

Fluid Dynamics And Turbo Machines.
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Part D.
Module-2.
Lecture-11.
Introduction To Compressible Flow.

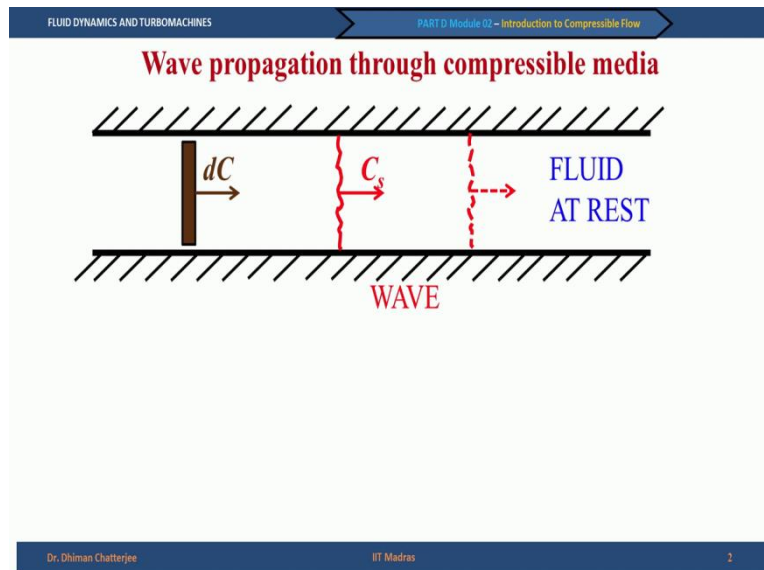
Good afternoon to you all for the 8th week's lecture on the fluid dynamics and Turbo machines. In the last week we have discussed about the hydraulic machines and in this week we will take up the steam and gas turbines. As I have mentioned in the beginning of this module on Turbo machines that the steam and gas turbines come under the classification of compressible flow machines or machines that handle compressible flows. In the fluid dynamics module, Dr Bakshi has talked about the incompressible flow. So today what I will do is, I will give a brief introduction to compressible flow.

Please note that this is only an introduction which is necessary to better appreciate steam and gas turbine. This by no means is a complete lecture on compressible flow. For that you should refer to specialised courses and textbooks on compressible flow. So let us first take the physical example of this compressible flow. You have already studied in fluid dynamics that fluid flow can be compressible or incompressible. And you have also learned that the Mach number of the fluid flow should be less than 0.3 in order to treat the flow as incompressible.

We will know this, we know these things and they will talk about some more aspects of the compressible flow in today's lecture. First we will start with wave propagation through compressible media. We should note this term compressible media. In reality there is no medium which is completely incompressible. A medium which is completely incompressible, in that the speed of sound as we can we will show it, will go to infinity. But all the fluid medium, the solid medium, whatever we can think of are compressible.

Only the extent of compressibility as was discussed in the module A will vary from gas to the liquid to solid. So when we talk about wave propagation through compressible medium, what we have to keep in mind is that we will have a finite speed of the disturbance or finite wave speed or sound speed. So how do we understand this?

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Let us say we have a long channel which is filled with fluid initially at rest and we have a piston and that piston suddenly starts to move. So we give this piston a small, infinitesimally small velocity DC . C as you have already been aware in this module, we have used it to refer to any velocity. Now if this medium had been an incompressible, an idealised incompressible medium, the moment the piston is set into motion with this DC , this disturbance would have propagated through the entire fluid and the entire fluid should have been in motion.

But because it is a compressible medium, so what will happen is the fluid faraway from the piston will still be at rest, however the fluid near the piston will start acquiring the velocity. And then this information or the disturbance that there is a small velocity DC been imparted by the piston will propagate. And we can say that this disturbance propagates as the wave with the speed of sound CS through the medium. And you can see that for the left of the fluid, from this wave, this region as shown here will be already under the influence of the motion.

Whereas the fluid to the right is actually at rest even in this instant. Next instant you can see that this wave would have come to some other location. Now if we want to analyse the flow from this stationary frame, then what we see is this is an unsteady flow because at one instance this region which is shown between the solid wave and this dashed wave lines, it was in rest and then after this wave has passed, then we find that this would have also experienced the motion. So inaudible analyse this problem, we can try to frame it in a slightly different but equivalent way.

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FLUID DYNAMICS AND TURBOMACHINES PART D: Module 02 – Introduction to Compressible Flow

Conservation of mass yields: $\rho C_s A = (\rho + d\rho)(C_s - dC)A$

Neglecting second order terms: $\rho C_s = \rho C_s - \rho dC + C_s d\rho - d\rho dC$

$\rho dC = C_s d\rho$

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We can say that this is a same long channel and we fix our axis system X and Y and we say this is the wave, now however we make the wave as stationary. And we impart a relative velocity that way the fluid to the right moves to the left with the velocity C_s and the fluid to the left which was already under the motion because of the piston will have a velocity $C_s - DC$. DC I remind you is the motion or the velocity due to the piston. Now the fluid which is downstream, which is now to the right will have this undisturbed pressures P and ρ .

