitute into this, you already know your expression for friction factor coming from the previous section, because the Poiseuille number is also a function of the a ratio of a by b so, you know. Now, therefore, what is your fully developed friction factor know you Heisenberg factor. So, corresponding to some x which is greater than l h you can therefore, calculate what is your delta p?

So, for the entire pipe, so, you can therefore, replace your x with the length of the pipe. So, this is what is now going to be different in micro channel case, in a macro channel case many a time you will be ignoring the Heisenberg factor you just knock out the second term and only calculate the delta p form the first term, but for micro channels you should also include this second term, this becomes quite important there are also some other standard correlations to get the overall pressure drop as a function of only x, without bothering this separate contributions of fully developed flow and Heisenberg factor. So, one such correlation looks like this it is a function of x non dimensional x which is your x by d by r e. So, if you simply substitute your non dimensional x, you get your value of non dimensional pressure drop. So, here already the effect of the Heisenberg factor is implicitly there. So, these are all empirical correlations from different experiments right. So, and these are applicable equally for both macro and micro channels, does not matter only that the effect of developing length since they become more significant at for micro channels.

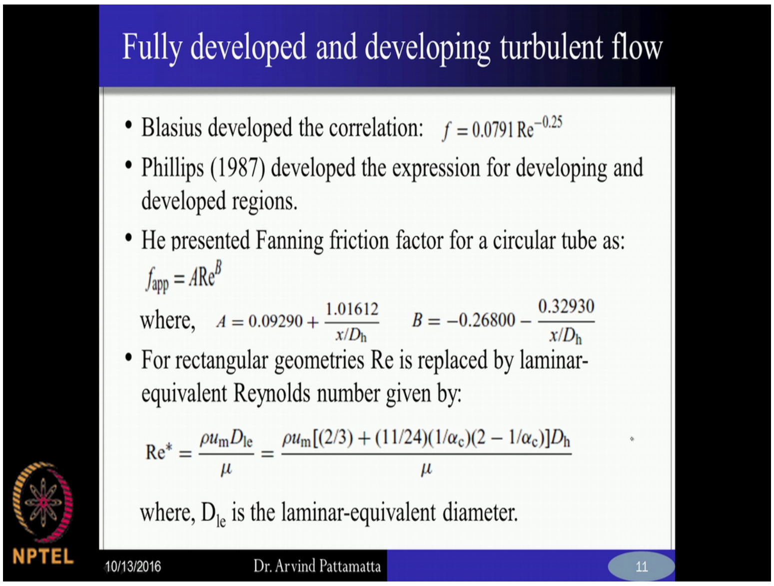
(Refer Slide Time: 30:38)



So, this has to be rigorously followed. So, if you plat this apparent that you are calculating here based on your f and Heisenberg factor and you plat it as a function of the local x plus, may be something of this particular form. So, you find that for values of. So, the x goes from the right to left here the smallest value is on the right and then it increases towards the left. So, in the developing region, so, that is in this particular region this is where you find there is a lot of difference between the different aspect ratios. So, for different aspect ratios your value of k will become quite different and in the developing region is where this parameter will become important compare to this particular factor.

So, therefore, effect of the aspect ratios becomes more significant in the developing region, but after you go deeper. So, for values of x is greater than larger than your l d developing length. So, the effect of the Heisenberg factor becomes smaller and smaller and your fully developed friction factor becomes the most important contributor, and all this different aspect ratios will collapse in to a single line. So, this is how the plots look.

(Refer Slide Time: 32:03)

