Micro and Nanoscale Energy Transport

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Lecture - 30
Gas Flow and Heat Transport in Microchannels, Part 1

Very good morning, today we will start new topic. So far, the last 30 lectures or so, we focused only on the Nanoscale transport of energy and now in the remaining part, portion of about 12 lectures, we will look at Microscale transport. Now I mean, as we discussed in the very beginning, the introductory class you know Microscale phenomena is not so very different from the micro scale and you have to be careful with what kind of fluids you are dealing with. Accordingly the Microscale phenomena can become complicated.

If you are probably talking about gases, these are once which are usually rarefied when you confined the distance between the channels and so on and in this case, the Knudsen numbers can go quite high, sometimes if the order of 1 and therefore, the use of continuum transport can be questioned, but in other cases where liquids are used, even in Microchannels. There is no problem using the continuum equation.

In that case, it is not different from the Microscale, but you observe several physical interesting phenomena happening in the Microscale regime, which is probably more relevant than, when you observe this in the macroscale. These kinds of interesting phenomena are what we are trying to bring out, when we study the Microscale energy transport.

The first part of this Microscale transport will be focused on gas flows because you know if at all you want to say anything about deviation from continuum in Microscale, I think it is for primarily gases and therefore, this is the first thing that we will deal with and mean as the titles says, you know you have gas and this is the single phase system and you are also looking about not just a simple gas flow, because simple gas flow could be anything from a continuum, all the way up to sub continuum, but we are particularly concerned about gas flows and Microchannels. This is where the Microscale transport of energy can be studied.

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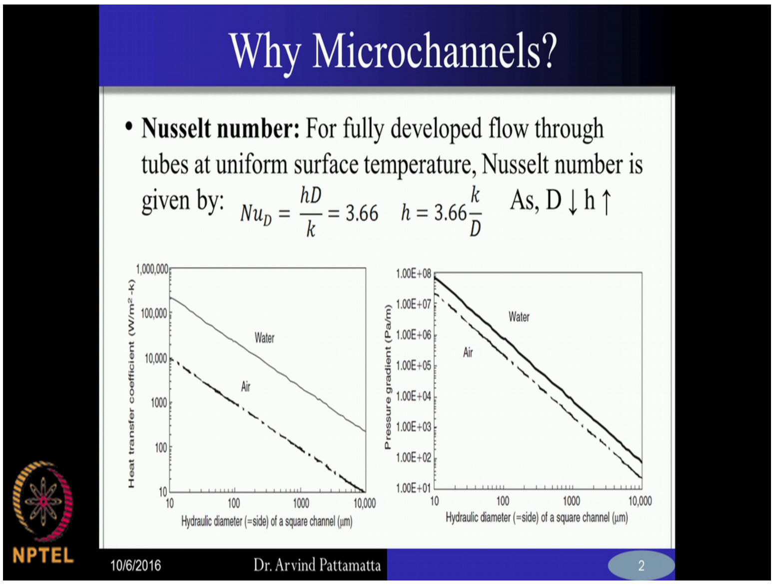

Why Microchannels?

- **Nusselt number:** For fully developed flow through tubes at uniform surface temperature, Nusselt number is given by: $Nu_D = \frac{hD}{k} = 3.66 \quad h = 3.66\frac{k}{D}$ As, D ↓ h ↑

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Now coming to why, first of all we need to look at you know Microchannels. Microchannels are increasingly becoming popular devices for effective cooling applications, especially if you talk about electronic cooling. In such a case you know what is different about Microchannels compared to Microchannels you know. If you look at the fully developed flow through tubes and you also assume the boundary condition to be a uniform surface temperature. The Nusselt number is a constant in the laminar flow regime for the case of a uniform surface temperature. It is 3.66 whereas, for uniform heat flux, it is 4.66.

Therefore, you have a fixed value of Nusselt number, especially in the laminar regime. Now the question is, now what is the effect of heat transfer coefficient? What happens, when you reduce the channel diameter? And what is it is effect on the heat transfer coefficient?

From this expression, it is clear that even if Nusselt number is remaining a constant, as you reduce your channel diameter, heat transfer coefficient is going up. It is inversely proportional to the channel diameter.

For example, if you want to get an idea, if you plot the heat transfer coefficient as a function of the hydraulic diameter and you know if you go from the right towards the left that is from a large Macrochannel, which is a say 10 10000 micrometer, to the case of 10 micrometers. This is plotted on a large scale and you plot the corresponding values of heat transfer coefficient on the Y axis for 2 different fluids. 1 for air the other for water; obviously, you know for water which is having a higher value of thermal conductivity.

Although the Nusselt number is the same for both the fluids, you are talking about you know higher thermal conductivity for water compared to air and therefore, water 10d to have a higher heat transfer coefficients, but not only that as for both of both these fluids as we reduce the size of the channel, from your Microchannel, Microchannel all the way to Microchannel of the size of 10 microns.

You see that the heat transfer coefficient has also gone up 3 orders of magnitude, from 10 all the way to 10000. This is an inversely proportional proportionality that you observe here is what is plotted here. This is very good because you know it is a D power minus 1 dependence, which is which means that if you reduce your channel 10 times, you are going to find an increase an h by factor of 10.

This is very advantages especially when you are looking at a cooling system such as the electronic cooling, where you have to focus on very small surface areas and you have to at the same time dissipate large amounts of heat. Although the surface areas are very small, the amount of heat that has to be dissipated or the heat flux that has to be removed is very large and you know. The heat flux if you want to remove such large amounts of heat flux and maintain this component at a fixed temperature, your heat transfer coefficient has to be very high. That is how the convection cooling can be affective.

Herefore, from this simple analysis we can conclude that your, you know Microchannel is way more efficient in removing overall heat transfer rate, compared to conventional Microchannels and at the same time you also have a penalty to pay in terms of pressure drop. It is not only that you know the heat transfer coefficient is going up simply for free. At the same time you have to put in more pumping power to pump the same amount of liquid to maintain the same Reynolds number for example. That is again the relation between for a fixed value of friction factor.

If you do the calculation, again you have a problem where if you reduce the amount of the size of the channel, your pressure drop will keep going up. That is again delta P which is inversely proportional to the diameter. Therefore, you on 1 hand it is good from the heat transfer point of view, but from the fluid dynamics point of view, it means it is a requiring additional pressure drop to pump the, to maintain the same Reynolds number at the inlet. This is again a big penalty now as you can see on the plot for pressure drop as you reduce the channel diameter by 3 orders of magnitude, you have you know an increase by not just 3 orders, but about 7 orders of magnitude.

This is the pressure gradient Pascal per meter; you know this is the kind of pressure penalty that you have to pay. That is why this becomes a very interesting problem, if you want to optimize by moderately increasing the Nesselt number, keeping the pressure penalty increase to a minimum. We cannot directly jump from this to this and claim that this is very effective from hydrodynamic point of view, because your pressure drop is going several orders of magnitude higher than that of the heat transfer coefficient.

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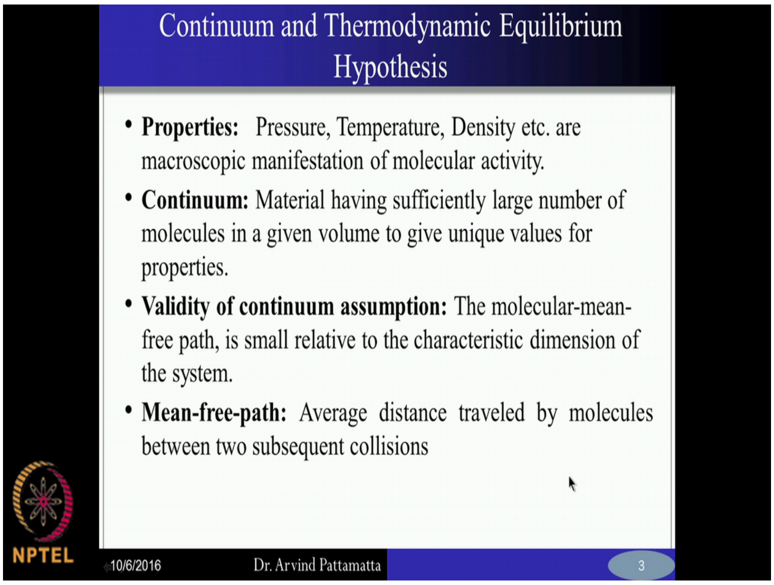

Continuum and Thermodynamic Equilibrium Hypothesis

- **Properties:** Pressure, Temperature, Density etc. are macroscopic manifestation of molecular activity.
- **Continuum:** Material having sufficiently large number of molecules in a given volume to give unique values for properties.
- **Validity of continuum assumption:** The molecular-mean-free path, is small relative to the characteristic dimension of the system.
- **Mean-free-path:** Average distance traveled by molecules between two subsequent collisions

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But nevertheless, I think when you are dealing with small devices and you have to cool them, innovatively you have to use Microchannels and therefore, you have to deal with this counter intuitive problem. Now, again just revisiting, when we talk about gas flows in Microchannels, we need to understand the basic definition of continuum, I think we have already done this. I will just quickly run over this. When we talk about therefore, properties, these are associated with the continuum such as the pressure temperature density therefore; these are the macroscopic manifestations of the molecular activity. These are valid only when you have a continuum with sufficiently large number of molecules to contribute to these averaged quantities that are defined above and once you are talking about you know the large mean free path, these continuum assumptions will break down. Therefore, you have to find validity of the continuum assumption. To in order to do that, we define the Nusselt number based on 2 characteristic dimensions, 1 is the mean free path, the other is the length scale of the domain, and mean free path you already know by now this is the average distance travelled by the molecules between 2 subsequent collisions.