And fluid to the left which is already under the disturbance will have a pressure let us say $P + DP$ and $\rho + D\rho$. Please note once again that the X direction for this picture is from right to the left. Now we can apply the conservation of mass across this control volume and we can say that ρ times C_s Times A , A is the area of the passage is equal to $\rho + D\rho$ multiplied by $C_s - DC$ multiplied by A . Now area is area constant area duct, constant area passage and hence area gets cancelled.

So we are left with ρC_s is equal to $\rho + D\rho$ multiplied by $C_s - DC$. Now we can open these brackets, multiplied the cross terms, that is $\rho C_s - \rho DC + C_s D\rho - D\rho DC$, we can do that and they can neglect the second-order terms. That is the term which contains the $D\rho DC$. If we neglect the term $D\rho DC$ because these are going to be small, we have assumed that DC is an infinitesimally small change in the velocity and then we can say that the corresponding small change in density is $D\rho$ and hence the product will be much smaller.

So this is a very standard practice in analytical expression, deriving analytical expressions in mechanics. So we can now amplify because we know that ρCS gets cancelled from both sides of the equation and finally we end up getting that ρDC is equal to $CS D \rho$. This is what the mass conservation or continuity equation gives us. If we now do the momentum conservation, we should note that on one side we have pressure P acting, on the other side we have a pressure $P + DP$ acting, but the area is same.

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FLUID DYNAMICS AND TURBOMACHINES PART D Module 01 - Introduction to Compressible Flow

Conservation of linear momentum yields: $\sum F_x = \dot{m}(C_2 - C_1)$

$-dP = -\rho C_s dC$ ← $PA - (P + dP)A = \rho C_s A((C_s - dC) - C_s)$

↓ $\rho dC = C_s d\rho$

$dP = C_s^2 d\rho$ → $C_s^2 = \frac{dP}{d\rho}$ → $C_s = \sqrt{\frac{dP}{d\rho}}$

Since there is only infinitesimal change across the wave, the process may be assumed as reversible and in the absence of heat transfer, we can consider it as **isentropic change**.

$C_s = \sqrt{\gamma RT}$ for perfect gas
= 332 m/s (for air at STP)

$C_s = \sqrt{\frac{K}{\rho}}$ For copper, it is 3722 m/s

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And we know that the rate of change of linear momentum is nothing but the force. So if we go for the conservation of linear momentum, then we know that ΣF_x which is a summation of all the forces is equal to $\dot{M} C_2 - C_1$. 2 is downstream side to the left, 1 is upstream side to the right according to this figure. Please keep the figure in mind and the forces that are acting along positive X is PA , the force that is acting along the negative X is $P + DP$ multiplied by A . We have not considered any friction, so fluid friction does not arise here.

And then we can say that $PA - P + DP$ multiplied by A is ρCS multiplied by A , that is the mass flow rate, ρCS multiplied by A is \dot{M} in C_2 is nothing but $CS - DC$ and C_1 is CS . So once we simplify, we get that $-DP$, see the 1st term gets cancelled PA with the 2nd term, with this the 1st of the bracket gets cancelled and we get $-A DP$ and we have A on the other side, so A gets cancelled, so we get $-DP$ is equal to $-\rho CS$ multiplied by DC . And we already know that ρDC is equal to $CS DP$.

So let us substitute ρDC in and put it as $CS DP$, then we will get that DP is equal to $CS^2 D\rho$. And hence we can write that CS^2 or the speed of sound square is equal to $DP / D\rho$ or we write in the usual way that CS is under root $DP / D\rho$. Now we cannot evaluate this expression $DP / D\rho$ unless we know what is the thermodynamic process. And to get an idea to the thermodynamic process, particularly the one that we are dealing with here, let us try to recollect what are the assumptions we have made.

We say that there is only an infinitesimal change across the wave which is caused by an infinitesimal velocity change in the piston DC . And the process may be assumed reversible, we are not talking about any friction and there is no heat transfer. Hence we can consider the change to be isentropic change or reversible, adiabatic change. And so we now know the relationship of how the pressure varies and the density dependence. So we can then substitute for perfect gas that CS is under root γRT and the usual value of 332 metres per seconds for air at STP can be obtained.

If we think about the liquids or gases or solids, the another way of talking about in terms of liquids and solids is in terms of bulk modulus and we can say that the CS is nothing but under root K / ρ where K is the bulk modulus of the medium and then we get that... For example for a solid like copper, it is 3722 metres per seconds. So what we get, we find that the more compressible the medium is, the more easily the medium can be compressible, the speed of sound is less. And it can also be shown that for an ideal incompressible medium, there is no change of density and hence we will have the speed of sound going to infinity.

