Fluid Dynamics And Turbo Machines.
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Part B.
Module-1.
Tutorial.
Integral Analysis.

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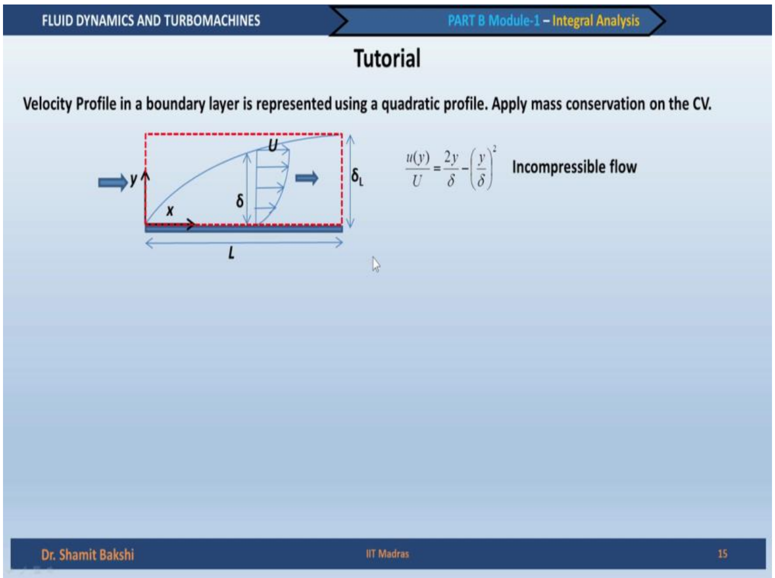

Good morning, so welcome to the tutorial session for the 2nd week. We will demonstrate some problems to look at the topics discussed in the 2nd week, that is integral analysis. So, the first problem deals with a velocity profile in the boundary layer. So, basically we have used this problem to demonstrate certain concepts in the first week also where we demonstrated how you can apply boundary conditions. For this week this tutorial, we, what we will try to do is we have taken, so this is a flat plate and there is a flow, uniform flow, so this arrow indicates a velocity U capital U and this is facing this area, so it is moving on top of this plate. As it passes the plate, a boundary layer is formed, as we have defined before the boundary layer is a region where the viscous forces are important.

So, in the boundary layer we have a velocity profile like this, we have taken, assumed a quadratic profile, you can again see here like in the problem demonstrated in the first tutorial that the boundary conditions are met even by this. Like at Y is equal to 0, the velocity is zero. At Y is equal to Delta, Delta is basically the thickness of the boundary layer at any particular point. So, Delta is a function of X, we have to note that. So, X and Y coordinates are demonstrated here, so at Y is equal to Delta, u is equal to U, that is also, that **co co** condition is also met. And if you take a derivative, you will find that du by dy will also come out to be zero at Y is equal to Delta. Of course that is just for our check but it is not the actual problem which we are trying to do here.

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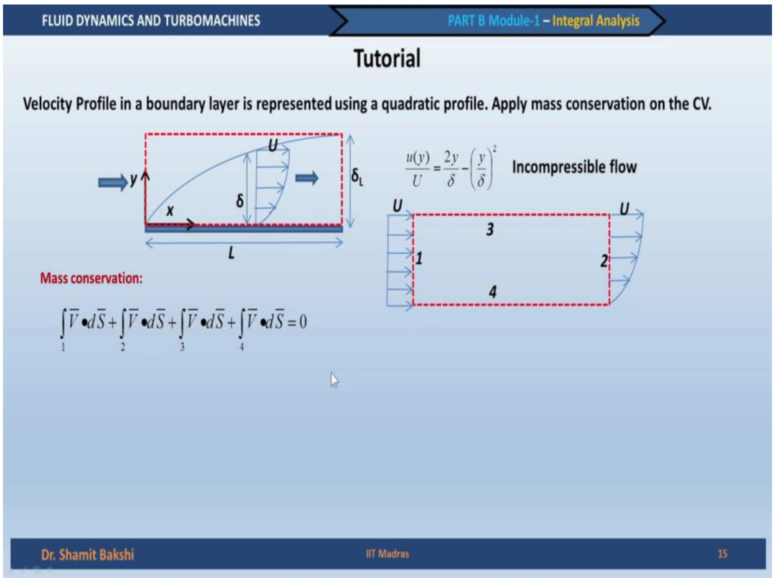