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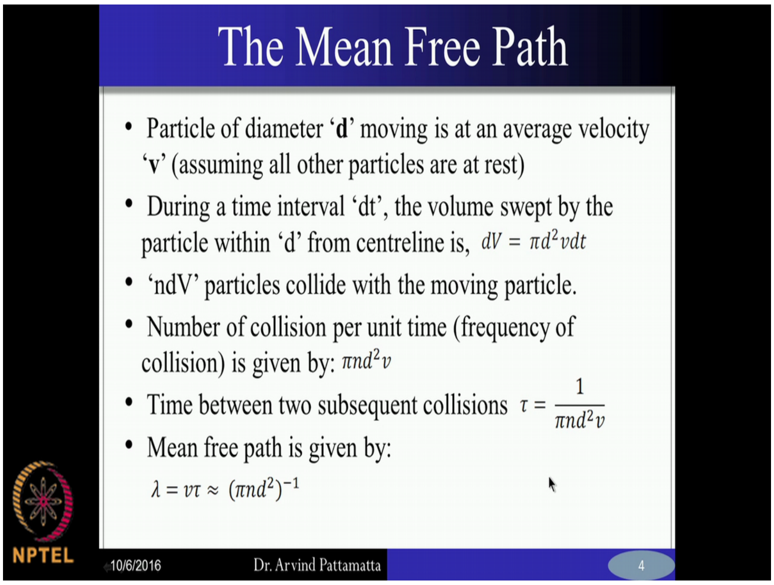

The Mean Free Path

- Particle of diameter '**d**' moving is at an average velocity 'v' (assuming all other particles are at rest)
- During a time interval 'dt', the volume swept by the particle within 'd' from centreline is, $dV = \pi d^2 v dt$
- 'ndV' particles collide with the moving particle.
- Number of collision per unit time (frequency of collision) is given by: $\pi n d^2 v$
- Time between two subsequent collisions $\tau = \frac{1}{\pi n d^2 v}$
- Mean free path is given by:

  $\lambda = v\tau \approx (\pi n d^2)^{-1}$

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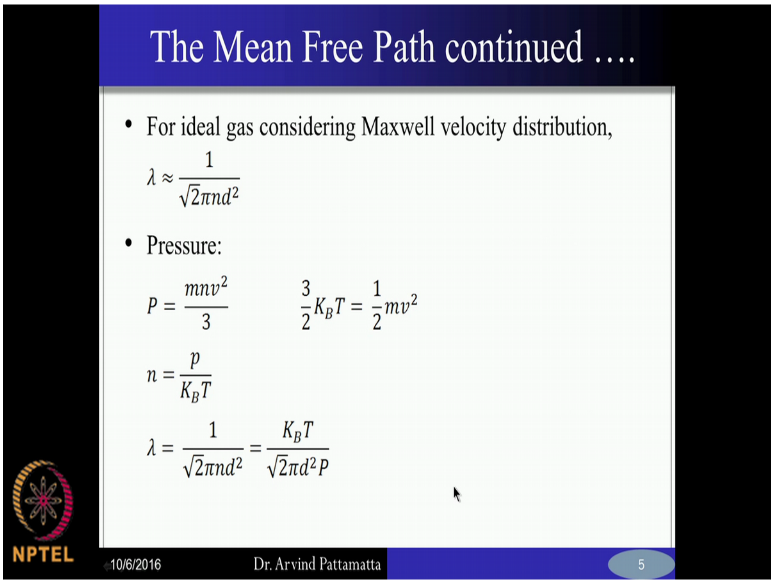

The Mean Free Path continued ….

- For ideal gas considering Maxwell velocity distribution,

  $\lambda \approx \frac{1}{\sqrt{2}\pi n d^2}$

- Pressure:

$$P = \frac{mnv^2}{3} \qquad \frac{3}{2}K_B T = \frac{1}{2}mv^2$$

$$n = \frac{p}{K_B T}$$

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{K_B T}{\sqrt{2}\pi d^2 P}$$

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And we have also under kinetic theory, we have seen how to derive the expression for the mean free path, very simple simplistic method tells you that if you how do I go back with this? Now this is interesting, does it go back like this? Yeah, it goes, yes. I think this works.

Mean free path, that what we have derived is 1 over pi 1 N D square and in order to also correct for the relative velocity between the molecules, we when we derive the mean free path we assume that 2 molecules are already in contact and now we are talking about the distance travelled to collide with the third molecules. That is the distance between 2 successive collisions. Already you have 1 collision, following that you want to see what is the distance travelled by this effective molecule and collides with the third molecule and that is how you get this. Now, but in that assumption, we are assuming the third molecule is fixed and it is static, but in reality this is moving with a Maxwellian distribution. In order to current correct for that we have a reduction in the, or correction factor that has to be introduced, which is a 1 over square root of 2.

From this definition, we also saw that both your temperatures and pressures are related to the kinetic energy of the molecule. This is a very very important fact, what we call as pressure and temperature at the continuum scale or nothing but the microscopic manager, the macroscopic manifestations of microscopic kinetic energies and also we therefore, get an expression for the mean free path by substituting for from the ideal gas equation of state. We can substitute for the number density of molecules and write that in terms of pressure and temperature.

Therefore, we have we know that at a given pressure and temperature and for a given kind of gas molecule, we know what will be the mean free path. This calculation we have already done.

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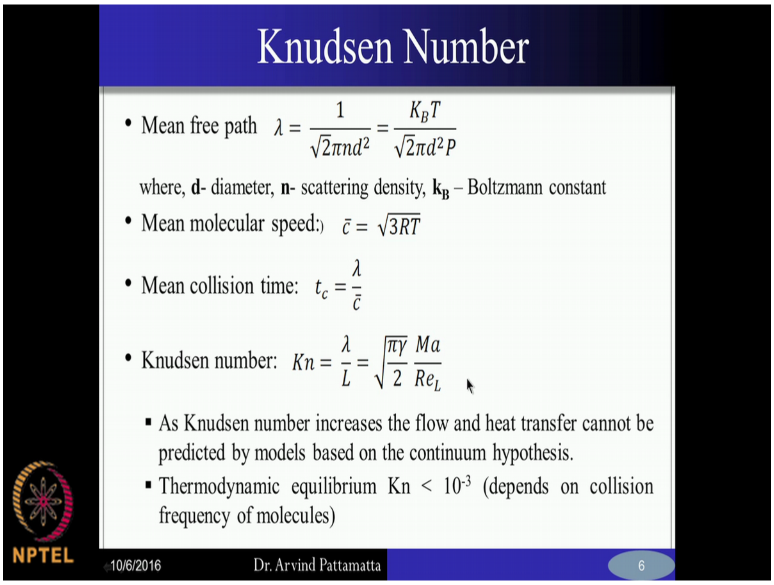

Knudsen Number

- Mean free path $\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{K_B T}{\sqrt{2}\pi d^2 P}$

  where, **d**- diameter, **n**- scattering density, $\mathbf{k_B}$ – Boltzmann constant
- Mean molecular speed:) $\bar{c} = \sqrt{3RT}$
- Mean collision time: $t_c = \frac{\lambda}{\bar{c}}$
- Knudsen number: $Kn = \frac{\lambda}{L} = \sqrt{\frac{\pi\gamma}{2}}\frac{Ma}{Re_L}$
  - As Knudsen number increases the flow and heat transfer cannot be predicted by models based on the continuum hypothesis.
  - Thermodynamic equilibrium Kn < 10^{-3} (depends on collision frequency of molecules)

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Now again you can also define the Knudsen number by using this mean free path that we have derived, takes this expression. You can also non-dimensionalize all of these parameters here and if you do that, you will find that there are 2 non dimensional numbers, which are coming in the definition of Knudsen numbers. This is simply turning out to be the ratio of Mach number 2 Reynolds number. Depending on the characteristic speed you know if you are talking about you know mean molecular speed, this is given by square root of 3 R T and you can define Knudsen number in terms of therefore, the non-dimensional numbers Mach number and Reynolds number and you can also for, when you are solving any problem with a non-dimensional group of numbers, you can use this expression rather than writing this in terms of dimensional quantities such as pressure and temperature.