We have discussed this with respect to the straight passage. Now what happens is in case of steam and gas turbine flow passages, the passages will have varying area. So the blades will produce the blade passage areas which are continuously varying. Of course that is a complicated geometry, so what we will discuss here is what is the effect of an isentropic flow through a channel of a simple varying area as shown now.

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FLUID DYNAMICS AND TURBOMACHINES PART D: Module 02 – Introduction to Compressible Flow

Isentropic flow through a channel of varying area

Conservation of mass yields: $\rho CA - (\rho + d\rho)(C + dC)(A + dA) = 0$

After dropping higher order terms and simplifying $\frac{dC}{C} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$

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So we are talking about a flow from left to the right and hence we call these structures as diffusers. We could have done this similar analysis with the flow area reducing in the direction of flow, in that case we call it a nozzle. So for this geometry, whether it is a nozzle or a diffuser, it does not really matter, we assume the same like we have done for the straight passage that we are assuming an isentropic flow, that means that is no question of fluid friction or viscosity and there is no heat transfer, so reversible, adiabatic flow.

And we take a control volume, we write down the variables, the flow variables to the left is C , ρ and A , C is the speed, ρ is the density and A is the area at that location. On the right side what leaves is $C + dC$, the corresponding density of the fluid is $\rho + d\rho$ in the area of person has changed, this is the difference from the previous example we took of a straight duct, in this case the area is not constant, area becomes $A + dA$.

So if I take this control volume I out and try to find out the force, the force on the left which is acting in the flow direction, the flow direction in this case is from left to right is my positive X direction and we can say that the force acting on the left area is P times A , on the right the force acting is $P + dP$ times $A + dA$. Also there will be a force acting on the additional area which is on the side and since it is a very small length dx we are talking about between the left and the right ends of the control volume, we can approximate the pressure as $P + dP$ by 2.

So the force that acts on the surfaces, on the slant surfaces are is $P + dP$ by 2 multiplied by dA . So we can do the conservation of mass like the way we have done for the last time but

here please note that the area is not constant. So we can say that ρCA is the mass flow rate entering the control volume and $\rho + D\rho$ multiplied by $C + DC$ multiplied by $A + DA$ equal to is the mass flow rate which leaves the control volume. And for a steady flow you know that ρCA must be equal to what leaves from the right side.

Now no mass can leave from the slant surface because wall is present. So we can follow the similar analysis of expanding these brackets, and ranging terms which are higher-order and dropping those terms, simplifying terms and removing terms which can get eliminated and once we do this simplification, I suggest that you do the simplification as well parallelly, I am not doing the all the steps. So if you do the simplification, you should get that DC by $C + DA$ by $A + D\rho$ by ρ equal to 0.

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FLUID DYNAMICS AND TURBOMACHINES
PART D: Module 07 - Introduction to Compressible Flow

Conservation of linear momentum yields:

$$PA + \left(P + \frac{dP}{2}\right)dA - (P + dP)(A + dA) = \rho CA((C + dC) - C)$$

After dropping higher order terms and simplifying $dP + \rho C dC = 0$

$\frac{dC}{C} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$

$\frac{dP}{\rho C^2} + \frac{dC}{C} = 0$

$-\frac{dP}{\rho C^2} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$

$dP - \rho C^2 \left(\frac{dA}{A} + \frac{d\rho}{\rho}\right) = 0$

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You could have got this relationship simply from the fact that ρCA is constant, you could have taken log and differentiated and you will get the same relationship. But here I am doing from the control volume approach from the 1st principle. So the mass conservation for a, for an isentropic flow through a channel of varying area gives me letter DC by $C + DA$ by $A + D\rho$ by ρ equal to 0. The conservation of linear momentum then will yield that PA loss $P + DP$ by 2 multiplied by DA . Please note that PA acts from on the left surface, so it is acting on the direction of positive X . And so is $P + DP$ by 2 times DA .

Whereas $P + DP$ which is acting on the right face is acting in the negative direction of X and hence this sign comes as $-$. So we can say that the total force acting on this control volume is P times $A + P + DB$ by 2 whole multiplied by $DA - P + DP$ ball multiplied by $A + DA$. And

that must be equal to rate of change of momentum, so ρCA as before is the mass flow rate and the velocity on the right-hand side when it leaves is $C + DC$ and the velocity with which it enters, the flow enters the control volume is C .

So we can say that ρCA multiplied by $C + DC - C$. And we can simplify this left-hand side and the right-hand side, ignore the second-order terms and when we simplify, we get that $DP + \rho C DC$ equal to 0. This result is important and we can rewrite this expression as DP by ρC square + DC by C equal to 0. Why am I doing it is because I have a term DC by C in the mass can duration and I am trying to eliminate one of the terms. We did not mean that we can eliminate DC by C , we can, we should eliminate one of the terms.