What we want to do in this case is we by taking this as an incompressible flow we want to apply mass conservation on this control volume, the control volume is the rectangle marked by this red dashed line. So, let us apply mass conservation for this control volume. So we take out this control volume and look at the flux or look at the quantities, relevant quantities, relevant quantities, the mass flow rate coming in and going out. We define these 4 control surfaces 1, 2, 3, 4, then as you can see here the velocity at the inlet, that means the velocity at the left control surface is uniform velocity. That means if we move along the surface, the velocity is constant, it is same as capital U.

On the right-hand side we have a velocity which is not constant along this control surface, which is changing. So, we have to utilise the integral form here. So, we look at that. But just to look at all the velocities at the inlet it is uniform, at the outlet it is the boundary layer profile as given this equation. Now let us apply the mass conservation equation. For the mass conservation equation in the integral form, it is like this that dM by dt should be equal to 0, that is for a system and for this control volume of course this is incompressible flow and the steady flow. So, for this incompressible the time variant term is not appearing here. The boundary layer, this is a steady boundary layer, so the time variant term is not appearing, only term which is appearing here is the integral, integral of V bar dot dS over the entire control surface, the control surface can be now is written into 4 parts, the first, 2nd, 3rd and 4th control surface.

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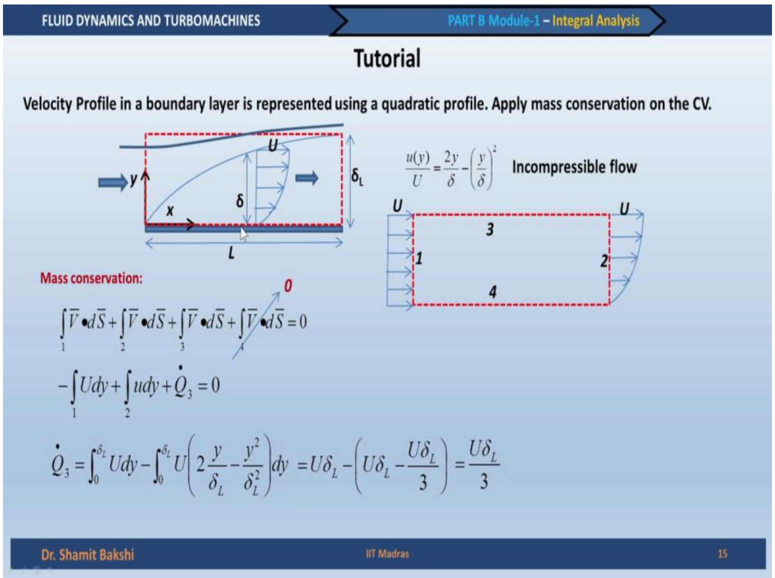

So, integral of V bar dot ds over the control surface 1 plus that for the control surface 2, 3 and 4 should be equal to 0. Basically this is the statement of the mass conservation form of Reynolds transport theorem. So, let us apply this for this particular case. If we see a 4th one, see this is V bar dot DS, so this is, this considers the velocity component perpendicular to the surface, if you consider the 4th surface, perpendicular to this surface there is no velocity because this is a wall. So, the perpendicular to the wall the velocity is zero, this component disappear, so we can write this in the form as shown here. The first term is written as integral over 1 minus U dy. So how does this become minus, this is this comes with a negative sign because the direction of velocity, the magnitude of velocity is uniformly capital U the direction of velocity is positive, that means in this direction, in the positive X direction.

But the direction of the surface vector, the area vector, sorry, the area vector, so the direction of area vector is perpendicular to this surface, surface 1. So, the area vector is directed opposite to the velocity vector. Of course that is always true for an inlet like we showed in the application of Reynolds transport theorem previously also. So, U and the area vector makes 180 degree with each other and if they are one, making 180 degrees with each other, so this negative sign will come. So, this is just by directly applying this equation. Secondly, for the 2nd part, that means the 2nd surface, integral over the control surface 2 U dy, so this is again, this will be positive because the direction of the velocity is same as the direction of the area vector which is perpendicular to this surface.

