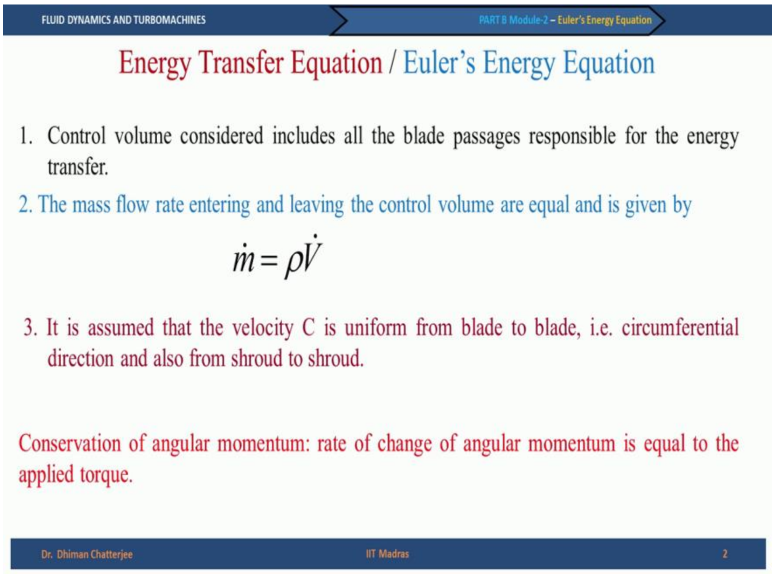
Fluid Dynamics And Turbo Machines.
Professor Dr Dhiman Chatterjee.
Department Of Mechanical Engineering.
Indian Institute Of Technology Madras.
Part B.
Module-2.
Lecture-5.
Euler's Energy Equation.

Good afternoon, I welcome you all for week 6 the 2nd lecture on fluid dynamics and Turbo machines. In the last class we talked about the Turbo machine and the different velocities that we come across, namely the absolute velocity, the relative velocity and the lead peripheral speed. So in this class we will talk about Euler's energy question or how the energy transfer takes place. I use the usual control volume approach which means we will talk about the blades by blades I mean the impeller blades or the rotating blades or rotor which is responsible for the energy transfer.

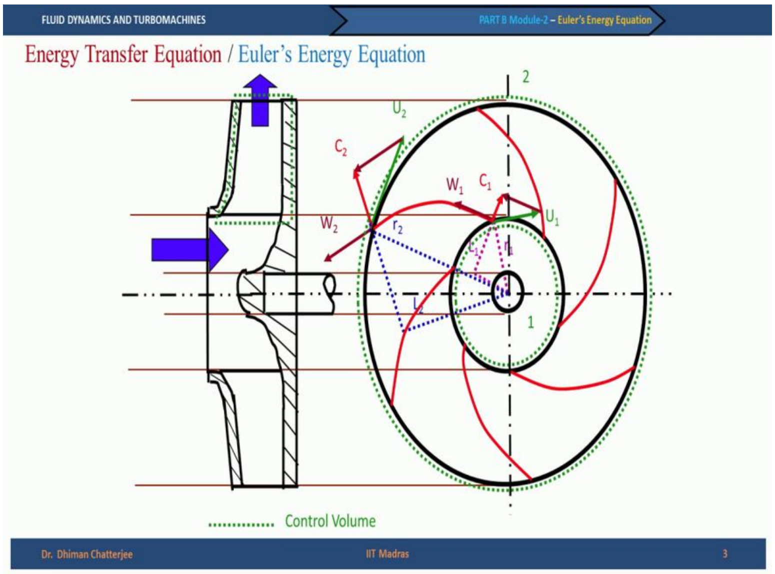
(Refer Slide Time: 0:47)



We will construct a control volume around all the blade passages, then we will consider the mass flow rate entering and leaving the control volume which is given by M dot equal to rho V dot. It is assumed that velocity C is uniform from blade to blade, that is along the circumferential direction and also from shroud to shroud. If you remember in the last class we talked about the vane congruent flow which means the flow is identical in all vanes which also mean there is a flow uniformity as I showed you in the last class from the blade to the blade.