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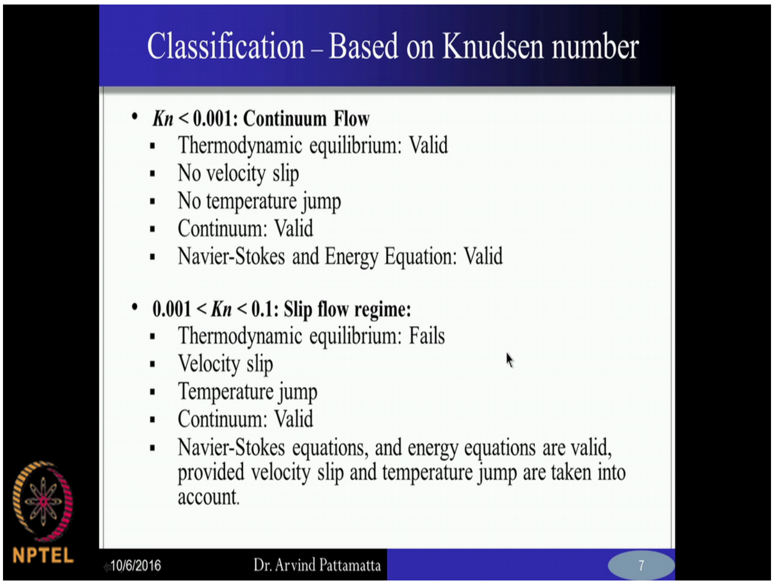

Classification – Based on Knudsen number

- **$Kn < 0.001$: Continuum Flow**
  - Thermodynamic equilibrium: Valid
  - No velocity slip
  - No temperature jump
  - Continuum: Valid
  - Navier-Stokes and Energy Equation: Valid
- **$0.001 < Kn < 0.1$: Slip flow regime:**
  - Thermodynamic equilibrium: Fails
  - Velocity slip
  - Temperature jump
  - Continuum: Valid
  - Navier-Stokes equations, and energy equations are valid, provided velocity slip and temperature jump are taken into account.

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We have already looked at different regimes of fluid flow, where continuum approximation is valid and where you have to go to sub continuum, typically when Knudsen number is much smaller than you know point 1. That is we are talking about point 0 0 1 and lesser than that. This is your real continuum flow and in this regime you have a validity of thermodynamic equilibrium, all your equilibrium that you are talking about and there is no problem, there is a perfect you know no slip condition for velocity and temperature at the boundaries and you solve your conventional Navier stokes equations.

Now when you talk about moderate Knudsen numbers between point 1 and point 0 0 one, here also to some extent, you can use continuum, there is no problem all this is with respect to gas flows. Mostly with gas flows you can reach up to you know Knudsen number of 1 10, but with liquid flows you hardly cross the second regime. When you are talking about continuum assumption that is valid; however, the concept of local equilibrium will not be satisfied everywhere. Especially close to the walls. This is where the scattering of these molecules with the boundaries happen and if you look at them, even Knudsen numbers of the order of 0.1. The scattering between the gas molecules will become more rarefied compared to the boundary scattering and therefore, locally near the boundaries, there will be non-equilibrium. Therefore, the thermodynamic equilibrium concept will not be valid throughout the fluid domain.

Now, this can be accounted for, at the walls by bringing into water called a slip effects. There are 2 kinds of slip, 1 for the momentum and the other for energy. We will see that in the next few slides, how the slip boundary conditions are coming up, but you still solve your Navier stokes equations in the broad sense, but instead of giving a no slip boundary condition, you account for the local non equilibrium at the walls, by means of the slip boundary condition. The slip is only our model, I mean in the real experiment I mean if you go to the microscopic picture; it is basically a local non equilibrium. This local non equilibrium cannot be simply accounted for in the continuum equations. It is difficult to patch the you know the transition from the local non equilibrium near the wall to an continuum away from the wall.

It is difficult to pass, that is why, people resort to using this kind of slip boundary condition models which can account for you know in some arbitrary manner, with some kind of empirical constants. You just model this non equilibrium by another continuum approach, but with some empirical constant which might contain some information, but this is not a very rigorous method. Therefore, what happens if you continue increasing Knudsen numbers? Even your continuum assumption, you know in the global scale will completely fail. Even when you are talking about regions which are away from the wall, you cannot use continuum approach and again we also have problems with the near wall conditions as well, but overall you cannot use your Navier stokes and energy equation.

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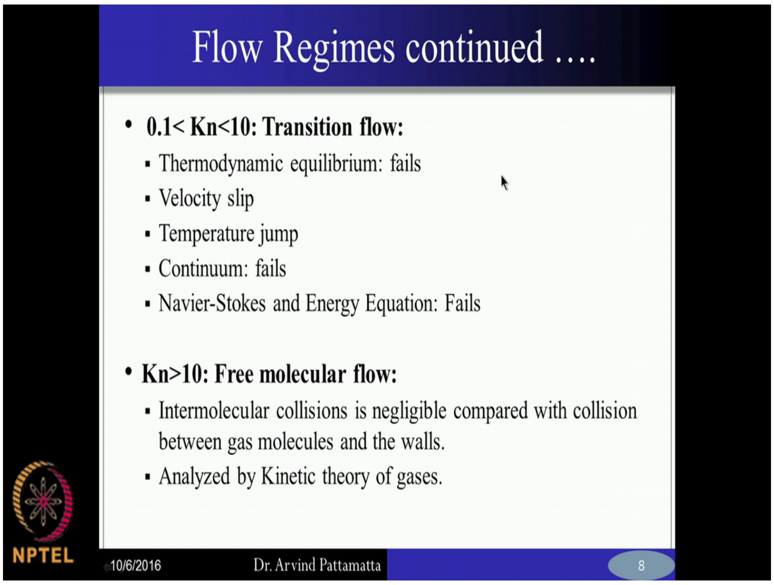

Flow Regimes continued ….

- **0.1< Kn<10: Transition flow:**
  - Thermodynamic equilibrium: fails
  - Velocity slip
  - Temperature jump
  - Continuum: fails
  - Navier-Stokes and Energy Equation: Fails
- **Kn>10: Free molecular flow:**
  - Intermolecular collisions is negligible compared with collision between gas molecules and the walls.
  - Analyzed by Kinetic theory of gases.

In this case, we have to use, what we call as no Burnett equations. These are coming from expanding or the Boltzmann transport equations to several terms and retaining some of the lower order terms. The lowest order term will be the Euler equation, followed by Navier stokes, and then the relatively higher order term will contain information, more information about higher Knudsen numbers. This kind of equation is called the Burnett equation and this can be used for the transition flows. This is a, I mean regime where definitely your Navier stokes equation will not be applicable right away and with gases, it is quite likely that you can, many applications also in rarified gas flows you can reach this particular regime.

Now, for Knudsen numbers crossing 10 you know, in that case you are now looking completely into the molecular picture. You have very few gas molecules and it is all mostly gas molecular collision, center molecular collisions and this will also be very negligible compared to the collision between the gas molecule and the wall and then how do we analyze this? Even the Burnett equations cannot be applicable here, and then we have to resort to something like the Boltzmann transport equation again. Real sub continuum pictures Boltzmann's have transport equation and there is the simplest assumption is a relaxation time assumption.

In the rarefied gas dynamics are there for the B T with the relaxation time approximation is used when you are in this particular regime, Knudsen number greater than 10.

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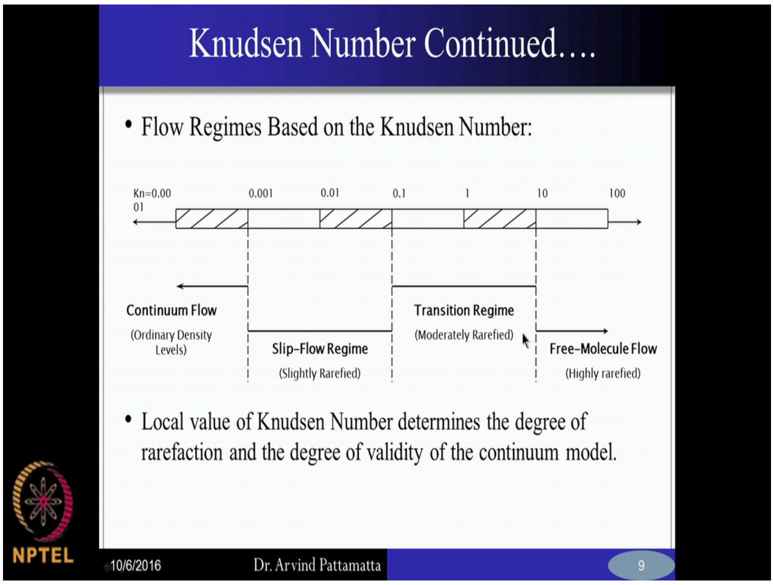

Knudsen Number Continued….

- Flow Regimes Based on the Knudsen Number:

Kn=0.0001 0.001 0.01 0.1 1 10 100

Continuum Flow (Ordinary Density Levels)

Slip-Flow Regime (Slightly Rarefied)

Transition Regime (Moderately Rarefied)

Free-Molecule Flow (Highly rarefied)

- Local value of Knudsen Number determines the degree of rarefaction and the degree of validity of the continuum model.