So, this remains positive and we have written small u here indicating this is a velocity profile, not a constant velocity, whereas in this case this is capital U which is a constant. Now, the important part to notice here is that the 3rd surface, we cannot actually directly write an expression, from the given information we cannot directly write an expression for velocity here, we do not know the velocity here. So, what we can do by applying mass conservation we can find out if there is any velocity in the 3rd boundary. V bar dot DS is actually the volume flow rate, so this is in the scalar farm, this is velocity into multiplied by area. So, this is basically the volume flow rate.

So, we write this as volume flow rate plus Q 3 dot is equal to 0. So, this is our equation, let us see if there is any volume flow rate or volume fluid flowing out or flowing into this control volume through the 3rd boundary. So, if you do this, simplify this further, you will get Q3 dot is equal to we take it in this right-hand side then this becomes U dy and this becomes like this where we have plugged in the velocity profile from here into this. So, U is equal to, capital U multiplied by this into dy. This is quite useful because this is useful also to find out the volume flow rate through a surface but the velocity is not constant, it is varying along the surface. If that is the case, then you can directly plug in the component of the velocity and integrate. Now the integration is with respect to Y, so the limits of Y is 0 to Delta L.

See the way the control volume is taken is Delta L, Delta is actually the control sorry the Delta is actually the boundary layer thickness at any X point, Delta L is the boundary layer thickness at the end of the plate. So, this is basically Delta L. So, we integrate it between 0 to delta L U dy 0 to delta L this one. So, let us integrate these 2 functions and let us see, so this will be directly capital U into delta L and this one comes out as minus U delta L minus, sorry minus of U delta L minus U delta L by 3. So this is basically the final expression. So, finally we get this as U delta L by 3. So, this is a very interesting information which says that the Q3 dot is actually not zero. In fact it is one 3rd of U into delta L, U into delta L is actually the volume flow rate coming into this control volume, U multiplied with this area.

Of course while writing the expression for area here, I have missed out one point, I have written there are DS bar as dy, what we assume here is that we take, this is a two-dimensional situation, so we take dy multiplied by 1. 1 is the unit length in the direction perpendicular to the slide. So, in the direction perpendicular to the slide we take a unit length, thereby we get, actually if you to write this, you can write it as U dy multiplied by 1 or some constant, let us say the width W, but this will come out of the integral as it is a constant. So, now considering this is, if you consider this expression which we have obtained here applies, if you consider unit length perpendicular to the control volume.

If you consider a length let us say W perpendicular to the control volume, then this will be multiplied by W, that is the only difference. Now, coming back, we see that the Q3 dot is actually 1/3 of the volume which is coming into this control volume. So, 2/3 goes out here through the other end of the plate, 1/3 actually goes out through this part of the plate. This is very interesting. Now when can you have velocity going out through the control surface 3, that means that there is a velocity component perpendicular to the surface. So, which means if you draw a streamline, so this is blue thick line is a representative streamline. So, if you draw a streamline through this, it will actually bend.

Why it has to bend, because by the definition of the streamline we know that if we draw it and into the streamline, it should show the direction of velocity. Now the fact that on this control surface you have a V component of velocity means that the streamline should bend, it cannot be parallel to the plate, it should bend. So, in the free stream, it is from this simple application of the mass conservation we can see that in the free stream outside the boundary layer, the streamlines will slightly bend to account for the small velocity present exiting the control volume taken within the boundary layer within little portion outside the boundary layer. So, this bending of the streamline is nicely demonstrated through this problem.

But if you go further above then the streamlines will be straight because they will represent the uniform flow in the free stream. Very near to the boundary layer there will be bending of the streamline, although the entire region is basically inviscid. So, this is the first problem, here we see in case of this flow past a flat plate or flow over a flat plate, the application of the mass conservation.