So here I find that DC by C can be written in terms of DP , ρ and C square. And this is what we have got, DC by $C + DA$ by $A + D\rho$ by ρ equal to 0, we have already obtained from mass conservation. So if we now use these 2 relationships and eliminate DC by C from them, what we get it is $-DP$ by ρC square which is my DC by C term from here + DA by ρ which is coming from the mass conservation + $D\rho$ by ρ equal to 0. Or we can write it as $DP - \rho C$ square whole multiplied by DA by $A + D\rho$ by ρ equal to 0.

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FLUID DYNAMICS AND TURBOMACHINES

PART D Module 02 - Introduction to Compressible Flow

$$dP - \rho C^2 \left(\frac{dA}{A} + \frac{d\rho}{\rho} \right) = 0 \quad \& \quad C_s^2 = \frac{dP}{d\rho} \quad \frac{dC}{C} (Ma^2 - 1) = \frac{dA}{A}$$

$$dP - \rho C^2 \left(\frac{dA}{A} + \frac{d\rho}{\rho C_s^2} \right) = 0 \Rightarrow dP \left(1 - \frac{C^2}{C_s^2} \right) = \rho C^2 \frac{dA}{A} \Rightarrow dP(1 - Ma^2) = \rho C^2 \frac{dA}{A}$$

Ma < 1

$dA < 0 \Rightarrow dC > 0, dP < 0$

Ma > 1

$dA > 0 \Rightarrow dC < 0, dP > 0$

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Let us see what we can get from here if we move further. We can write the same expression again in the next slide, I write that $DP - \rho C$ square whole multiplied by DA by $A + D\rho$ by ρ equal to 0. This is essentially the same expression which I have derived in the last slide. And we know that the speed of sound is given as C_s square is DP $D\rho$. And we can now club these 2 relations, that is the one on the left-hand side which we got from momentum

conservation and the speed of sound and then we can say that $DP - \rho C^2 \frac{DA}{A}$ remains, we now replace $D\rho$ and write in terms of DP .

So we get the DP by ρC^2 square equal to 0. And once we write it, we can club the expression which are connected with DP together and we get that DP whole multiplied by $1 - C^2$ by C^2 is equal to $\rho C^2 \frac{DA}{A}$. Now C by C^2 as you know is Mach number, this was already covered in the fluid dynamics module of these lectures on fluid dynamics and Turbo machines. So I replace C by CS as Mach number and hence C^2 by CS^2 as Mach number square and I get that DP times $1 - \text{Mach number square}$ is equal to $\rho C^2 \frac{DA}{A}$.

We will now take special cases of Mach number less than 1, subsonic flows and Mach number greater than 1, supersonic flows separately. 1st we will start with Mach number less than 1 and we can have 2 situations, the 1st situation can be the flow in a converging passage or nozzle. We can also discuss the flow in a diverging passage or diffuser. First let us look at a subsonic flow through a nozzle. In the direction of flow, this direction of flow is given by the blue arrow, in the direction of the flow, there is a reduction in the area.

Converging passage or nozzle as a reduction in area. So this reduction and area implies that $\frac{DA}{A}$, this term, $\frac{DA}{A}$ here is going to be less than 0. Now if $\frac{DA}{A}$ is less than 0, what will be the sign of DP ? $1 - \text{Mach number square}$ for Mach number less than 1 is positive. So that means the sign of DP should be the same as the sign of $\frac{DA}{A}$. And hence we should note that for this case $\frac{DA}{A}$ is less than 0 which means DP is less than 0 and if we had eliminated pressure and written in terms of speed, then we could have written it as DC by C times Mach number square -1 is equal to $\frac{DA}{A}$.

I told you that you can eliminate DC by C or you could have done it DP by P , so now we have removed DP and we have substituted in terms of DC . So if you look at this expression, this bracketed term, Mach number square -1 is less than 0 for Mach number less than 1. Now this is negative, $\frac{DA}{A}$ is also negative, that makes DC to be positive and we get an accelerating flow in the nozzle when Mach number is less than 1. This is our usual experience of course.

When we take a diffuser, the scenario just reverses because we have area change positive in the flow direction, area has increased, which means in the relations connecting DP with $\frac{DA}{A}$, $\frac{DA}{A}$ is positive and hence DP has to be positive and in the relation connecting C with A , we

find that DC has to be negative. And hence the flow reduces or decelerates for the case of a diffuser in a subsonic flow. The situation changes if we go for the supersonic flow.

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The slide contains the following content:

- Header: FLUID DYNAMICS AND TURBOMACHINES | PART D Module 02 - Introduction to Compressible Flow
- Equations: $dP(1 - Ma^2) = \rho C^2 \frac{dA}{A}$ & $\frac{dC}{C}(Ma^2 - 1) = \frac{dA}{A}$
- Central text: **Ma > 1**
- Left diagram (Converging passage): Shows a narrowing passage with a blue arrow pointing right. Below it, the text reads: $dA < 0 \rightarrow dC < 0, dP > 0$.
- Right diagram (Diverging passage): Shows a widening passage with a blue arrow pointing right. Below it, the text reads: $dA > 0 \rightarrow dC > 0, dP < 0$.
- Footer: Dr. Dibhan Chatterjee | IIT Madras | 8

Let us look at it again carefully. Let us have the same expressions in front of us which we have done in the last slide. So DP into 1 - Mach number square is rho C square times DA by A and DC by C times whole multiplied by Mach number square -1 is DA by A. But in this case we are considering Mach number greater than 1 and we are considering a converging passage and the diverging passage. For Mach number greater than 1, for the pressure relationship we have these bracketed terms as negative.