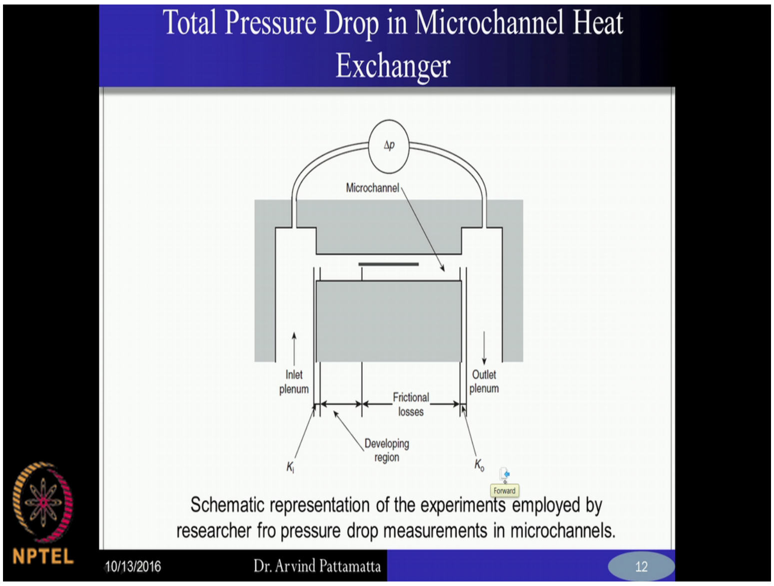


So, similarly, all these discussion we had for laminar flows. So, what happens, when flow becomes turbulent, and in that case how these correlations become different? So, for the generic case for turbulent flows, one of these often used correlation is developed given by Blasius which is given like this, f is equal to 0.0791 Re power minus 0.25. So, this is the simplest correlation for estimating the friction factor.

Now, there has been modification for the case where you have both developing and developed regions. So, there is one person Phillips who developed an expression for the turbulent flows where you have both the developing and developed region to be accounted. So, in that case, he has modified the correlation. So, that the f apparent can be directly calculated as a function of r e, so, this r e is any where fixed, but the parameters which has changing on the values a and b. So, these, a and b parameters or functions of the non dimensional position. So, based on the non dimensional position x by d you can substitute into this expression, this is again another empirical correlation developed by Phillips you can get the value directly for the f apparent. So, here you do not have to worry about adding separately the Heisenberg factor.

So, again for a case of rectangle geometrics, you know you can use slightly modified definition of Reynolds number. So, you can just plug it directly into this expression a get the apparent friction factor. So, over all you know the discussion is that whatever correlations have been developed for macro channels. So, far is now used as it is for micro channel only thing that we give more importance to the developing region right.

(Refer Slide Time: 34:30)



So, whether it is laminar or turbulent does not matter. So, now, what are the other additional effects which become significant for liquid flows in micro channel? So, now, that we know that within the channel itself you have to account for the developing region and we have therefore, use the appropriate correlations depending on whether they are laminar or a turbulent and not only that most of this micro channels are fade by means of manifold. So, you do not usually have just to single micro channel, but several time s you have a parallel system of micro channels, these are small diameter channel.

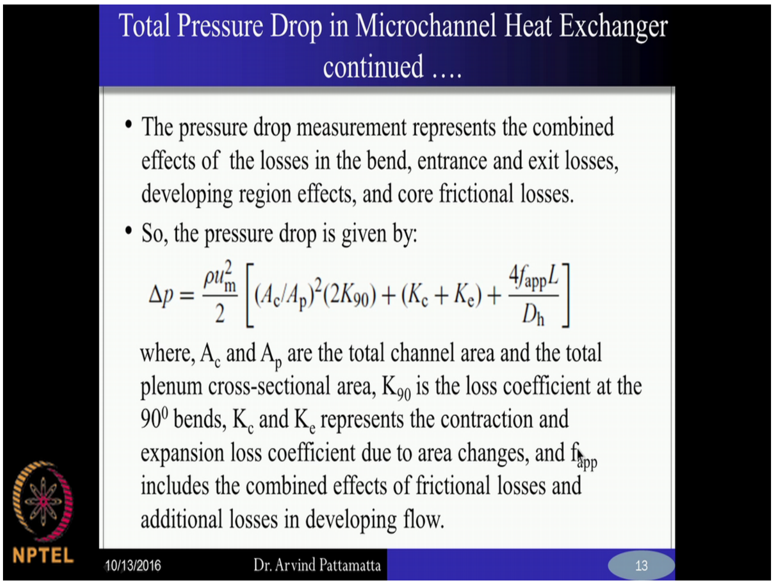
So, therefore, in order to cover a particular surface you want to use many numbers of this channel. It could be 5, 10, 20, and 100. So, could be several of these channels start parallelly, and in that case you have to have a feeder mechanism, which supplies the liquid to all these channels and at the end of the channels they collect the liquid and then take them out. So, these are this can be refer to as the manifolds inlet and outlet manifold are inlet and outlet plenum. So, this is similar to any heat exchanger in any heat exchanger you supply the liquid through a manifold distributed to. So, any tubs of the heat exchanger and then collect it in an outlet manifold and take them out.