Further we are going to talk about the flow is uniform and identical from shout to the shroud. The principal we are going to apply is conservation of angular momentum and we all know that rate of change of angular momentum is equal to applied torque. In fact it is, at this point it is worth remembering that Dr Shamit Bakshi in the first half of the course where he talked about fluid dynamics, he had applied the same principle of conservation of angular momentum to derive Euler's energy equation or energy transfer equation of a Turbo machine. We are going to do it, the same expression we will get but in a slightly different way.

(Refer Slide Time: 2:24)

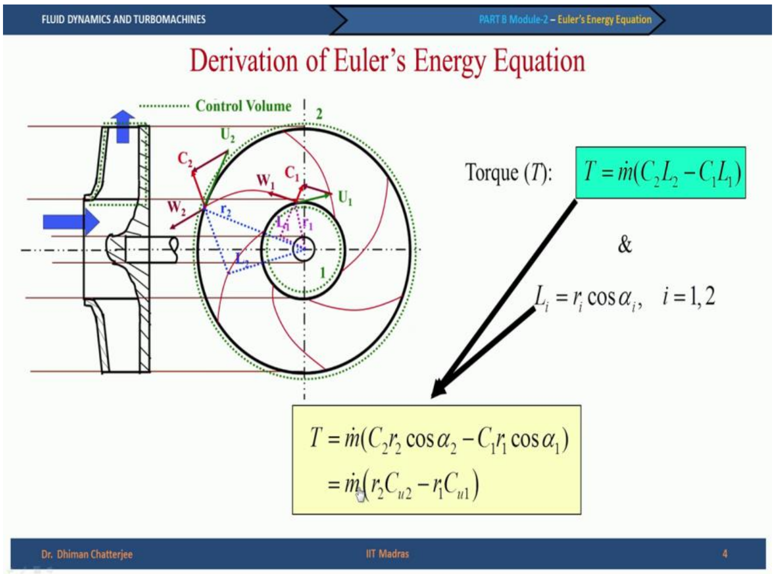


So let us look at it. In the last class we have given this description, this is end to end or blade view and we are talking about the Meridional views, we say, let us say it is a pump, the fluid comes from the low radius and goes out of the impeller at a higher radius as shown by the arrows. What we do is we construct a control volume all around the blade. This dotted line which is shown clearly here is actually my control surface and which is shown in both the views. So what is encompassed in all the blades is the control volume.

The fluid enters the control volume at the radius 1 and leaves at radius 2. So we construct the velocity triangles assuming of course the vane congruent flow and hence you can see that relative velocity is tangential at the inlet and also leaving the blade tangentially and we have given the names as C1, W1 and U1 and C2, W2 and U2, just like we have done in the last class. So this is the control volume were talking about, the mass flow rate of M dot which enters is blade passages and leaves, in the process there is an energy transfer.

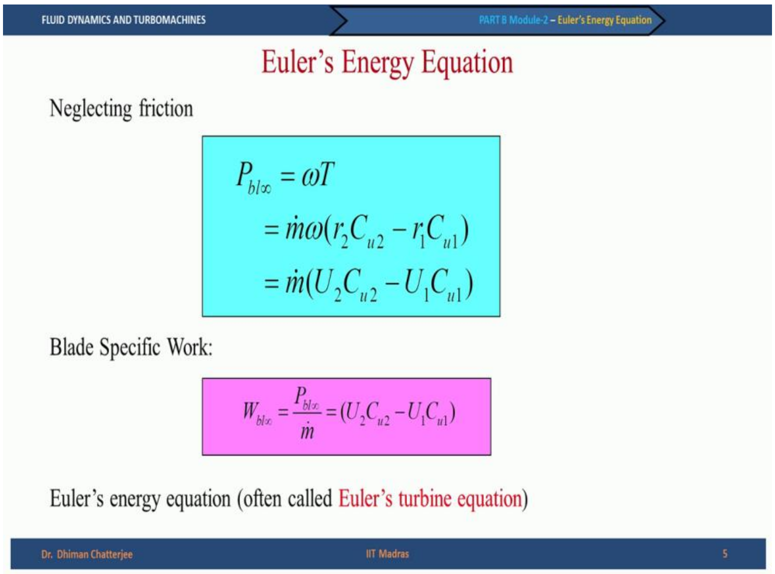
So we take this diagram to the next slide and we talk about the conservation of angular momentum. In order to get the angular momentum we need to extend C1 and we draw a perpendicular such that the length is given by L1, of course the inner diameter is given or radius is R1 as shown and the C2 will come corresponding lengths will be L2 and R2.