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Therefore, I mean just a very important parameter again is the Knudsen number. Based on the value of Knudsen number, you can actually go anywhere from continuum flow all the way to free molecular flow and you have to know, since we are for you know gases we still use the continuum approximation from here till the slip flow, we have to be careful, what happens when we go slightly beyond you know point 1?

There is a very thin separation between these regimes. Sometimes people also use the slip flow regime models for Knudsen numbers of 0.3 or 0.5, but these are not very very clear. What is the limit of the validity of the continuum models?

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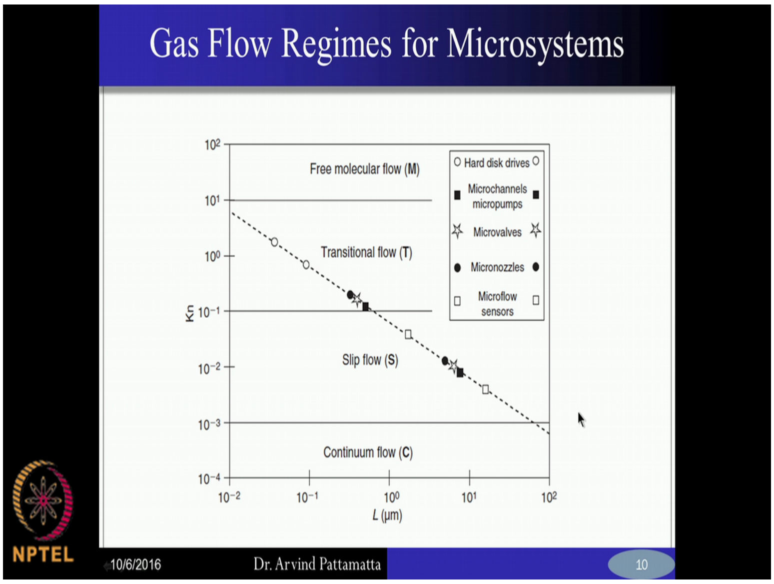

Gas Flow Regimes for Microsystems

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Yeah. This is again I think I showed this figure in the very beginning in the introduction, just to show their different kinds of micro systems and the corresponding the length scales and what is the Knudsen number range that you can encounter in them. It is not just simply anything which is small will have a Knudsen number; you know it depends on the working fluid. We have therefore, 2 parameters, 1 is the mean free path, and the other is the length scale. This is therefore important, what is the kind of application you are looking at. In terms of Knudsen number, we have all these you know, horizontal lines here, which classifies the transition from 1 regime to the other. The bottom most is your small Knudsen number regime, which is continuum and the top most is your free molecular flow.

Now, if you move all the way from a larger length scale, which is of the order of say 100 microns because 100 microns now could be mostly you know towards very small Knudsen numbers and as you are going towards the Nanometers you are talking about 10 power minus 2 microns and smaller than that. These are the order of few Nanometers. In that case you are definitely going to be dealing with large Knudsen numbers.

In general as you move from the right towards the left, most of your applications will have will see an increasing value of Knudsen number.

Therefore, this move like a straight line, in the straight line you are transitioning your flow regimes as you move from the right towards the left. If you are talking about hard disk drives for example, these are 1 case where you have very large Knudsen numbers because the separation between the disks will be less than a micron and at the same time you are talking about gas flows, air which is an air gap between the disks. Therefore, you are talking about Knudsen numbers of the order of 1 and above. This is in a transitional flow. If you are looking at an application involving analyzing the, you know air gap between the hard disk drive. This clearly cannot be solved with the continuum equations.

In this case, your Navier stokes equations will fail to predict, whatever phenomena is happening you have to at least resort to the Burnett equations with the slip boundary conditions. Whereas, if you are talking about micro channels with gas flows, it again depends, what is the fluid? If you use these Microchannels with gas flows, then you are talking about slip flow regime. You are in the micron length scale. Therefore, you can still use your continuum equations, but only account for the correction of local non equilibrium at the wall. Whereas, if you are talking about liquid flows this will completely come down to the continuum regime. You just solve your continuum equation with your normal no slip boundary conditions and this will predict a perfectly well, no problem.

Therefore, when people talk about Microchannels, you should not get carried away. There is nothing fancy in that, if you are talking about most of the Microchannels use liquids, because as you saw the heat transfer coefficient is higher for liquids compared to gases and therefore, practical cooling applications liquid is generally preferred and in such a case that is nothing to worry about in terms of modeling, perfect continuum assumptions can be used. If you also involve other applications such as micro nozzles and so on, they might fall in the transition or the slip flow regime you know. These are the pro, these are the gray areas, where it is not clear whether you have to go to the Burnett equations or use a continuum equation. They are somewhere you know present in that regime, where it is difficult to classify what kind of model has to be employed.

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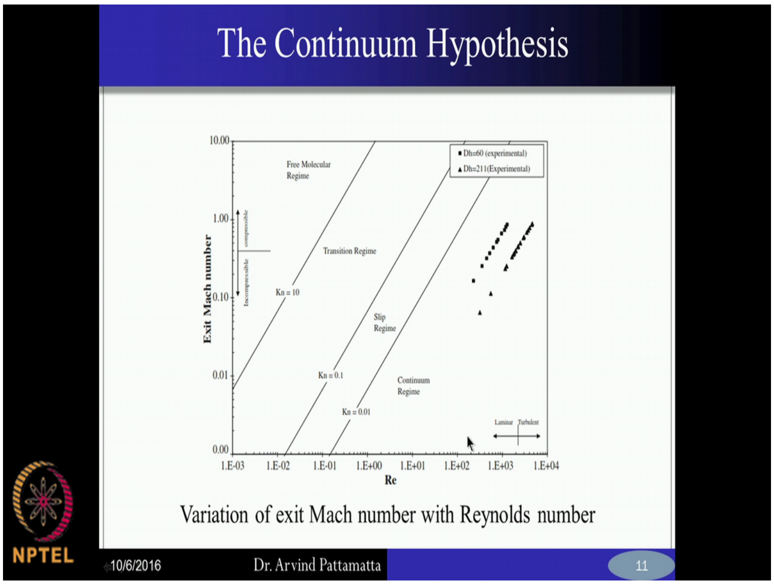

The Continuum Hypothesis

Variation of exit Mach number with Reynolds number

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Now again, I mean we saw that the Knudsen number is nothing but a function of your Mach number to the ratio of Mach number to Reynolds number. We can again classify all these regimes rather than looking at only Knudsen number, we can plot Mach number on 1 axis and Reynolds numbers on the other and we can also classify these regimes and not only that it also depends on what kind of equations you want to solve.

The Navier stokes equations can be a compressible or incompressible. We did not discuss about that. That can be analyzed when you also plot the Mach number and then decide now where you are standing. Therefore, now what is interesting here as you suppose you know you are in the laminar regime, you know we are talking about Reynolds numbers which are less than a 1000, and most of these Microchannels since you have very small diameters, mostly you will not cross the laminar regime, 99 percent of the cases.

Even when you are talking about very high velocities as you can see here, the Mach number could cross you know sometimes even 0.31 and so on, but the corresponding value of the Reynolds number will be always less than a 1000. These are very small diameter tubes. In terms of Reynolds number you are doing good. You do not have the problem of analyzing a turbulent flow. It is a simple laminar flow regime, but you have to be careful what kind of equations you are solving. For example, if you look at high Knudsen numbers. There is no problem if you are talking about Knudsen numbers less than 0.01. In such a case you know even if you are talking about a Reynolds number of you know close to a 1000, you are mostly in the incompressibility regime.

If you just draw a vertical line from here and a horizontal line towards the Y axis, we will find your Mach numbers are reasonably smaller enough to make an incompressible assumption, but whereas, if you increase your Knudsen number, even if you go to Knudsen number of point 1 if you are in the slip flow and you are operating at Reynolds number of say 100. And if you just draw a vertical line and then a horizontal the Mach number 10 Ds to be above 1 which means that you have a supersonic flow happening in the channel Microchannel. This is this means that although you are in a laminar flow regime, now you will see all these compressible phenomena happening.

The exit Mach number can go up depending on the Knudsen number, the higher the Knudsen numbers, this value will go up even for smaller Reynolds numbers. For example, if you go to Knudsen number of 10, you are going into the compressible regime for as small Reynolds number as how much? 10 power 5 into 10 power minus 2. That is like you know very very small Reynolds numbers. It might appear to you that this is hardly anything, but the velocities are very high now you are talking about velocities, which are greater than how much, at least 120 meters per second. So, that is your Mach number of 0.3.