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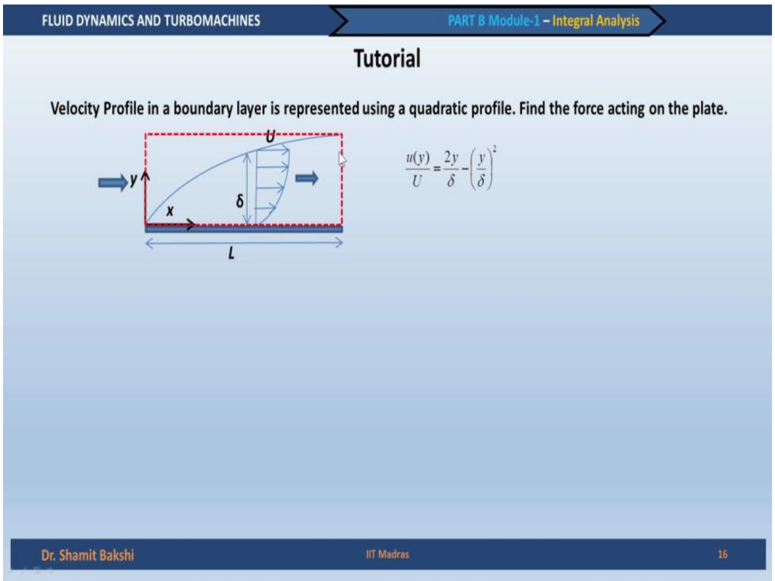

The 2nd problem is an extension of the first problem, this deals with the application of the momentum conservation to the same problem. We saw in the last problem, the application of mass conservation, in this problem we see the application of momentum conservation. So, the velocity profile is same, what we want to find is that what is the force acting on the plate. So, this is the plate, we want to find out what is the force which is acting on the plate. In other words, we can say what is the force required to keep this plate stationary. Like we saw in the previous application of the momentum conservation, how do you, what is the force required to keep a plate which is oriented perpendicular to the flow, how to keep that plate stationary, here we see if the plate is aligned along the direction of the flow, what is the force required to keep that plate stationary.

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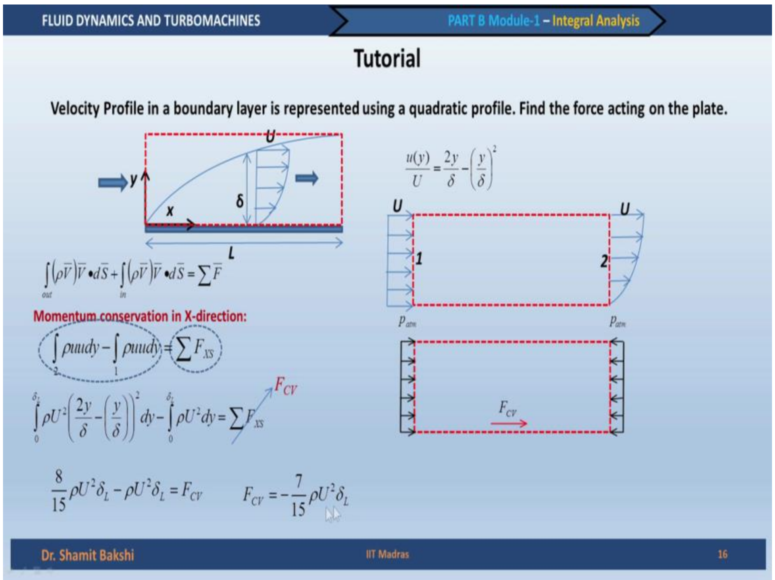

So, we again take the same control volume as in the last problem, then take over this control volume and then apply the momentum conservation equation. The general form of the momentum conservation equation is this for a steady flow. The unsteady flow is already dropped because this boundary layer which we are talking about are steady boundary layer. It is a steady flow, it is a time invariant flow. Let us apply momentum conservation in X direction because they want to find out the force acting on the plate in the X direction. We name these 2 control surfaces 1 and 2 and then try to apply this equation, so if you apply this equation, what you get is basically for the exiting surface, so this is basically the exiting surface, 2nd surface is basically the exiting surface.

Rho U multiplied by U dy integral of that, so basically this V bar becomes U because we are talking about momentum conservation in X direction and V bar dot DS is again U into, it is a scalar quantity and it is U into dy. Of course, even in this case, we have assumed a unit length perpendicular to the control volume. We can take it as some other constant quantity also and multiply this expression with that quantity. Okay, so for the, this is for the exiting surface, for the incoming surface you can write a similar expression and this is basically the sum of all the forces acting in the X direction, sum are all the surface forces acting in the X direction because body forces are neglected here and they are also unimportant in this case.