So, in this particular case, as it is illustrated you can see that if you measure the pressure drop between the inlet and outlet manifold. Now this is how your pressure tapings are and you connect it to a differential pressure transmitter, and you try to measure the pressure drop. So, this gives you the overall pressure drop of the micro channel. So, you do not exactly put a pressure trapping here, and here you want to measure the overall pressure drop from the inlet of the manifold to the exit manifold, plus the pumping power required is to overcome this over all pressure drops. So, in this case apart from the developing region and frictional losses you also have two other losses which are coming into picture one is the entrance losses the other is your exit losses. So, in the entrance you also see that the flow has to turn 90 degrees.

So, you also have additionally bend losses. So, therefore, three other factors contribute to the overall pressure drop if you take from here to here. So, the other the other two apart from the developing and frictional losses are entrance exit losses and you also have the bend losses, and if you want to see a complete value of pressure drop that is from the inlet of the plenum to the exit of the outlet manifold. So, then you also have to include the pressure drop right across the inlet plenum and also the outlet plenum. So, those also are not very small values, because usually the length of this plenums are also usually large correct because you are supplying flow it to. So, many of these micro channels, so, depending on the number of micro channels the length of the inlet and outlet plenum also can be significantly large and therefore, these pressure losses within the plenum also will become important.

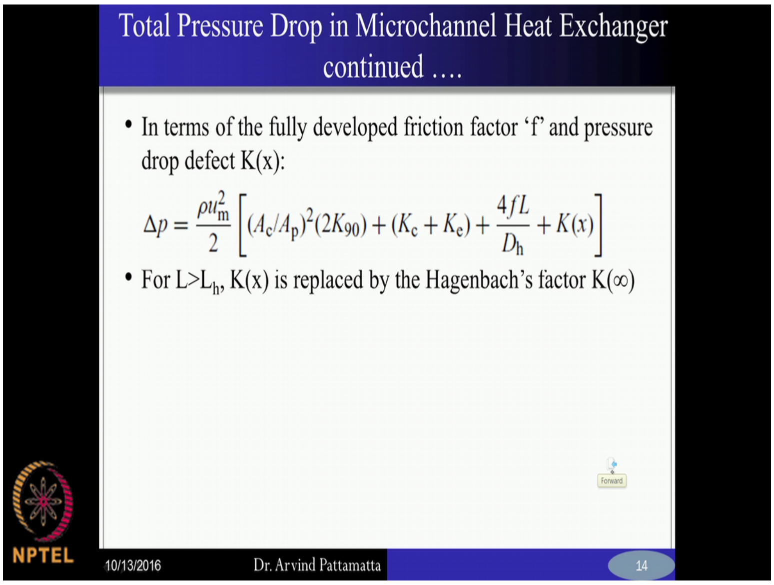
So, therefore, in the case of micro channel all the. So, called negligible pressure losses which you consider for a macro channel will become more significant such as the entrance losses, exit losses, bend losses.

(Refer Slide Time: 38:35)



And the pressure losses through the manifolds all of this will become important, and therefore, when you calculate your pressure drop we have to account for every small value which you might other vise neglect. So, therefore, apart from your 4 times f apparent l by d, this is within the channel you also have bend losses which is given by this particular parameter call the loss coefficient due to bend. So, this case, your loss coefficient and similarly you have loss coefficient due to constriction and expansion. So, that is at the inlet you have a constriction. So, there is a small vena contractor which is forming. So, there are constriction losses there and are again when it is exiting you having a sudden expansion. So, there is also expansion loss, so, apart from therefore, this particular term you also have to account for the loss coefficient due to constriction expansion and the bend losses.