And for the convergent passage DA is also negative which makes that DP is greater than 0. For the diffuser, the opposite situation prevails and we get DP less than 0. So we can say for Mach number greater than 1, a converging passage acts like the decelerating flow passage, the speed reduces, whereas a diverging passage or a diffuser acts like a accelerating flow passage and the pressure reduces. So now if you have to construct a geometry in which you want the flow to continuously increase from subsonic to supersonic, how will you do that?

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FLUID DYNAMICS AND TURBOMACHINES

PART D: Module 02 – Introduction to Compressible Flow

$Ma < 1$ $Ma = 1$ $Ma > 1$

In the absence of work and heat interaction, it was shown earlier that

$$h_0 = \text{constant}$$

where, $h_0 = h + \frac{C^2}{2} \Rightarrow T_0 = T + \frac{C^2}{2C_p} \Rightarrow \frac{T_0}{T} = \left(1 + \frac{C^2}{2C_p T} \right)$

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You can also keep in mind that if Mach number is equal to 1, we get no change in area because if Mach number is 1, then you see DA by A goes to 0, so we should have a flat region. This is shown here. So in this case the flow starts from the left with the arrow shown here, the flow is initially subsonic and as we have discussed just now that in the convergent passage, the flow will accelerate and reaches the straight portion which is called the throat. This arrangement of a convergent portion followed by a very narrow or a straight portion and then a diverging portion is called a CD nozzle or converging diverging nozzle or D level module.

Now this nozzle or CD nozzle to be more precise is used in steam turbines for example. Why, because what happens is in a converging passage, you cannot exceed Mach number of 1. At the most what you can get it is a sonic velocity at the throat of the minimum region, area, minimum area portion. But having connected a diverging portion, it is possible that the velocity can accelerate and we get Mach number less than 1 in the converging portion, Mach number equal to 1 in the throat and Mach number greater than 1 in the diverging portion.

And hence the, in the steam turbine as you will see in the next lecture, the CD nozzle is used to accelerate the fluid. In the absence of work and heat interaction, it was shown earlier that H_0 is equal to constant which is the stagnation enthalpy is constant. We have discussed this as an example problem in thermodynamics when we talked about flow in a nozzle or a diffuser. So starting from there we can say H_0 or the stagnation enthalpy is nothing but static enthalpy + C square by 2 which can be written in terms of temperature assuming C_p to be

constant as T_0 equal to $T + C^2$ by $2C_p$. And we can write that T_0 by T is $1 + C^2$ by $2C_p T$ times T . So what does it lead to?

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The slide contains the following mathematical derivations:

$$\frac{T_0}{T} = \left(1 + \frac{C^2}{2C_p T}\right) \quad \& \quad C_p = \frac{\gamma}{\gamma - 1} R \quad \Rightarrow \quad \frac{T_0}{T} = \left(1 + \frac{\gamma - 1}{2} \frac{C^2}{\gamma R T}\right)$$

$$= \left(1 + \frac{\gamma - 1}{2} \frac{C^2}{C_s^2}\right)$$

$$= \left(1 + \frac{\gamma - 1}{2} Ma^2\right)$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}} \quad \Rightarrow \quad \frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{\frac{\gamma}{\gamma - 1}}$$

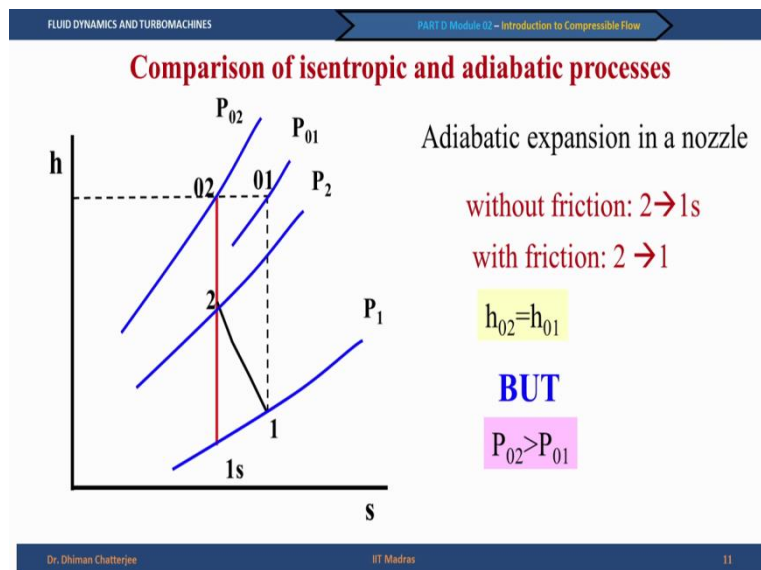
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Let us look in the next slide. We know that, we have already just found out that T_0 by T is $1 + C^2$ by $2C_p T$. And we know C_p is nothing but γR by $\gamma - 1$, these relationships you know from thermodynamics as well. So if we now combine this we can get that T_0 by T is $1 + \gamma - 1$ whole divided by 2 times C^2 by $\gamma R T$. And we can replace $\gamma R T$ as C_s^2 or speed of sound and hence we get ultimately that T_0 by T equal to $1 + \gamma - 1$ by 2 times Mach number square.

