Fluid Dynamics And Turbo Machines.
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Part A.
Module-1.
Lecture-4.
Tutorial.

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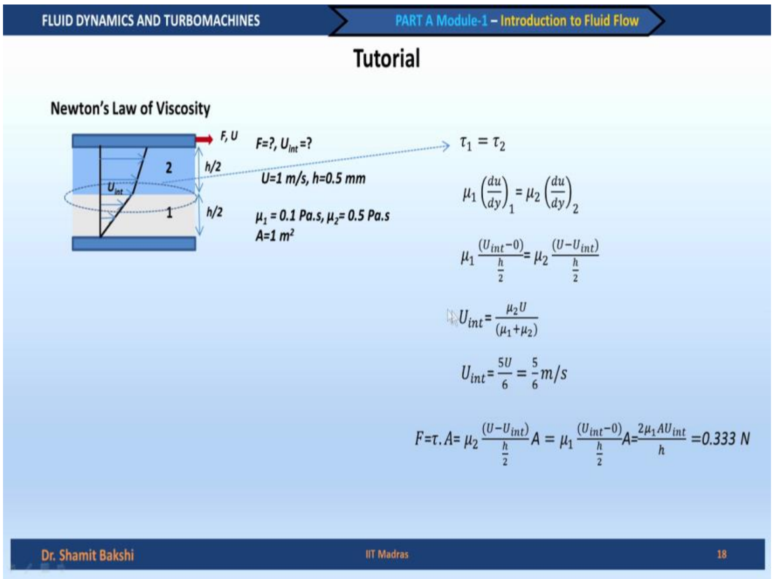

Today we are going to demonstrate some tutorial problems from the part A of this model which covers lecture 1, lecture 2 and lecture 3. So, the first problem is on Newton's law of viscosity. So, the problem statement is like this. You have 2 plates, the bottom plate is shown here, the top plate is shown here and there are 2 fluids, fluid 1 and fluid 2. The top plate is moved using a force F at a velocity U and the height of the first layer of fluid, that is fluid one is H by 2 till the half of this gap and the rest is again H by 2.

So, for this problem, these are the parameters which are given. The velocity U is given as 1 metre per second, H is 0.5 millimetre, viscosities of the first and 2nd are given, 2nd fluid are given, 2nd layers of fluids are given, 0.1 Pascal second and 0.5 Pascal seconds. So, the 2nd fluid in this given problem is more viscous than the first one. Area, the bottom area of this plate is 1 metre square.

So, what we have to find is what force is required to make the plate move at the given velocity which is 1 metre per seconds and another thing is also given here. We can see that the velocity profile, the velocity variation in the direction perpendicular to the plate is not continuous because of the different viscosities of this fluid. So, there is a change in the gradient of velocity at the interface. What is required is to find out what is the velocity of the interface at the interface. So, let us see how we solve this problem. We have to find out the force for moving the top plate at a velocity of 1 metre per second and also the velocity at the interface. While we discuss, it will be clear, why that, why is that there is a gradient discontinuity at the interface.

So, for looking at this, we look at this interface more closely. So, if we look at this interface, the shear stress on the common both the sides have to be same, there has to be the gradient of velocity can be discontinuous but the shear stress should be continuous because the shear stress is transferred through the fluid layers. The shear stress transferred at the interface by fluid one should be same as the shear stress transferred to the fluid 2. Or it is actually reversed, shear stress transferred by fluid 2 at the interface should be same as the shear stress transferred to fluid one at the interface. So, continuity of shear stress is the most important highlight of this particular tutorial problem. So, we make tao 1 as tao 2, tao 1 being the shear stress at the interface for fluid 1, tao 2 is the shear stress at the same point at the interface on the fluid 2 side.

So, if you write it in terms of, now if you use newtons law of viscosity, you can write this as mu 1 du by dy at 1 is equal to mu 2 du by dy at 2. So, you can also see from this expression itself that the gradient discontinuity in the velocity profile, that means the slope of the line in the fluid 1 has to be different from the slope of the line in the fluid 2 because the relationship which has to be maintained is the continuity of the shear stress. So, du by dy at the, in the fluid 1 side cannot be equal to du by dy in the fluid 2 side unless mu 1 and mu 2 article. So, this is the most important point in this problem, shear stress quantity, in fact there are lot of other things which you can also say. Like if one of the viscosities is very close to 0, then the velocity gradient could be very small. So, we will not talk about that problem here but let us say this is a stress continuity condition.