So, let us look at the left-hand side first. The left-hand side like in the previous case you have a uniform flow at the inlet and at the exit you have a velocity profile like this. So, how to find out the momentum of the fluid is coming in and the flow is going out? So, again we integrate the control surface extends from 0 to Delta L, delta L is basically the thickness of the boundary layer at the exit of the plate, that is the quantity which you need to know to solve that problem, like we utilised that in the last problem also, in the demonstration of the mass conservation also.

So, we, this is the 2^{nd} one, so the 2^{nd} one plug-in the value of this velocity profile here, you can notice that the square of this velocity profile this expression in the velocity profile comes into this quantity. Here this part is constant because this is a uniform flow. Now we let us talk about the right-hand side. What is the right-hand side? So, let us take it in a different control volume. So, in the right inside, what are the quantities on the left surface, you have atmospheric pressure working, so basically this is all the surface forces essentially the surface forces constitute of the pressure force and the friction force. So, you have uniform, so this is open to atmosphere, so you have atmospheric pressure, at the exit also same atmospheric pressure.

In fact what you can do is to consider the surface forces, you can always talk about gauge pressure, that means the pressure difference between the static pressure and the atmospheric pressure. And if you talk about gauge pressure, both these pressures will be zero. Okay but in this particular problem we have taken atmospheric pressure, so let us proceed like that but even the results will be same. So, what we see here, the area is same, the pressure is also same so, these 2 actually and the direction is always perpendicular and going into the control volume for the pressure force, that is always the case. So, this, for these 2 control surfaces, the pressure forces will oppose each other or compensate for each other, so what we are left out with is a force acting on the control volume from the plate side, from the top side there is no force in the X direction, so from the plate side let us say FCV is the forces acting from the late on the control volume.

So, we replace this with just F CV because the pressure force cancels each other. Now, if you integrate this path, you will get this expression 8 by 15 rho U square delta L, this is of course very simple to integrate, it is rho U square into delta L and the right inside is F CV. So, if you do that, simplify this further, what do you get, you get that FCV as -7 by 15 rho U square delta L. So, basically this is the force acting on the control volume. So, what is the force acting on the plate? What we are interested in is to find the force acting on the plate.

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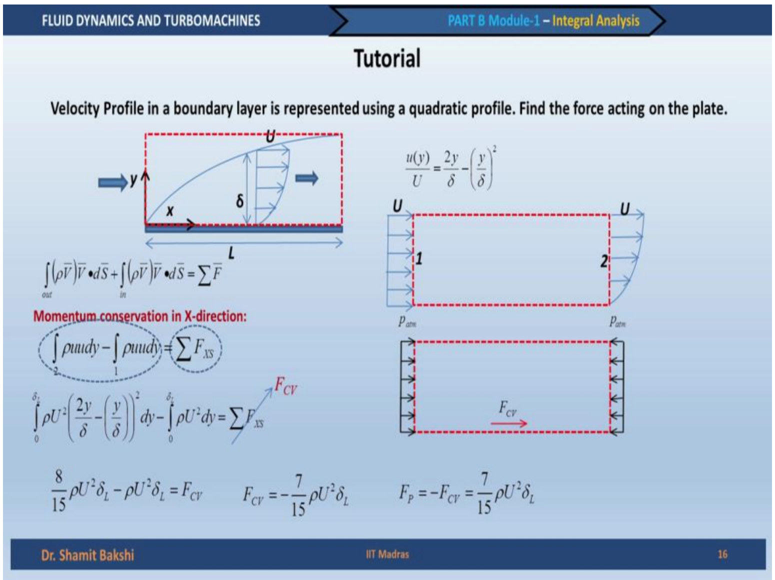

So, if you want to find that, it will be negative of, minus of the force acting on the control volume, so this is 7 by 15 rho U square delta L. So, systematically we can find out the force acting due to the shear. So, this basically this force is basically due to the shear on the surface. It experiences, the plate experiences efforts in this direction. Actually you can think about it, this is about getting to a quantitative value for the force but physically if you look at it, what happens here, let us understand the situation. So, we see that this is the control volume, so in this control volume there is a fluid momentum or there is a momentum of the fluid which is coming into the control volume. So, this transmits a force in this direction and momentum of the fluid which is exiting the control volume, in this particular situation, the momentum of the fluid exiting the control volume in X direction is actually less than the momentum of the fluid is coming into the control volume because the fluid actually decelerates due to the viscous forces as it comes into this control volume, as it comes **comes** into the top of that, the area on the top of the plate.