When we connect pressure with temperature, then we can say that P_0 by P , the corresponding stagnation pressure is P_0 by P is equal to T_0 by T whole to the power γ by $\gamma - 1$. And when we substitute T_0 by T as we have got just now, we can write that P_0 by P is $1 + \gamma - 1$ by 2 times Mach number square whole to the power γ by $\gamma - 1$. And we can also say that in the limit of Mach number being very low, in the incompressible flow limit you can show, I leave it as an exercise for you to show that P_0 is nothing but $P + \frac{1}{2} \rho C^2$.

Try to prove it yourself by taking that Mach number is very small. I leave it to you as an exercise and show that Mach number in the limit of small Mach number or low values of Mach number, you will get the incompressible flow relationship connecting stagnation pressure and static pressure.

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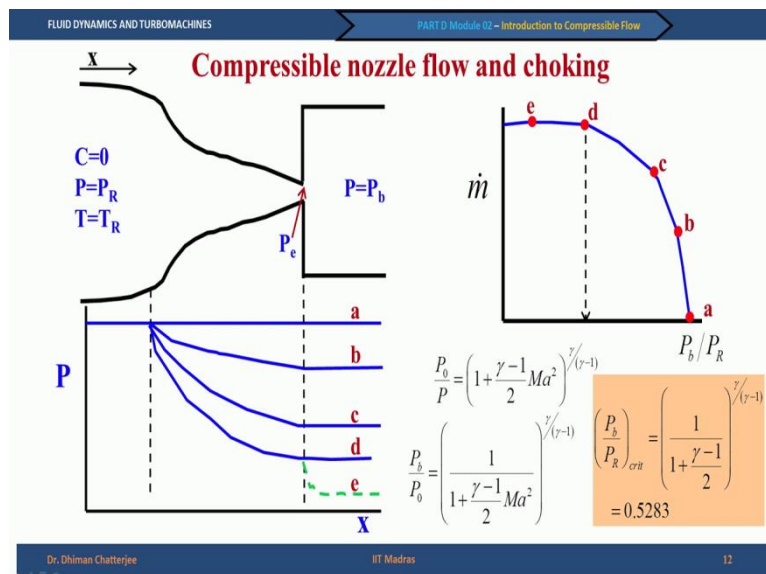
We can also do a quick check on the comparison of isentropic and adiabatic processes. So far we have discussed only isentropic processes, we have not considered any friction. I am not going into details of what the friction can bring about because that is a lecture in itself and one can see these in advance books or compressible flows and fluid mechanics. But here what I am just trying to show you an aspect of pressure and temperature for isentropic and adiabatic processes which is going to be useful for the better understanding of the steam and gas turbine.

So we say that it is in adiabatic expansion in a nozzle, 2 is the nozzle inlet, remember we have talked about that higher the number we assign, the higher is the pressure, higher is the enthalpy, so nozzle inlet as higher pressure, higher enthalpy, there is an expansion, so we have given it the number 2, number 1 is given to the nozzle exit. 1 is to remind you is nothing but the ideal or isentropic expansion in the nozzle, 02 corresponds to the stagnation state, 01 corresponds to the stagnation state corresponding to nozzle exit.

So we know that H_{02} equal to H_{01} , we have got it from the 1st law of thermodynamics. But if we consider now 2 cases, isentropic and adiabatic, that is with friction and without friction but boat without any heat transfer, then we can say that without friction in the idealised case the expansion would have should have taken place from 2 to 1s, however due to friction, due to reversibility, we get it from 2 to 1. And from the 1st law we have said that H_{02} equal to H_{01} , if there is an ideal gas then they can say for the perfect gas T_{02} equal to T_{01} and this is shown here in the horizontal line.

But you also note that the pressure the point 02, the state 02 lies on a pressure letter P0 2 which is not same as the P0 1. We find that P0 2 is greater than P0 1, that is because there is a drop in pressure because of friction. So we have to consider these effects and later on when discussing the steam and gas turbines we will define a nozzle efficiency to account for this non-isentropic or adiabatic cases of involving steam turbine.