You are at least talking about that much velocity, when you are dealing with Knudsen number of 10. You have to be very careful although Knudsen number of 10 is not a very common application even with gases I mean because if you just go back I mean, 1 of the common application is the hard disk drive. You are talking about Knudsen number of around 1, you are still not gone into the Knudsen number of 10. Even if you look at Knudsen number of 1, which will be somewhere here, it may not be Reynolds number of 10 power minus 2, but somewhere around 10 power minus 1. Even for that Reynolds number, you might end up with compressible flow. You have to be therefore careful with gas flows that mostly you will be dealing with compressible flows, even for small Reynolds numbers if your Knudsen number is greater than 0.1.

As long as you are you know less than 0.01, there is no problem, but if you are even in the slip flow regime, there is quite a likelihood, if you are operating of the order of even Reynolds number of 1, you see that if you look at Reynolds number of 1 and above. That can go into the compressible flow regime.

Therefore, most of the models to continuum models for the gas flows in Microchannels, are solved with the compressible form of Navier stokes equation.

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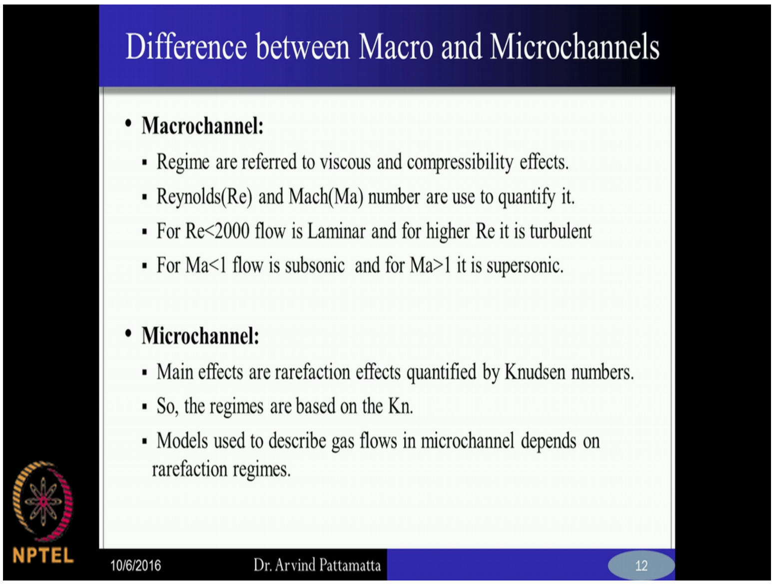

Difference between Macro and Microchannels

- **Macrochannel:**
  - Regime are referred to viscous and compressibility effects.
  - Reynolds(Re) and Mach(Ma) number are use to quantify it.
  - For Re<2000 flow is Laminar and for higher Re it is turbulent
  - For Ma<1 flow is subsonic and for Ma>1 it is supersonic.
- **Microchannel:**
  - Main effects are rarefaction effects quantified by Knudsen numbers.
  - So, the regimes are based on the Kn.
  - Models used to describe gas flows in microchannel depends on rarefaction regimes.

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Let us just summarize therefore, what would you do, if you had a Microchannel versus Microchannel gas flow? In the case of Macrochannel gas flow, we know generally, what we will do is we check the Mach numberand and if you are usually in the laminar flow regime that is less than 2300 is the Reynolds number and you correspondingly look at what is the Mach number. Usually it will be less than 0.3 and more likely that you will use only incompressible models in the case of Macrochannel, very rarely it will simply exceed 0.3 and in the laminar regime, it is very unlikely whereas, if you look at Microchannels, most of the times even for very small Reynolds numbers, you are in the compressible flow regime.

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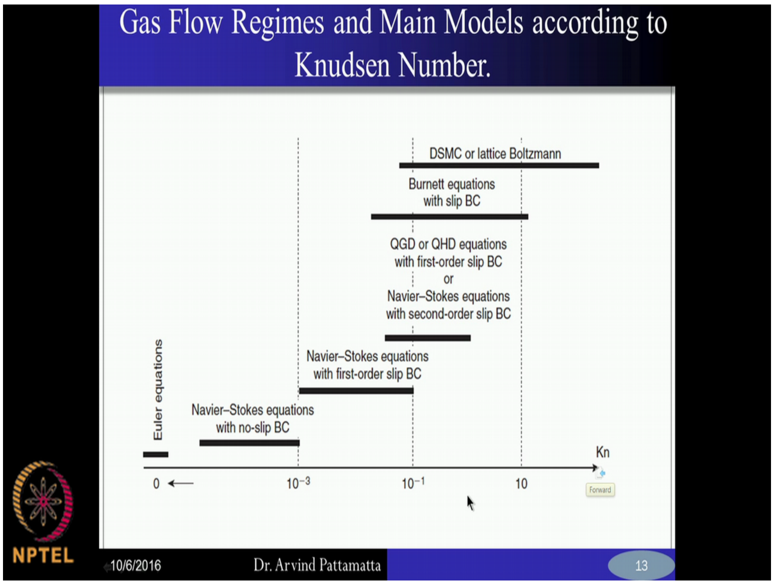

Gas Flow Regimes and Main Models according to Knudsen Number.

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And therefore, not only rarefaction effects, rarefaction is the high Knudsen number effect, but also compressibility effects will also be become important. Therefore, what are the kinds of equations that we can use to describe the gas flows? As you go from Knudsen number, which is almost 0 to very high Knudsen numbers and Knudsen number of 0 by the way exactly 0 although it is continuum, but it indicates that there is no viscosity also, it means 2 things. We are talking about either a very large domain l going is infinity, which is not usually possible or mean free path going to 0 is also not possible,, but in terms of viscosity, you know the viscosity is 0.0.

That means, it is completely described by in viscid theory or Euler equations and then you talk about your finite Knudsen numbers, it could be 0.000001, but nevertheless; that means, it is a having a finite viscosity. Viscosity effects are important and it is continuum and therefore, you solve the Navier stokes equation with no slip boundary condition and then slowly in the slip flow, you start encountering the slip flow regime between Knudsen number 10 power minus 3 to 10 power minus 1, where you use your Navier stokes equation with what we call as the first order slip. There are different slip boundary conditions depending on again the Knudsen number.

People have tried to still use the Navier stokes equations to as much higher Knudsen number as possible, only modifying the order of the slip at the wall. They were desperate to hang on to it, although it is not required. If you are still going above 0.1, people have developed what is called a second order slip boundary condition. They still use the Navier stokes equation primarily outside and near the wall at the wall, they use the second order slip, but you need not do that.

There are problems there with the numerical solution with the second order slip. You can as well directly solve the Burnett equations with a slip bond with the first order slip boundary condition and even if you go to smaller no length scales Knudsen numbers of the order no 10 and above you have to go to Boltzmann transport equation.

These are the different levels or different kinds of models that you will be encountering as you go towards higher and higher Knudsen numbers.

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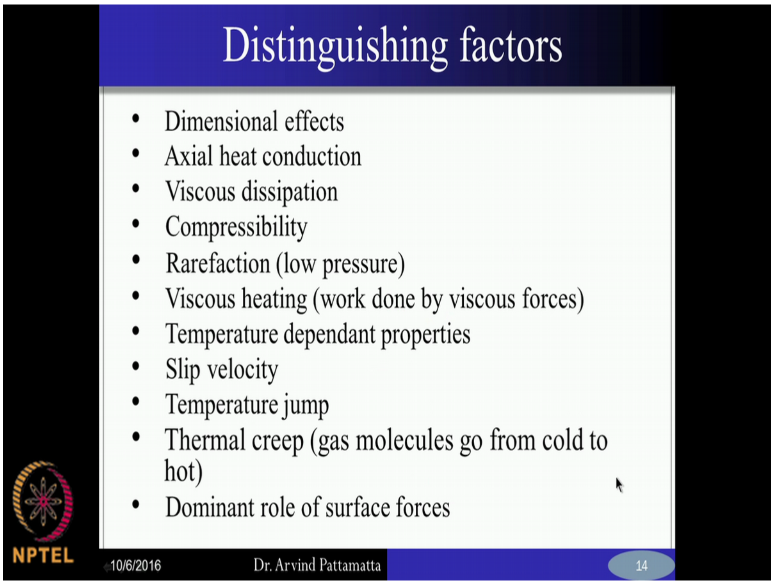

Distinguishing factors

- Dimensional effects
- Axial heat conduction
- Viscous dissipation
- Compressibility
- Rarefaction (low pressure)
- Viscous heating (work done by viscous forces)
- Temperature dependant properties
- Slip velocity
- Temperature jump
- Thermal creep (gas molecules go from cold to hot)
- Dominant role of surface forces

What are the basically distinguishing factors again? We have very important compressibility effects from the heat transfer point of view; we also have what are called as viscous dissipation effects because you know if you look at heat transfer course. If you are talking about the viscous dissipation, viscous dissipation is governed by the ratio of Eckert number to Reynolds number.

Eckert number is U square U square term; that means, as the velocities are becoming higher, the Eckert number will be larger and when you are talking about compressible flows the viscous dissipation also becomes very important. Therefore, along with the viscous dissipation, along with the compressibility effects for the heat transfer point of view, the viscous dissipation becomes important. This will tell you how the work the forces which contribute to the work are finally, dissipated as heat in the fluid.