So as the fluid decelerates, it exits the plate at a lower momentum and this momentum deficits, that means the incoming momentum, the incoming liquid momentum and the outgoing liquid momentum, the difference between that is transmitted as a force to the plate. This also means that if you can have a momentum gain of the fluid while it crosses the control volume, then you can derive a propulsive force, a propulsive force means it is you can derive a force which can move the plate or move the body in the direction opposite to the direction of the flow. And this is a important aspect in the case of fish propulsion where actually develop a momentum at the exiting surface which is more than the incoming momentum, that it does by moving it its fins in a appropriate manner.

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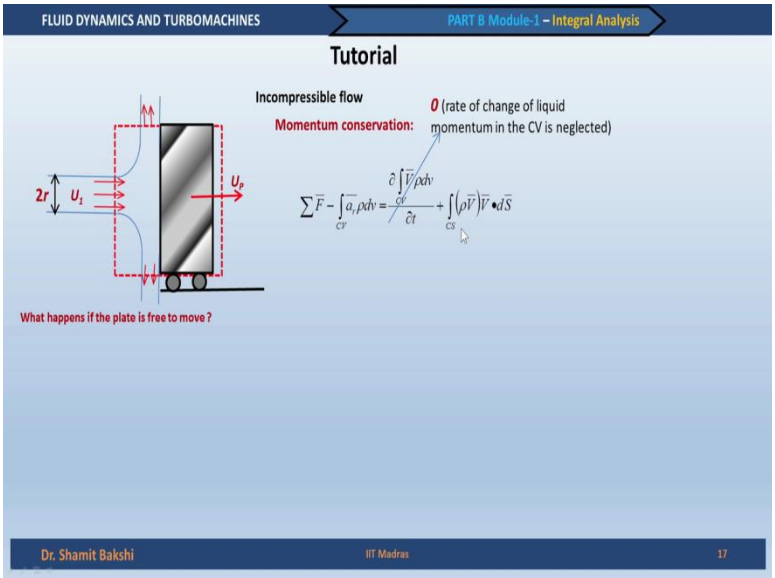

In the next problem, so what we do again this is an extension of the problem which we have demonstrated before. The problem which we had taken before is to find out the force is on a plate. So, this is basically something like that plate but we have added rollers on the bottom so that this can move in this, it is free to move in the X direction. And these rollers are frictionless, so it can, it does not give rise to any resistance to the motion of the particular object. We know what happens when we keep this object stationary, it imparts a thrust onto the plate. Now, what we want to see in this problem is what happens is the plate is free to move like this.

So, as we can imagine the force will make the object to move in the direction of the flow and finally it will come to the flow velocity. Let us see if we can mathematically represent this situation. So, this, as this object is free to move, so it has a velocity vector in this direction, let us take a control volume around the object. So this control volume is taken in such a way that both the sides we have atmosphere. We have avoided like we have demonstrated before, we avoided taking the control volume on the plate surface because there we have to consider the plate for and again draw a free body diagram of the object. Instead of doing that, we have taken an object which actually has atmosphere on both the ends. So, the difference between this case and the other case is that here the control volume is moving and not only that it is moving at a, not at a constant velocity. So, it is accelerating.

So, we take incompressible flow and we see how we can apply the momentum linear momentum conservation equation for a accelerating control volume. The reference frame as we are saying again and again is always fixed to the control volume. So, this is a problem with a non-inertial frame of reference. So the momentum conservation can be written as this form, this is what we derived while we derived the expression for the the force in the case of a non-inertial frame of reference not in the general form but for a control volume moving at, moving with the linear acceleration. So, this is the extra term which come there which is basically the inertia force acting on the control volume. So, acceleration of the control volume rho into dv.