Now, by applying this condition, as the profiles are given as linear, we can find out the intermediate velocity first. So, for that mu 1 multiplied by this, is basically, for a linear profile, you can easily find out the gradient as U interface end of side 1 U interface -0 divided by H by 2 and this is U minus U interface divided by H by 2. So, now following this you can find out the value of interface velocity in terms of the velocity of the top plate and the viscosity ratio. So, for the given conditions, if you apply the given condition, the interface velocity is given as 5 by 6 metre per seconds. For the plate moving at one metre per second, this is 5 by 6 metre per second.

So, now we can find out the force required, that is quite simple because the force is basically shear stress into the area. So, shear stress we already obtained and the shear stress in any layer is same. The same shear stress is actually transferred from the top plate till the bottom plate. So, anyway is the shear stress is same, so you can write this as mu 2 into this U minus U in by this or mu 1 into this and then you can find out the value of the force required to move this plate at a velocity of one metre per second.

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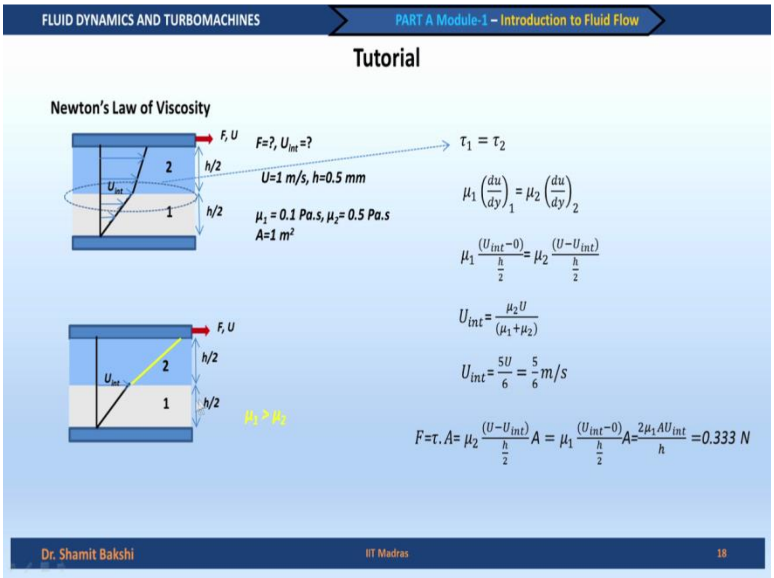

From this expression, another thing we can comment is that let us say we saw a situation where, because now we have a parametric relationship between U intermediate and U, we can say, what happens when mu 1 is less than mu 2 which is the given case. That viscosity of fluid 1 is less than the viscosity of fluid 2, viscosity of fluid 1 is 0.1 and that of 2 is 0.5. So, that means you need a very small change in velocity in fluid 2 because the viscosity is high there, you need a very small change in fluid 2 than compared to the fluid 1.

So, the velocity profile will look like this. The velocity of the plate is very close to that at the interface because the viscosity 2 is high. If you look at mu 1 is equal to mu 2 which is the same fluid condition, of course then you have just a linear profile like this. You can think about mu 1 being greater than mu 2, that means the lower fluid has a higher viscosity than the fluid at the top mu 2. Under this condition, you will have a velocity to file as shown here.

That means, for a given value of U int which also means that the given value of shear stress, because see, of course if the viscosity, if we keep mu 1 same and if we reduce the value of mu 2 for the same shear stress, then you have to, what it means is you have to move the plate at a higher velocity. You cannot move the plate at 1 metre per second and get an U int required U int for generating this shear force, you need to move it, which is of course understandable because high viscous fluid can transfer shear stresses more effectively. If 2 is less, viscosity of the 2nd fluid is less, it cannot transfer shear stress so effectively. So, this is the first tutorial.