(Refer Slide Time: 4:18)



So if I take this now, all the lengths and velocity vectors, we can find out that the top T is given by M dot by C2 L2 minus C1 L 1 where we can say that L I, that is L1 equal to R1 plus cos Alpha 1 and L2 equal to R2 cos alpha 2. So in short we have written it as L I is equal to R I cos alpha I. Let us substitute this expression for L I in the torque and what do we get, we get that torque is equal to M dot C2 R2 cos alpha 2 minus C1 R1 cos Alpha 1 which will give me M dot R2 C U 2 minus R1 C1 or in other notations we can say that M dot multiplied by R2 C2 U minus R1 C1 U.

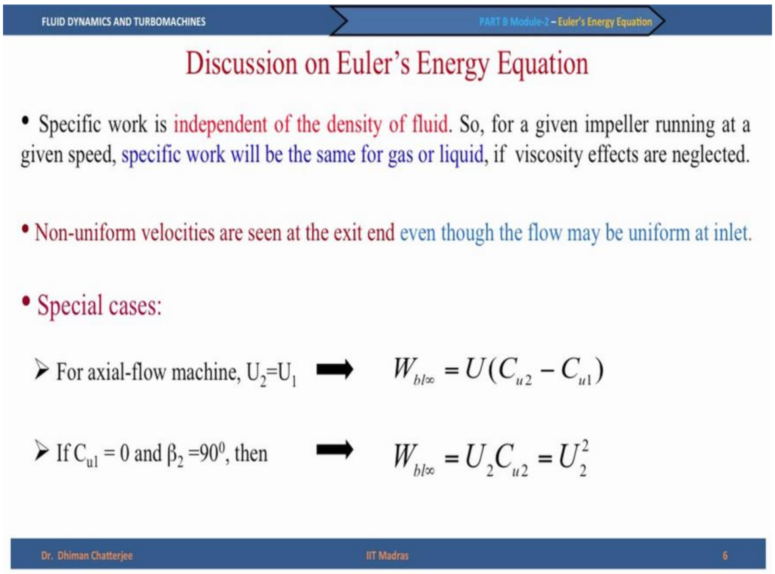
(Refer Slide Time: 5:18)



And then, when we are talking about energy transfer, we have to also keep in mind that this is an idealised case, hence there is no friction, so we can say that P BL infinity, P here stands for power BL for the blade, that is the power transferred by the Blade and infinity is an indication of the vane congruent flow, the idealised flow condition we have discussed earlier. So P BL infinity is the idealised power transfer which is omega T which will be equal to M dot omega times the torque already we have found out and that is R2 C U 2 minus R1 C U1 which will be given as M dot U2 C U 2 minus U1 C U1.

And we can say that the blades specific work W BL infinity, here again the BL infinity I want to remind you refers to the blade and infinity the first to vane congruent flow, actually you can keep this in mind in a similar way like we have talked about earlier that vane congruent flow takes place when there are infinite number of blades. So this is an idealised condition as you can appreciate, so W BL infinity is corresponding to the vane congruent flow or the idealised case or the case of infinite number of blades and that is given as U 2 C U2 minus U1 C U1. This is known as Euler's energy equation, often called Euler's turbine equation and this is exactly the same relationship if you recollect was obtained earlier by Dr Bakshi in the fluid dynamics lecture.

(Refer Slide Time: 6:59)