This becomes important, if you have either a very highly viscous fluid or a compressible fluid and of course, you also are dealing with the rarefaction, low pressures in the case of Knudsen number flows and you can also have problems because of temperature dependent properties, if you have large temperature gradients, for example, because large temperature gradients indicate higher local non equilibrium. You can also have variation of properties with temperature and at the walls you have problem is the boundary conditions, you have to resort to using the slip velocity and for temperature also you need a slip boundary condition and there is also something called as thermal creep or thermal transpiration in which the gas molecules go from you know 1 end to the other due to a temperature gradient.

This also has to be accounted for in the slip boundary condition you know that, this is a correction factor that we usually employ at high Knudsen numbers. Again dominant role of surface forces, if you neglecting any surface force of the macro scale they might become very important at high Knudsen numbers.

You have to be doubly careful before you ignore any surface force whereas, the volumetric forces like gravity and all my might not be so important here. These are the different complexities.

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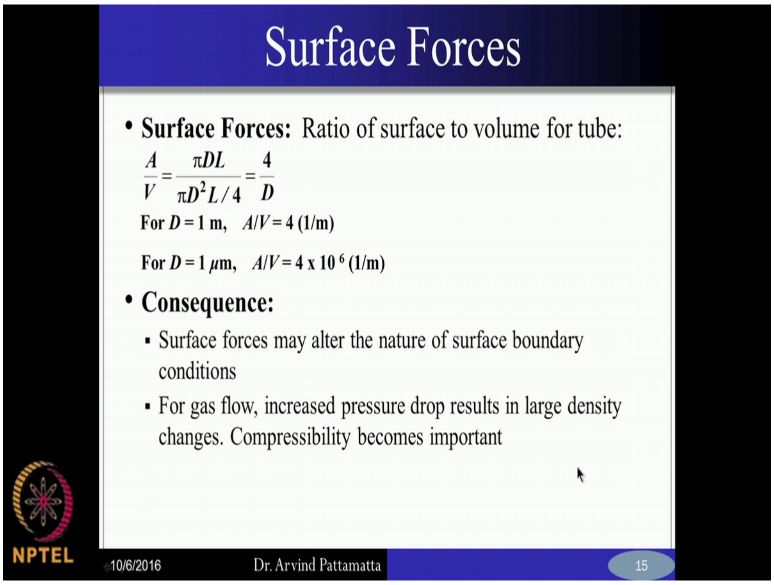

Surface Forces

- **Surface Forces:** Ratio of surface to volume for tube:

$$\frac{A}{V} = \frac{\pi DL}{\pi D^2 L/4} = \frac{4}{D}$$

For $D = 1$ m, $A/V = 4$ (1/m)

For $D = 1\ \mu$m, $A/V = 4 \times 10^{6}$ (1/m)

- **Consequence:**
  - Surface forces may alter the nature of surface boundary conditions
  - For gas flow, increased pressure drop results in large density changes. Compressibility becomes important

When you go to the Microchannel gas flows and again you know this ratio of surface area to volume and we have done this simple problem also. If you are reducing the dimension, what happens to this ratio?

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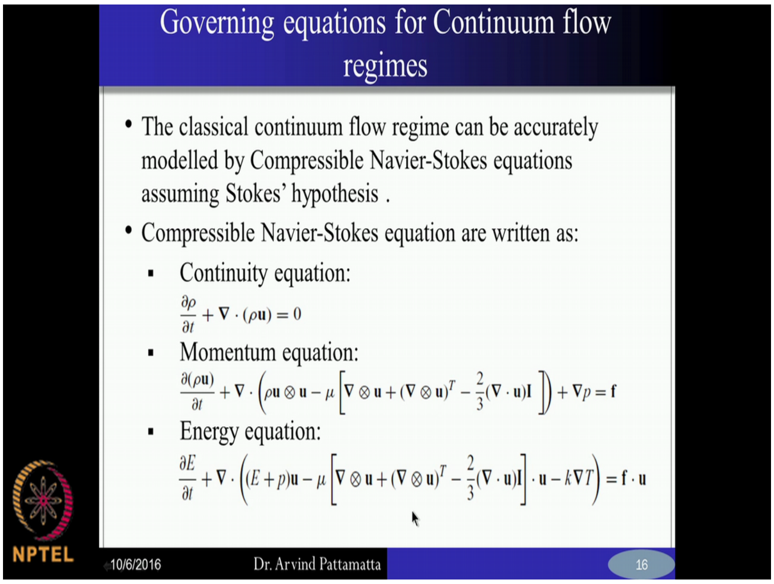

Governing equations for Continuum flow regimes

- The classical continuum flow regime can be accurately modelled by Compressible Navier-Stokes equations assuming Stokes' hypothesis .
- Compressible Navier-Stokes equation are written as:
  - Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

  - Momentum equation:

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} - \mu \left[\nabla \otimes \mathbf{u} + (\nabla \otimes \mathbf{u})^T - \frac{2}{3}(\nabla \cdot \mathbf{u})\mathbf{I} \right] \right) + \nabla p = \mathbf{f}$$

  - Energy equation:

$$\frac{\partial E}{\partial t} + \nabla \cdot \left((E + p)\mathbf{u} - \mu \left[\nabla \otimes \mathbf{u} + (\nabla \otimes \mathbf{u})^T - \frac{2}{3}(\nabla \cdot \mathbf{u})\mathbf{I} \right] \cdot \mathbf{u} - k \nabla T \right) = \mathbf{f} \cdot \mathbf{u}$$

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Therefore, the overall governing equations is if you are looking at the Knudsen number range up to 0.1, we use the continuum approximation and we solve the compressible Navier stokes equation. The compressible Navier stokes equation which is written in a coordinate free representation. You have the continuity equation; you have the momentum and energy equation. In the momentum equation, you are having this term which is 2 by 3 del dot U. For the incompressible flows that will be 0. That is the additional term coming in the momentum equation due to compressibility and in the energy equation; you have this term here to by 3 del dot U. For the in compressible flows that will be 0 and that is the additional term coming in the momentum equation due to compressibility and in energy equation, you have this term here 2 by 3 del dot U and apart from that, you also have the pressure work this is your del dot P U.

If you are solving for energy, therefore, in the incompressible regime, you do not have this pressure work term del dot P U, will become 0 and also this term will become 0. This is your heat conduction term, this is your advection term, this is your energy transfer due to work discuss work. Therefore, your stresses and pressure will contribute to work which will therefore, affect the internal energy. These are your contribution of these stresses or viscosity to the work and this is your pressure work term, this is your energy heat conduction and there could be some external body force, which can also do work on the system and they can change the energy of the system.

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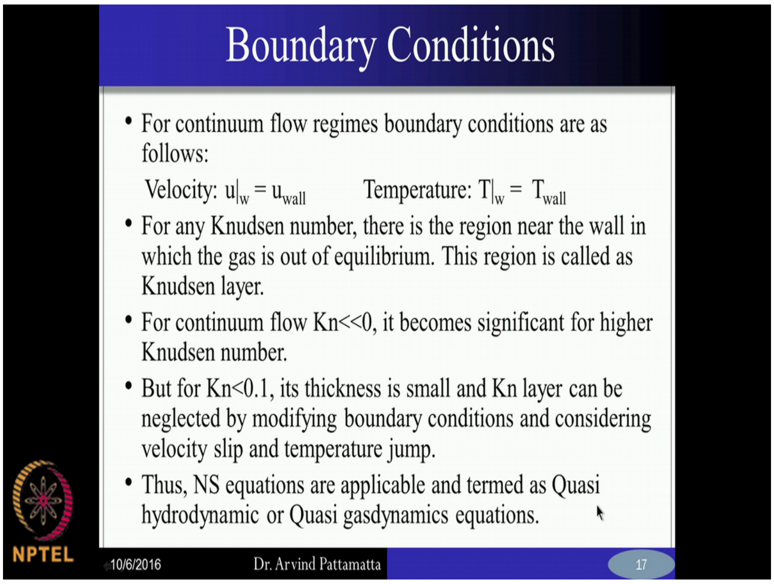

Boundary Conditions

- For continuum flow regimes boundary conditions are as follows:

  Velocity: $u|_w = u_{wall}$ Temperature: $T|_w = T_{wall}$
- For any Knudsen number, there is the region near the wall in which the gas is out of equilibrium. This region is called as Knudsen layer.
- For continuum flow Kn<<0, it becomes significant for higher Knudsen number.
- But for Kn<0.1, its thickness is small and Kn layer can be neglected by modifying boundary conditions and considering velocity slip and temperature jump.
- Thus, NS equations are applicable and termed as Quasi hydrodynamic or Quasi gasdynamics equations.

Therefore, this is the juries are the generic Navier stokes question that you will be solving for gas flows in micro channel. Therefore, in the case of continuum you just simply apply the no slip boundary condition.