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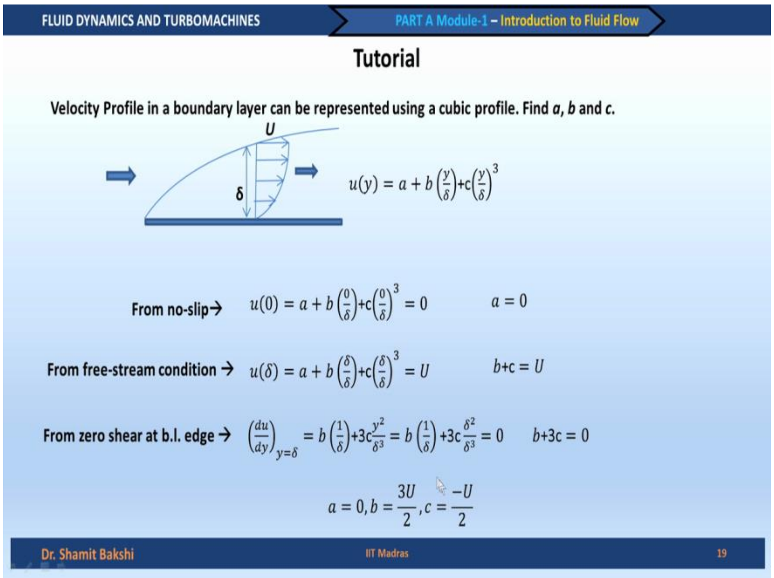

The 2nd problem is about velocity profile in a boundary layer, so we have introduced boundary layer while talking about the difference between viscous and inviscid flow. It is a region within which the viscous effects are important. Now, let us say the velocity profile in the boundary layer is given as this UY, U is a function of Y, it is given as A plus B into Y by Delta plus C into Y by Delta Q. Delta is basically the thickness of the boundary layer. So, this changes as we can see here, this changes if we move along the plane. So, let us say you have this cubic profile. The question is can we find out A, B and C, these 3 constants. So, in some sense, this is also related to 2 concepts, application of the no-slip boundary condition and Newton’s law of viscosity.

So, no-slip boundary condition is of course first thing is, we can say from no-slip, we make U0 as the velocity of the plate, the plate is not moving here, so it is 0. So, U0 is zero and you can immediately, if you plug in the value of Y, so the Y equal to 0 here, you get immediately A is equal to 0. 2nd condition is from the free stream condition. What is a free stream condition? It has to match the velocity in the free stream, free stream means the velocity at which the fluid is coming towards the plate. So, it should have the same velocity at the edge of the boundary layer. So, if you apply that condition, what you get is that the U at Delta is equal to this, so that means B plus C and U. A is already zero from first condition, B plus C is equal to U.

The 3rd condition is more interesting condition, it, you have to apply a zero shear at the boundary layer edge, the edge of the boundary layer should have a shear stress, zero shear stress. Why is that, because after the boundary layer it is inviscid, there is no shear stress transferred to from inside to outside the boundary layer. So, shear stress has to be zero here. If the shear stress is zero, by applying Newton's law of viscosity, what we can say the gradient of velocity is zero. Mu into du by dy at Y is equal to Delta should be zero, mu cannot be zero, so du by dy at Y is equal to Delta is zero. If we apply that condition, you get a 3rd condition B plus 3C is equal to 0, now you can solve between these 2 equations, you can get the values of A, B and C.

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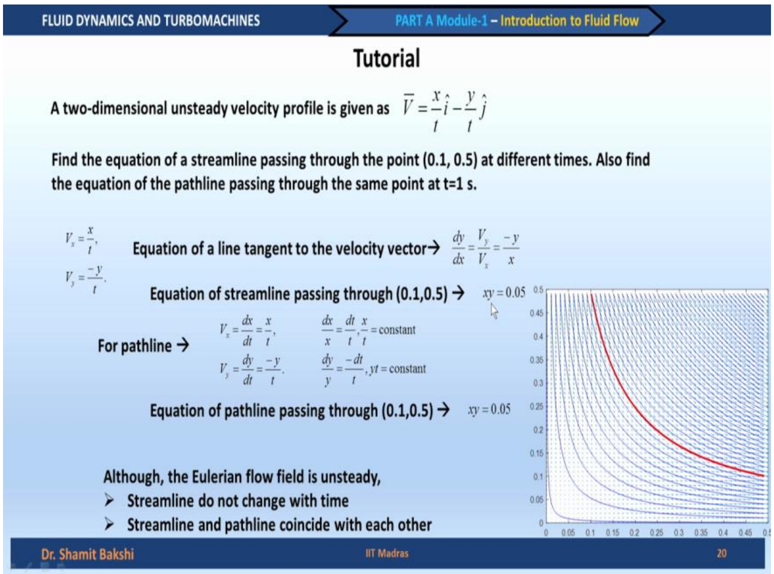