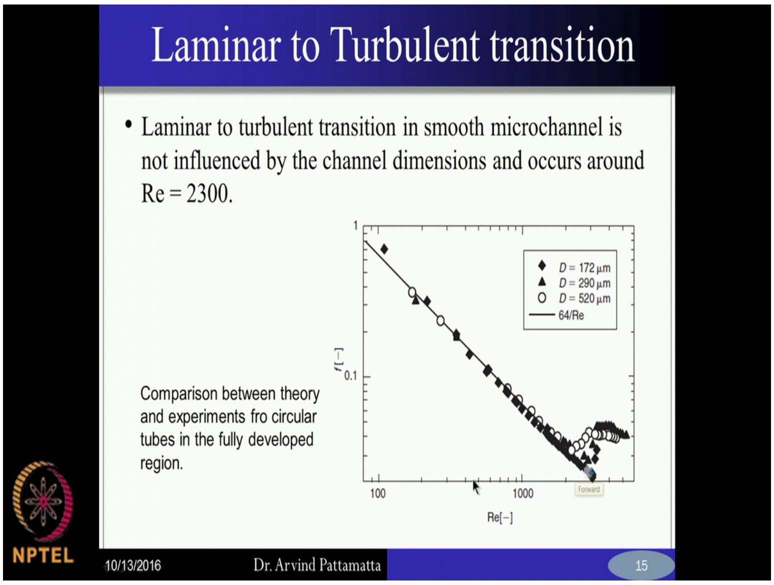
(Refer Slide Time: 39:49)



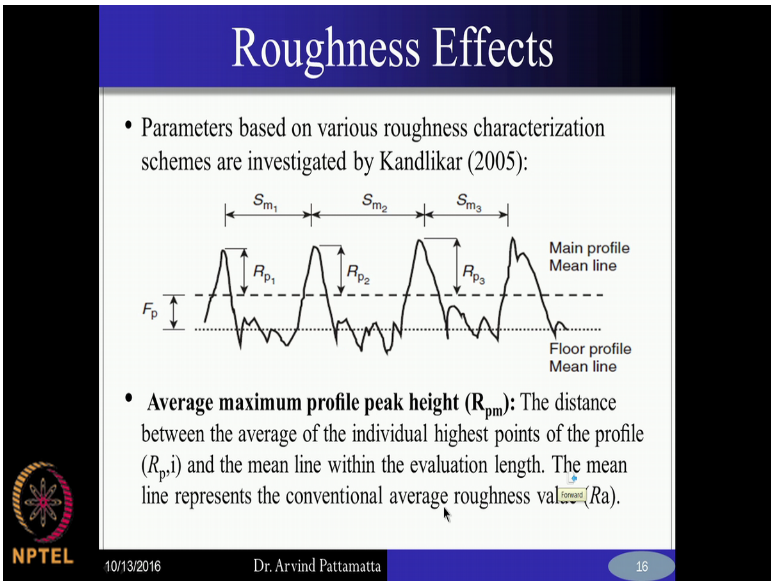
So, therefore, the expression for pressure drop becomes more complicated now with all these effects and you can also substitute for f apparent in terms of the Heisenberg factor, and you will therefore, get at least 1 2 3 4 5 terms totally right.

So, this is the expression that is generally used to estimate the pressure drop between the inlet and the outlet of a micro channel. So, this till does not account for the pressure loss in the manifold. So, this is as the figure shows only the pressure tapping here and here, if you are putting one pressure taping here and one, one here and one here then that should account for the pressure losses in the manifold.

(Refer Slide Time: 40:41)



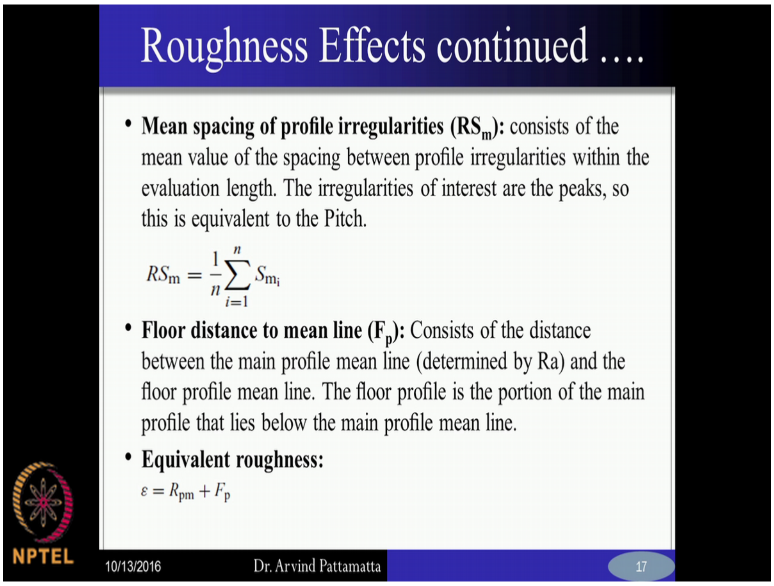
(Refer Slide Time: 40:45)