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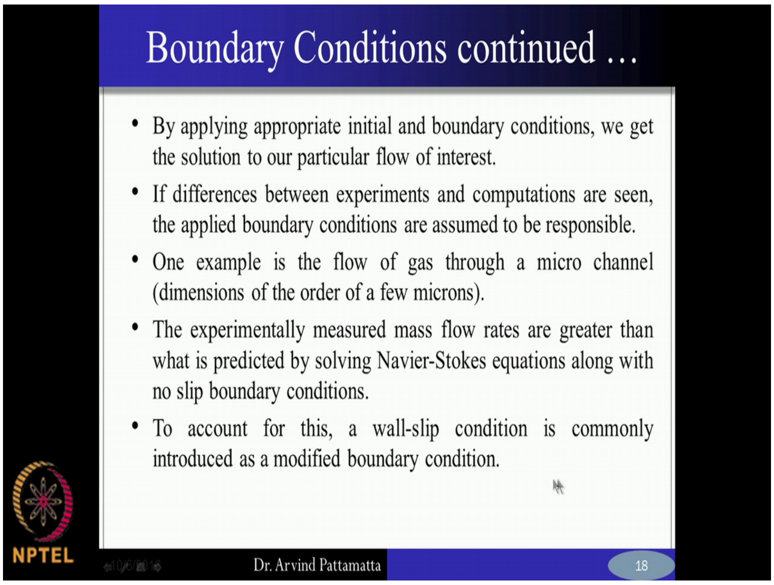

Boundary Conditions continued …

- By applying appropriate initial and boundary conditions, we get the solution to our particular flow of interest.
- If differences between experiments and computations are seen, the applied boundary conditions are assumed to be responsible.
- One example is the flow of gas through a micro channel (dimensions of the order of a few microns).
- The experimentally measured mass flow rates are greater than what is predicted by solving Navier-Stokes equations along with no slip boundary conditions.
- To account for this, a wall-slip condition is commonly introduced as a modified boundary condition.

But you know now as we are going towards larger and larger Knudsen numbers, we will see that particularly there is a region near the wall in which the gas molecules are out of equilibrium with the wall itself. We have seen this in the Nanoscale very clearly. We have a temperature jump at the wall. Similarly there is a velocity jump also and this becomes significant for higher Knudsen number. What do we do? How do we then account for this kind of sub continuum phenomena into a continuum picture?

This is done a little bit you know empirically. By introducing what is called as a wall slip boundary condition.

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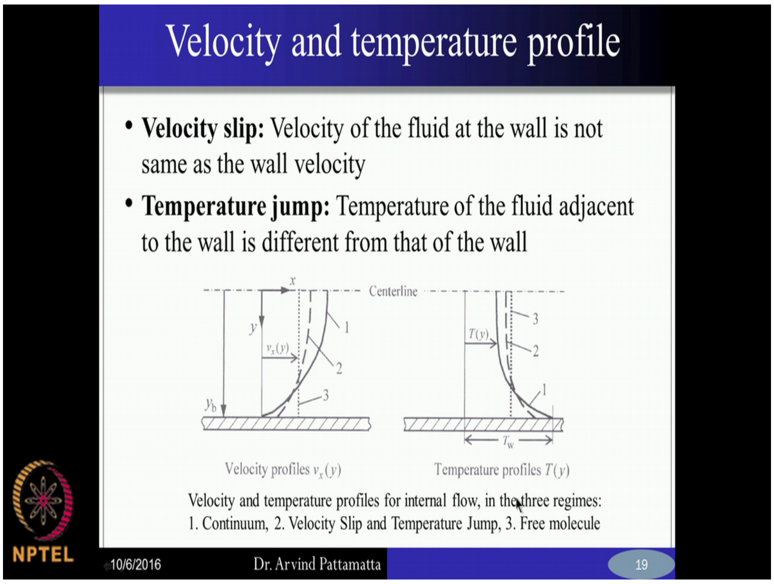

Velocity and temperature profile

- **Velocity slip:** Velocity of the fluid at the wall is not same as the wall velocity
- **Temperature jump:** Temperature of the fluid adjacent to the wall is different from that of the wall

Velocity and temperature profiles for internal flow, in the three regimes: 1. Continuum, 2. Velocity Slip and Temperature Jump, 3. Free molecule

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There are 2 kinds of slip, if you are talking about momentum transport, that is your velocity slip and how does it look? In your conventional flow between 2 parallel plates, if you draw from the center line to 1 end you have a profile which is a parabolic profile for example, but when you increase your Knudsen number, at say Knudsen number of 0.1, you will not see a profile which is becoming 0 at the wall, but it will look like it to have a finite velocity. This is your velocity jump similar to the temperature jump that we derived in the Nanoscale transport and temperature jump is also and; obviously, a consequence of this. That is because at this particular region in this region, you do not have equilibrium in momentum and energy between the gas molecules and the wall.

There are going to be hardly you know the mean free path is no the Knudsen numbers are large, the mean free path is large and therefore, the gas molecules are likely to collide with the wall, more likely to call it with the wall than themselves and therefore, you know they travel with a certain information of momentum and energy and when they encounter the wall they see that there is a jump. Therefore, they cannot adjust suddenly to that and therefore, the jump remains. This jump is there as you can see here and what happens to very very high Knudsen numbers? Finally, you see the jump becomes very sharp. You have a velocity profile which is maybe a vertical line and then from there it has to jump to 0 at the boundary, similarly with the temperature. The profiles therefore, shift as you are looking at higher and higher Knudsen numbers from a more gradual variation to a very sharp jump.

This is usually observed even with the Knudsen numbers of 0.1 and therefore, you know although this is a completely molecular phenomena, it is very difficult to understand how to incorporate this in a continuum framework. See 1 of the earlier earliest models to incorporate this into continuum came from Kundt and Warbury, 18750.

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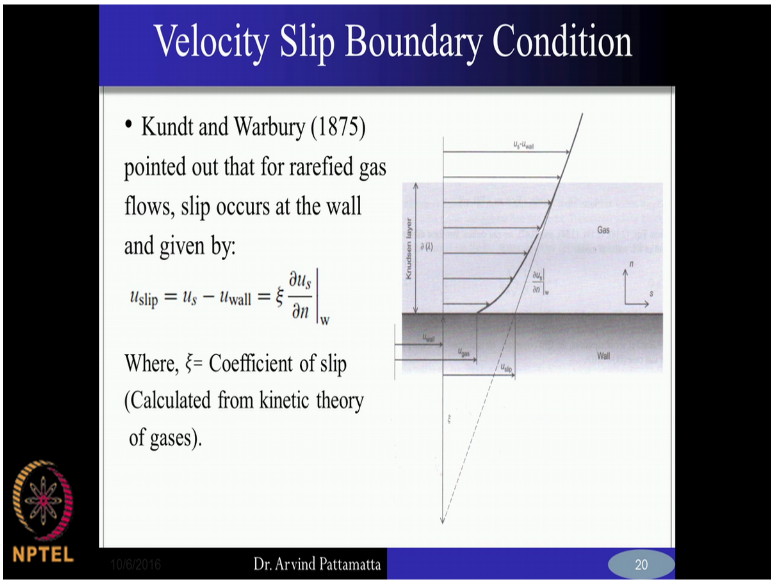

Velocity Slip Boundary Condition

- Kundt and Warbury (1875) pointed out that for rarefied gas flows, slip occurs at the wall and given by:

$$u_{\text{slip}} = u_s - u_{\text{wall}} = \xi \left.\frac{\partial u_s}{\partial n}\right|_w$$

Where, ξ= Coefficient of slip (Calculated from kinetic theory of gases).

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They just suggested that you know this slip, it appears that is an apparent slip length; that means, if you join this profile velocity profile at the wall and you continue and extended to inside the solid it will become equal to 0 somewhere at a depth of zeta from the no edge of the wall. This value of zeta is called the slip length. It is some kinds of virtual origin. We can say from which the velocity profiles start you know from value of 0 and then they continue to increase. In the case of small very small Knudsen numbers less than 0.001, this slip length will be approaching 0.

Therefore, directly the apparent origin will be directly from the wall. Now, for higher Knudsen numbers this origin or virtual origin seems to shift into the wall because of the slip effects and therefore, the slip length zeta seems to be a finite value and this increases with your Knudsen number. How do we incorporate therefore, the slip effects, we just give a value of slip velocity at the wall, which is the slip factor slip length times the gradient of velocity at the wall. This is a kind of dimensional analysis. You know the gradient at the wall gradient of velocity you multiply it by the slip length. This will become the units of velocity and therefore, this is an ad hoc you know a very simple ad hoc manner to incorporate the effects of slip now this coefficient of slip is going to become an empirical parameter. We do not know how to exactly evaluate this.

Therefore, people do experiments for different fluid and solid combinations they observe what is this slip length; that means, they measure this velocity profile and they extrapolate it and check where the virtual origin is and they obtain the value of the slip length and that is used in the numerical model. There is no rigorous theory for obtaining this, but there are some empirical ways of determining the slip length. People for example, Maxwell in 1879 have developed some expressions for inflating the slip length. 1 of the expression is that your slip.