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The next thing that we need to know is compressible nozzle flow and choking. So let us say that I have a reservoir here, in the reservoir, the velocity of flow is 0, pressure of the in the reservoir or the tank is PR and the temperature is TR. Since the velocity is 0, you can take that as an stagnation values. And this tank is connected to a convergent passage and then we have the back pressure PB. This PE we are distinguishing between P and PB, P is the pressure at the exit of the nozzle. And we will soon see why PE needs to be different from PB.

So let us do a thought experiment first. We say that there is a reservoir connected to back pressure environment and by a nozzle. Initially let us say that the PB or the back pressure is kept same as the reservoir pressure. Reservoir pressure does not change in this entire thought experiment that we are going to do. So we keep reservoir pressure as PR and we will gradually reduce PB. So when we say that we have PR equal to PB, there will be no flow and flow by the way will take place in the direction X shown here.

So if I want to vary pressure with X, starting with P equal to PR, then in the 1st case I get there is no change because there is no flow. Now let us say we start reducing PB gradually.

Once we do reduce gradually, there is no change here because this is the tank portion and then the pressure reduces and at the exit, P_E is equal to P_B . We continue reducing P_B further and we get the curve C where again P_E is equal to P_B . And D, P_E is equal to P_B .

And then if we reduce P_B further, what we find is that the flow rate does not increase anymore, the flow rate would be increasing as we reduce pressure from P_B at A to P_B at D. But if we reduce pressure P_B beyond D, what we will see is that there is no increase in flow rate and the pressure P_E is not equal to P_B . What will happen to the flow is that after reaching the exit, there will be an expansion wave, I will not go into details of this expansion wave, there will be an expansion wave and then the flow will adjust the pressure P_B at some distance downstream which is given by the line E.

So if we want to now draw the mass flow rate versus the pressure ratio, that is P_B by P_R , we can show it like this. That mass flow rate \dot{M} versus P_B by P_R start from 0 when P_B by P_R is 1 and it gradually increases till D as we have discussed but beyond D the line is straight horizontal, which means it does not depend on this backpressure. So D and E have the same value. And the flow at this D is called the choking flow. So how do I get the state D? What is the pressure ratio at D, that is what is that critical value of P_B by P_R , if we reduce the pressure ratio less than this critical value, the flow is choked, there is no further increase in mass flow rate.

To get that we need to look into the relationship we have derived earlier, between the stagnation pressure and the static pressure and we can write that P_0 by P is equal to $1 + \frac{\gamma - 1}{2} \text{Mach number}^2$ to the power $\frac{\gamma}{\gamma - 1}$. And in this case please note that P_0 is nothing but P_R and P we are referring to P_B . So then we can say that $\frac{P_B}{P} = 0$ which is $\frac{P_B}{P_R}$ will be equal to 1 whole divided by $1 + \frac{\gamma - 1}{2} \text{Mach number}^2$ to the power $\frac{\gamma}{\gamma - 1}$.

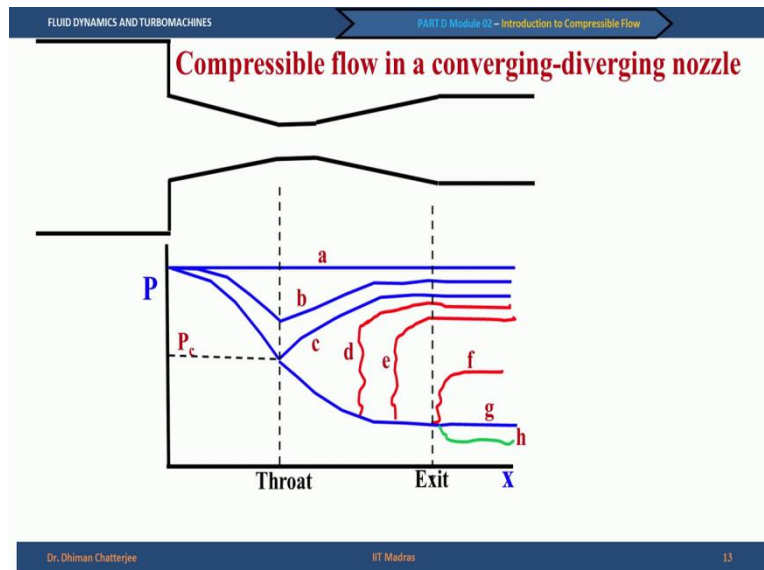
But in this case the Mach number becomes 1 because we told you already that in the convergent portion, the highest speed that can be attained is corresponding to Mach number 1 at the minimum portion area or the throat. At the throat the highest speed that can be obtained from a subsonic condition through a convergent passage is the Mach number 1 of the sonic State. So if we put Mach number 1 here, then we get $\frac{P_B}{P_R}$ for the critical case as 0.5283 if we use γ equal to 1.4.