So, therefore, now if you look at the other important effect at micro scale. As I said roughness becomes a very significant parameter. So, therefore, if you take a microscope put the surface of this particular duct and then you see how the roughness looks. So, you will be looking at these are the micro structures of the roughness. So, therefore, just for the sake of investigation you know Satish Kandlikar, has classified several parameters for including this roughness effect. So, one is that he has defined as called as a floor that is like a base from which all these roughness element seem to be procuring. So, that is like a, you know foundation for the roughness. So, this is called the floor profile and then he estimates the average height of this roughness and he draws another set of these dashed lines here. So, this is called the mean roughness height right. So, therefore, one of the important parameter that he characterizes is what is called the average maximum profile peak height. So, that is you measure from this mean profile what is the distance from the maximum point of each roughness element to that mean and then you take the average of all these values.

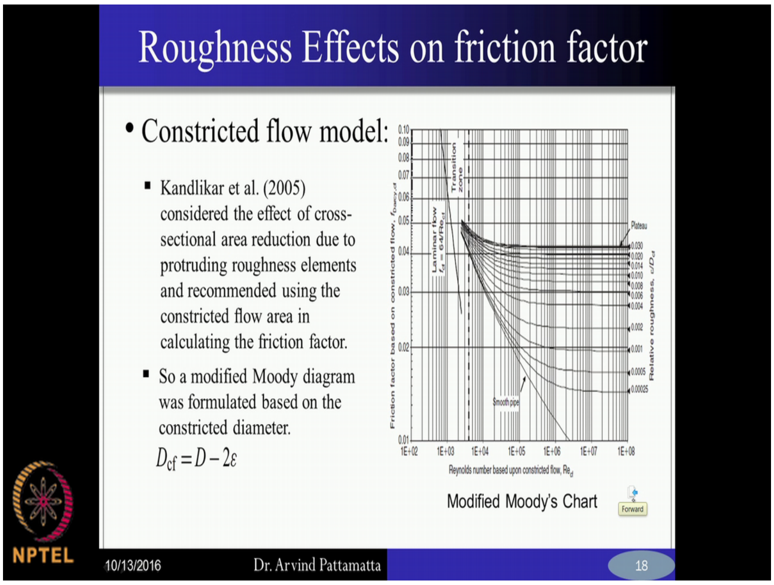
So, for example, you have for the first roughness element R p 1, second one R p 2, R p 3 and. So, on and you take a simple arithmetic average that will give you what is the average maximum profile peak height R pm. So, this is the maximum value of roughness, and you have to again since there is a variation between different roughness elements you have to average them and that give you an average value. So, this is one important parameter, the other important parameter is the pitch between the roughness. So, again if you take two roughness element the first to roughness elements the separation distances S m 1, similarly you have an S m 2, S m 3 and so on, you can take again an average arithmetic average and you have a mean spacing of profile irregularity. So, this is called as an RS m this is again an arithmetic average of the individual pitches.

(Refer Slide Time: 43:26)



So, therefore, the equivalent roughness will be what if you are measuring right from the floor. So, this distance of the mean profile from the floor is given by the distance f p, and you have a average maximum profile peak height R pm and therefore, you can calculated what is the equivalent roughness as the summation of F p and R pm. Therefore, for any surface you can defined what is f p, what is R pm, you can calculate and therefore, from which you can estimate what is the equivalent roughness because you cannot be choosy about going into the roughness equivalent roughness for each and every element you have to only define it for the average, that is why we calculate the average for r p. And we know that F p is anyway more or less a constant, because we take a base line for the floor right and we just take the summation of R pm and F p and that gives you the equivalent roughness, and this equivalent roughness is what is used in many correlations where the friction factor or nusselt number gets modified.