This small v of course is the representative of a volume, not of the velocity, we have to keep that in mind. Now, what we do to simplify this situation, we take this part as zero. But actually speaking, we, this is not strictly true. For the previous cases, this time but was zero because it was a steady problem, but in this case this is not actually zero but what we do, we neglect the rate of change of liquid momentum in the CV. So, by doing this, actually all this pertains to the liquid, the liquid velocity or the fluid velocity more generally and we neglect the momentum change of the fluid in that CV. So, by neglecting that we can remove this term, so that we can deal with a more simple situation. And this is the net rate of momentum exiting the control surface.

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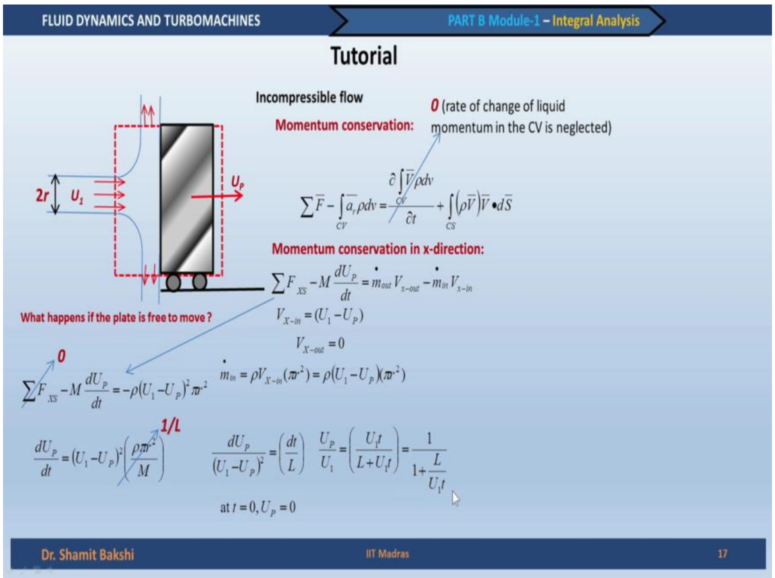

So, now let us write the momentum conservation in the X direction. In a more simple way, this can written as sum of all the forces should be equal to rate of, so this is basically the mass of the object of this body multiplied by its acceleration. So, this is basically acceleration of the object which is given by UP or you can say, it is like a thick plate, so you can say, we said that, UP as the, as its velocity. So, this is dUP by dt, M into dUP by dt, rate of change of momentum, M is not changing with time. In the rocket problem, it makes, it is little more complicated because M also changes with time. For this problem, M is time invariant. The mass of this plate is not changing with time. So, we get this term and then on the right-hand side it is actually the momentum of the fluid exiting the control volume subtracted to momentum of the fluid is coming into the control volume. So, M dot out multiplied by the velocity, outgoing velocity, M dot in multiplied by the incoming velocity.

Now, the incoming velocity, the velocity always have to be with respect to the control volume and the control volume is boring with a velocity with a velocity UP. So, if we have to write the velocity U1, right VX in, that means velocity inlet velocity is X direction, we have to write it in terms of the control volume fixed coordinate system. U1 is basically with respect to the ground fixed coordinate system, so VX in will be U1 minus UP, this was also true for a non-accelerating but moving control volume. So, let VX in is U1 minus UP, VXout is zero because even when the control volume is moving, with respect to the control volume there is no fluid is exiting the control volume, in this direction there is no fluid exiting the control volume in X direction.

Of course in Y direction it is exiting but that is not important for moment, when we are looking at momentum conservation in X direction. So, V Xout is 0 and we can write M dot in as rho VX in into pie R square, which is basically V dot DS here. So, VX in is U1 minus UP and we can further, then by plugging in this these 3 expressions into this equation we can write that sum of all the forces is equal to minus of this quantity which comes from here. So, if you see here, the outgoing part cancels, is zero and the incoming fluid momentum comes from the product of this and this, so that is minus of rho into U1 minus UP square pie R square. Pie R square is the area of the jet.

Now to go further, this part is zero because this is, this was present when we found out the reaction on the plate. But now as the plate is moving and we have taken this as the control volume, there is no force other than the inertia of the plate which is working on the, acting on the plate. So, this part is zero, if you look at this control volume, there is no X force acting, of course we could have taken the pressure multiplied by this area and pressure multiplied by this area, they will cancel each other like in the previous problem, so we have not taken that.