So, this problem demonstrates how to apply the boundary condition like no slip, free stream condition and zero shear condition to a velocity profiles. So, the velocity profile will look like this for the situation, for this situation, for the given conditions. We will demonstrate a 3rd problem, so this 3rd problem is related to the flow, the concept of flow visualisation, streamlines and path line. So, let us consider this velocity field, this velocity field, so now from our explanation of the velocity field, we can tell here that well this is a two-dimensional velocity field because, why is that, because the Z direction velocity is zero and not only that, the X and Y direction velocity are independent of Z. So, given that, this velocity field is a two-dimensional velocity field and it is an unsteady field because it changes with time, it reduces as the both the X components and Y component velocity reduces as time increases.

Now the problem is find the equation of a streamline passing through the point 0.1, 0.5 at different times and also find the equation of a, of the path line passing through the same point at T equals to 1 second. So, as we had seen that a streamline actually changes for an unsteady flow. But this is a very special situation, we will see what happens here, we have to find the streamline passing through this point at different times. So, X component like we saw here, the X component of flow, the flow velocity is X by T, Y component is minus Y by T. Equation, we have to write, by streamline we have to, we mean, we have to write an equation of a line tangent to the velocity vector. So, if we take the ratio of these 2, VY by VX, it should give the slope of the streamline at a location.

So, dy by dx is basically the slope of the streamline that X, Y which you can get just by dividing VY by VX and you see something interesting that T disappears. So, dy by dx is independent of T. So, even though the velocity, it is an unsteady field, the streamlines does not change because this is a, this for this situation it does not change with time. So, we can easily find equation of the streamline passing through the point point, passing through 0.1, 0.5. This can be integrated, it is dy by Y dx by X, if you take it together, it comes out to be dy by y plus dx by x is equal to 0, integrating that you can get X, Y equal to constant and you can ascertain the value of the constant by plugging in these values through which the streamline passes.

The streamline passes through 0.1, 0.5, you plugin these values here, you get 0.05, so that means X Y equal to 0.05 is equation of the streamline. So, the point to demonstrate here is that in this situation, even if the flow is unsteady, the streamline does not change with time. So, for path line, path line is a, we can see from here how to get the equation of a path line from the Eulerian velocity field like we have demonstrated the Eulerian velocity field means the velocity at a particular point, not of a particle. So, for path line we can say that VX is equal to, you can write it as dx by dt, so basically any particle passing through that point will have the same velocity at as that particular point. So, dx by dt of that particle will be given as X by T, dy by dt will be given as minus Y by T, so if we can integrate these 2 equations, we should be able to get the equation of the path line.

So, if you do that, you get X by T is equal to constant, is equal to constant and YT is equal to constant. So, if you look at the equation of the path line, again by using these 2, it is the same, XYZ is equal to 0.05. Of course this is understandable because if the streamlines are not changing with time, the particle or any particle introduced at a point has no other choice than to follow the streamline because perpendicular to the streamline there is no flow. So, it has to follow the streamlines only. So, the things which are demonstrated in this problem are that even though the Eulerian field flow field is unsteady, streamline do not change with time and streamline and path line coincide with each other.

So, this is the diagram which demonstrates the streamlines, the all the streamlines for this flow field, for this particular flow field. And the line marked in red shows the specific streamline which is asked for in this problem, that is XY is equal to 0.05. So, this, with this we come to the end of part A of the first module of this course. In the next part, that is part B, we will start with the integral analysis of fluid flow. Thank you.