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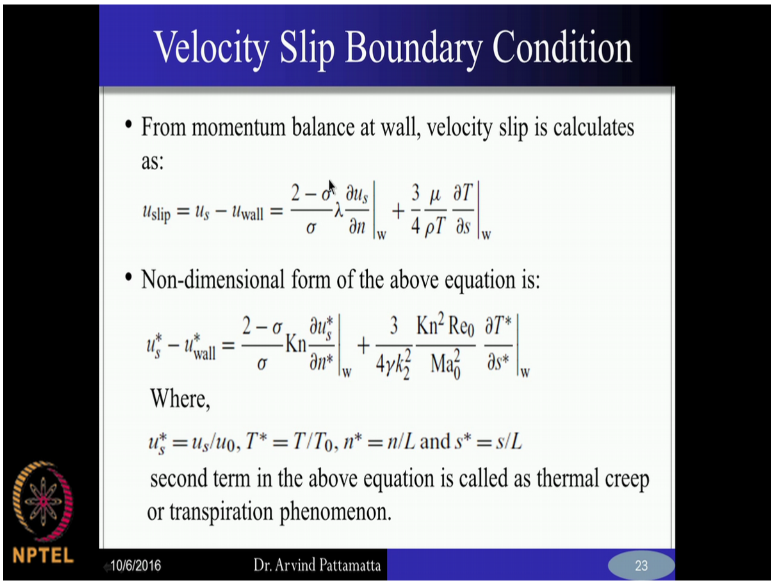

Velocity Slip Boundary Condition

- From momentum balance at wall, velocity slip is calculates as:

$$u_{\text{slip}} = u_s - u_{\text{wall}} = \frac{2-\sigma}{\sigma}\lambda \left.\frac{\partial u_s}{\partial n}\right|_{\text{w}} + \frac{3}{4}\frac{\mu}{\rho T}\left.\frac{\partial T}{\partial s}\right|_{\text{w}}$$

- Non-dimensional form of the above equation is:

$$u_s^* - u_{\text{wall}}^* = \frac{2-\sigma}{\sigma}\text{Kn}\left.\frac{\partial u_s^*}{\partial n^*}\right|_{\text{w}} + \frac{3}{4\gamma k_2^2}\frac{\text{Kn}^2\,\text{Re}_0}{\text{Ma}_0^2}\left.\frac{\partial T^*}{\partial s^*}\right|_{\text{w}}$$

Where,

$u_s^* = u_s/u_0$, $T^* = T/T_0$, $n^* = n/L$ and $s^* = s/L$

second term in the above equation is called as thermal creep or transpiration phenomenon.

Slip length can be expressed as a function of what are called as accommodation coefficient. This accommodation coefficient is denoted by Sigma and your X that zeta is equal to 2 minus sigma by sigma times Lambda where lambda is your mean free path here. We are now bringing in and again another constant we are rewriting the slip length in terms of the accommodation coefficient sigma. The question is how we then evaluate the accommodation coefficient.

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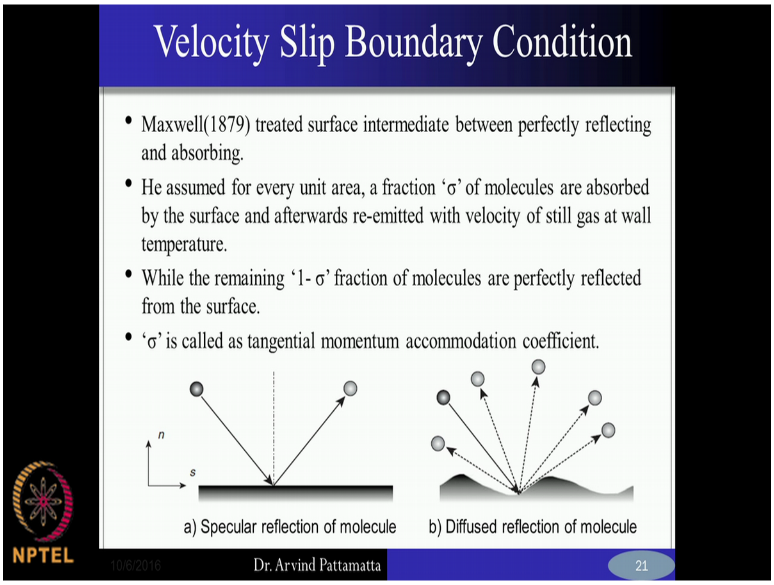

Velocity Slip Boundary Condition

- Maxwell(1879) treated surface intermediate between perfectly reflecting and absorbing.
- He assumed for every unit area, a fraction 'σ' of molecules are absorbed by the surface and afterwards re-emitted with velocity of still gas at wall temperature.
- While the remaining '1- σ' fraction of molecules are perfectly reflected from the surface.
- 'σ' is called as tangential momentum accommodation coefficient.

a) Specular reflection of molecule

b) Diffused reflection of molecule

Maxwell tells that you know depending on what kind of interface or boundary that you are dealing with, the value of the accommodation coefficient can be different.

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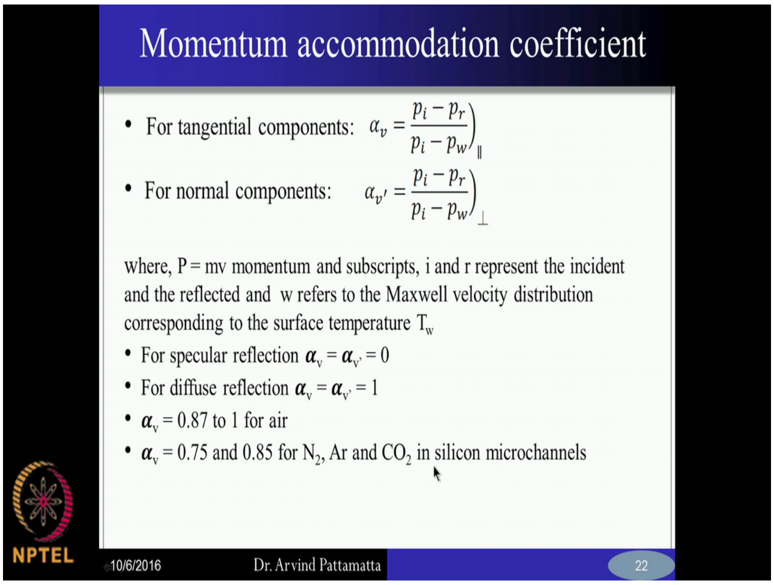

Momentum accommodation coefficient

- For tangential components: $\alpha_v = \left.\frac{p_i - p_r}{p_i - p_w}\right)_{\parallel}$
- For normal components: $\alpha_{v'} = \left.\frac{p_i - p_r}{p_i - p_w}\right)_{\perp}$

where, P = mv momentum and subscripts, i and r represent the incident and the reflected and w refers to the Maxwell velocity distribution corresponding to the surface temperature T_w

- For specular reflection $\boldsymbol{\alpha}_v = \boldsymbol{\alpha}_{v'} = 0$
- For diffuse reflection $\boldsymbol{\alpha}_v = \boldsymbol{\alpha}_{v'} = 1$
- $\boldsymbol{\alpha}_v = 0.87$ to 1 for air
- $\boldsymbol{\alpha}_v = 0.75$ and 0.85 for N_2, Ar and CO_2 in silicon microchannels

For example if you are talking about the specular reflection, that we discussed the last 2 classes, where you have a gas molecule coming in and reflecting with the same angle, but in the opposite direction, then you have a value of these accommodation coefficients. I think we have used different notation here. Let me just see I think we have used the different notation.

This alpha here should be nothing, but sigma. I think we have just changed the notation here. Please correct it. According to the case where your specular reflection is there, the value of sigma will be equal to 0 for diffuse reflection, the value of sigma will be equal to 1. Now, let us check if this makes sense when you put it yeah, when you put it to 1 for example, for diffuse reflection. So, this becomes this Lambda times D U S by D A and when it approaches 0, I think 0, Perfect effect 0 is not a practical case, but somewhat smaller than this value will be increasing. This will be a very large value, but most of the cases, where you are dealing for example, if you are talking about air, for this case the value of the accommodation coefficient can be somewhere between 0.87 to 1. Usually this is on the tending towards the diffuse reflection psi. In that case you know, it will be just close to Lambda times D U S by D N, where Lambda is the mean free path.

This was the model given by Maxwell. What he says that you know I think we do not have time will stop here, but yocu know, he looks at it in a very simplistic manner that what is the cause of the non-equilibrium at the wall. It is basically reflection of these molecules from the boundary and the way these molecules are reflected either they are specular or diffuse will govern the kind of non-equilibrium. Therefore, he just proposes the accommodation coefficient to define what is the deviation of the values of momentum from the equilibrium values just like you have F non equilibrium and F equilibrium? He just defines you know the values of momentum deviation from the Maxwell Boltzmann distribution function. Amount of being the more the amount of deviation, the higher is the non-equilibrium. We will just stop here.