Other than, other than this there is no surface forces existing for this control volume, this part remains as dUP by dt and other right-hand side we have this equation. So, we have a differential equation like this here.

In the previous problem when the plate was moving at a constant speed, the UP was not a function of time, now it is a function of time, so we get this differential equation, we can rewrite it in this form, that dUP by dt is equal to U1 minus UP, U1 is constant but UP is time variant. This quantity is also constant, so this has all constant qualities like the mass of the object, density of the liquid, radius of the jet, these are all constants, so we can replace this with one by L, we can write this particular form because if you find the dimensions of this quantity, it comes out to be one by reciprocal of length, so that is why we write it as one by L. Now, this is our equation, dUP by U1 minus UP square is equal to dt by L, so this is our differential equation, we integrate this equation with this initial condition.

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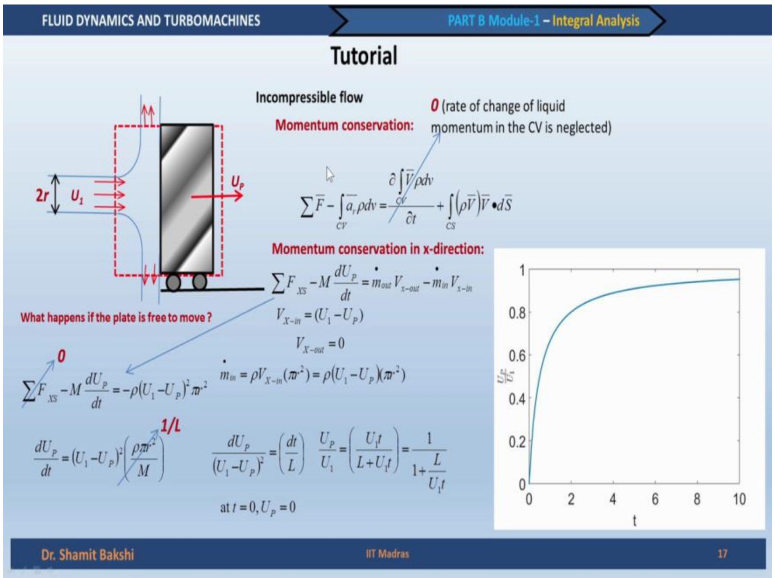

That means at T equal to 0 UV is equal to 0, the plate is that, the plate or the object is that rest. The solution which we get by integrating this is of the form, so UP by U1, U1 is constant, UP changes with time like this, we can plot this UP by U1 with respect to T, we will see what happens here as time increases. UP by U1 comes closer to 1, that means the plate accelerates and it comes close to the jet velocity. So the force acting on the plate gradually reduces because UP, what is important for the force is U1 minus UP and that value will reduce as UP comes closer to U1. So the plate start from rest, it accelerates, as it accelerates, the force acting on it reduces constantly. So as the force acting on it reduces constantly, the rate at of change of its velocity also reduces and it becomes finally asymptotic, because as it comes closer and closer as UP comes closer and closer to the jet velocity the force becomes very very small, so it comes very close to the jet velocity very quickly.

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But to come to exactly the, mathematically speaking, coming exactly to the jet velocity, it takes infinite time. So, this is basically the result of the analysis of momentum conservation applied for a non-inertial frame of reference. So, we hope that these 3 problems have demonstrated the concepts introduced in this chapter. So, there are 2 aspects of the concept of integral analysis introduced in this chapter. The first aspect is to look at the Reynolds transport theorem as an equation and then apply this equation mathematically to a particular situation which is very important. And the 2nd aspect which is probably more important for problem solving is to physically understand what is happening in this equation. This equation only tells us, this is probably one of the most important equations in the continuum fluid dynamics, it only tells us that when we consider any quantity like momentum or regular momentum or mass in a control volume, there is a rate of change term and there is a surface term.

So the sum of these 2 is equal to 0 for mass conservation or is equal to the net momentum transferred to the control volume or the force acting on the control volume in case of momentum conservation or torque in the case of angle of momentum conservation. So, this is what we have to keep in mind while solving the problems related to this particular chapter.

We will see the concepts introduced here will be useful to actually derive equation for differential analysis of fluid flow in the next chapter. Thank you.