So how do I understand, this is giving a mathematical way, how do I understand physically? Let me tell you a story. Let us say that you are sitting inside this reservoir and you are going to allow the flow to increase. And your friend is sitting in the back pressure here, in this portion, the moment every time he shouts, you increase the flow rate and the flow rate, the speed increases and your friend shouts further, you increase the flow rate, the speed increases further. What will happen is once the speed reaches the speed of sound, once the flow reaches the speed of sound, you will not be able to hear your friend's voice.

And then what happens, no matter how much or how many times your friend shouts, you will not increase the flow rate. So what happens is the same physical phenomenon is happening in this case. This pressure reduction in the PB is like a disturbance and let us assume for the time being that the PB is gradually reduced every time. We are reducing it slowly and gradually buy a small amount and this is like a disturbance we have talked. So the disturbance can propagate at the speed of sound but now the flow has also reached the velocity of sound.

So what happens, relative to the flow the disturbance cannot move up stream and the disturbance that is set in, the back enclosure will not reach the reservoir and hence there will be no further increase of flow rate. This value, the maximum value that the flow rate gets and we get a flat curve in the $M \dot{V}$ versus PB by PR curve is called the choking condition. So D is called the choking condition. Now what we will look into, what happens if we have a compressible flow not through a convergent nozzle but through a CD nozzle or converging diverging nozzle.

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Description is essentially the same, we have a reservoir and it is connected instead of a converging nozzle, in a converging diverging nozzle with some backpressure P_B and we want to do the same exercise of finding out pressure versus X , that is the distance. We can mark the region for the throat and Mark the region for the exit. Initially we start with reservoir pressure same as the backpressure, I am sorry I should say backpressure same as the reservoir pressure because reservoir pressure does not change, what I am changing is the backpressure.

And hence we get that there is no flow. Once we start reducing as I have done in the last case, the pressure reduces and then starts increasing, this is still a subsonic state. And then we get a critical point which is a sonic throat in case of B, the sonic speed has not reached in the throat. In case of C or G, the sonic speed has reached but the pressure at C, the backpressure corresponding to the curve C is such that the flow takes a subsonic flow and we get the upper lean of this curve. In case of G, the pressure is such that you get a supersonic flow in the diverging passage.

In both cases the pressure, the backpressure corresponds to the critical pressure which is P_C corresponding to C I have given and the sonic throat is reached, sonic throat condition is reached. But what happens if we have a pressure, backpressure ratio lying between the curves C and G. If the pressure is between C and G, then what will happen is the flow will continue in the supersonic region, this curve along G, then it comes here and tries to adjust the pressure.

Now to know that a supersonic flow in a diverging passage is always going to accelerate and it can never give you for an isentropic flow, I repeat it can never give you a pressure increase. We have just not seen that if Mach number is greater than 1, then if DP times $1 - \text{Mach number square}$ will be related with DA by A. And if area has increased, DA has increased, then DP has to reduce because $1 - \text{Mach number square}$ is going to be negative. So that means the pressure should reduce, which is exactly what the curve G gives.

But since we want the pressure which is higher than PG, so what happens is this isentropic flow condition breaks down and we get what is known as shock. So this curvy line is my attempt to show a shock stand there and then we get the pressure adjustment. If you reduce the pressure PB further, the shock travels downstream and instead of D, we get the shock at E and then the pressure adjusts. This process will continue till you reach the shock at the exit and then the shock no longer remains a normal shock.

We are not going into details of shock and the expansion whereas I have talked about earlier because in this course what we need is to appreciate the fact that a CD nozzle or a converging diverging nozzle is essentially useful to bring me a higher velocity than what a simple converging nozzle can give. And if the pressure is below G like H, then what will happen is the pressure will be adjusted by the form of expansion waves. For details about this you should see a book, good book on gas dynamics or compressible flows.

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FLUID DYNAMICS AND TURBOMACHINES

PART D: Module 02 - Introduction to Compressible Flow

Summary

- Wave propagation through a compressible medium is discussed
- Isentropic flow through varying cross-sectional area passage
- Flow in a nozzle and, in particular, converging-diverging nozzle is discussed

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So with this I come to a conclusion for the introduction, a very brief introduction to compressible flow. We talked about the wave propagation through a compressible medium.

We have talked about these isentropic flows through varying cross-sectional area passage and found out the relationships connecting the change of pressure, change of area and change of speed. Most notably we have noted that unlike in an incompressible flow, in compressible flow the nozzle or a diffuser can give rise to a change in velocity in either way.

That is in an incompressible flow, if whenever we see a nozzle, we know that the flow should accelerate. Whenever we see a diffuser, we know that the flow decelerates. But in case of a compressible flow we cannot conclude so simply. We need to know more, we need to know the Mach number. So Mach number less than 1 will behave in the same way as incompressible flow, we will get a converging portion for flow acceleration and diverging portion for flow deceleration.

Reverse situation arises when Mach number is greater than 1. And a CD nozzle in particular is useful to get a very high velocity coming out of the CD nozzle that can go into the rotor of steam and gas turbines. That discussion on steam and gas turbines will be taken up in the next lecture in this week and with this I conclude. Thank you.