(Refer Slide Time: 44:54)



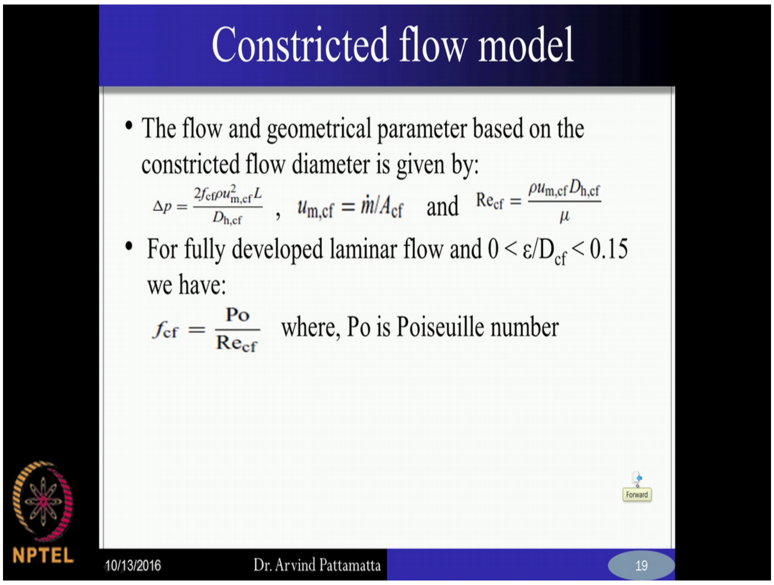
So, now one of the simples models that Kandlikar proposed. So, any real channel, the previous discussion is all for completely smooth channels, where you do not have any difference in adopting the a friction factor nusselt number from macro to micro channel, but now any real channel that you take will have this value of equivalent roughness and therefore, how do you see this effect now friction factor will definitely go up.

So, it is seeing a higher value of roughness the pressure losses will be more, so, how this is to be accounted for into the existing correlation. So, Kandlikar proposed the very simple model for this. So, this is called the constricted flow model. So, all he assumes is that, since you have calculated the equivalent roughnesses you can assume the diameter of the channel is now come down by this much length or this much dimensions. So, that is equivalent to 2 times epsilon. So, your actual diameter now or your constricted flow diameter is equal to your original diameter minus two times epsilon. So, the flow actually now sees a constricted dimension rather than the original dimension.

So, if you propose a correction for the constricted diameter and still retain all the expression for friction factor and. So, on the same expression only replace your Reynolds number in the original, case you have your d you replace now with d c f. So, then it seems to be working quite good. So, if you for example, modify the Moodys chart the original Moodys chart was based on with the actual diameter d you modified based on the constrictive flow diameter and you get a set of curves, and apparently if are all the micro channels they seem to be matching very well with this modified Moodys chart.

So, a very simple model, but never the less very effective and accurate, so, all we have to only consider now is the constricted diameter. So, therefore, accordingly your definition of Reynolds number gets modified based on the constricted diameter and again the velocity also gets modified correct.

(Refer Slide Time: 47:11)



So, originally the velocity was seeing a larger cross sectional area, now you have to calculate the constricted flow area which will be smaller therefore, the velocity will now go up right. So, the definition of Reynolds number will be now based on the velocity based on the constricted flow, and also the diameter also will b the constricted flow diameter, and based on this you still use the same friction factor which is dependent on the Poiseuille number. So, that remains the same only the values of u m and d h will become different and therefore, your pressure drop will become different, so, in the case, of micro channels with the roughness effects. So, a pressure drop gets modify by replacing your velocity and your conventional velocity and diameter with the constricted flow diameter and the constricted velocity. So, the other things still keep remaining the same all right.

So, we will stop here I think tomorrow we will talk a little bit more about this effect of constricted flows model and so on. So, I think we should be able to complete this chapter.

Thank you.