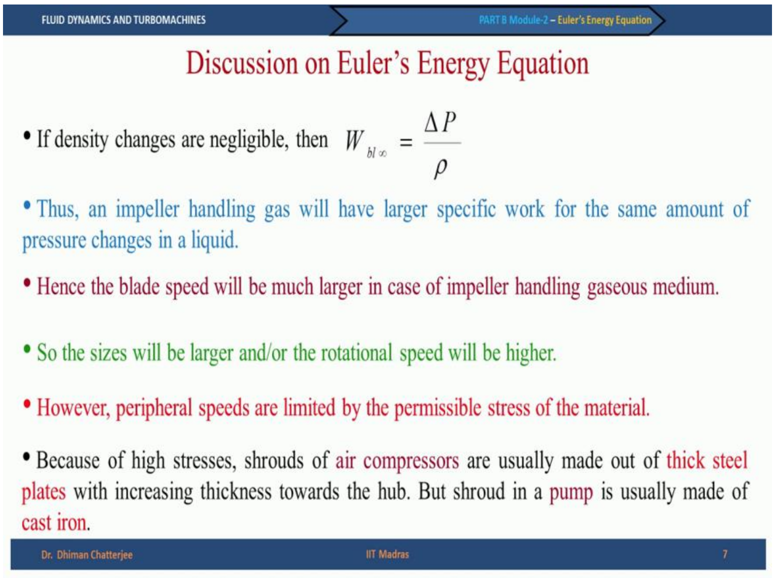


So now we will discuss a few aspects of this Euler's energy equation to get a better appreciation of Euler's energy equation. The first point is the specific work or to be more precise the blades specific work is independent of the density of the fluid. So for a given impeller running at a given speed the specific work will be the same for gas or liquid. Of course we are taking an idealised world that viscosity effects are neglected. 2nd, nonuniform velocities are seen at the exit end even though the flow may be uniform at the inlet which we are not considering here.

We will talk about the deviation from the vane congruent flow in the next class and we can talk about the special cases, the first special case comes from the axial flow machines, in case of axial flow machines U1 equal to U 2 and we will get that WBL infinity is U times C U2 minus CU1 where U is nothing but U1 equal to U2. And if in particular we take C U1 equal to 0 and beta 2 equal to 90 degree, then we can get the W BL infinity equal to U2 C U2 which is equal to U2. So first let us again recollect what we have talked about the implication of CU1 equal to 0.

CU1 equal to 0 means that inlet whirl component is zero in case of pump and the exit whirl component is equal to 0 in case of turbine. And this is not necessary but if we make this assumption, it is going to give us a little more inside. The same is with beta 2 equal to 90 degrees. We are taking a special case and in reality beta 2 need not be 90 degree but if we do that then we find that W BL infinity is U2 square. This has a very significant influence in the performance of the Turbo machines. We will see this aspect coming again and again.

(Refer Slide Time: 9:03)

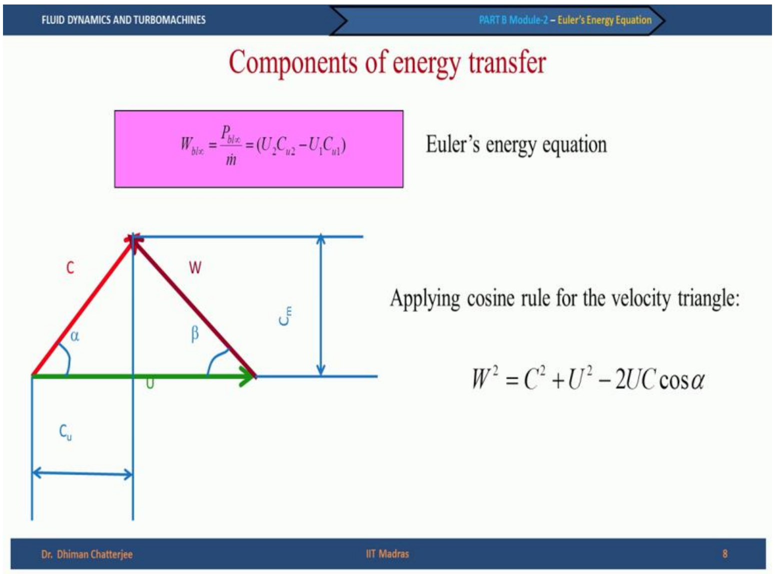


So we can say if density changes are negligible then W BL infinity is related Delta P rho and we can say that the pressure rise of the 2 Turbo machines, let us say one handling the gas, the other handling the liquid, if we compare, then we can say that for the same amount of pressure change in case of a gas, the gas will have larger specific work because we are talking about the density of the gas being small, okay. And then recollect the relationship we got W BL infinity is U2 square which is the Blade peripheral velocity square. So what does it mean?

If W BL infinity is high as we have just now discussed for gaseous medium or Turbo machines handling gaseous medium, then we can say that the blade speed will be much higher. Thus we can expect that in case of blowers or steam or gas turbines the speed will be higher than the corresponding, the hydro Turbo machines like pumps or hydraulic turbines. And if the speed is high, then we will as 2 options, one in order to get U high, either I can make the rpm high or I have to make the size high.

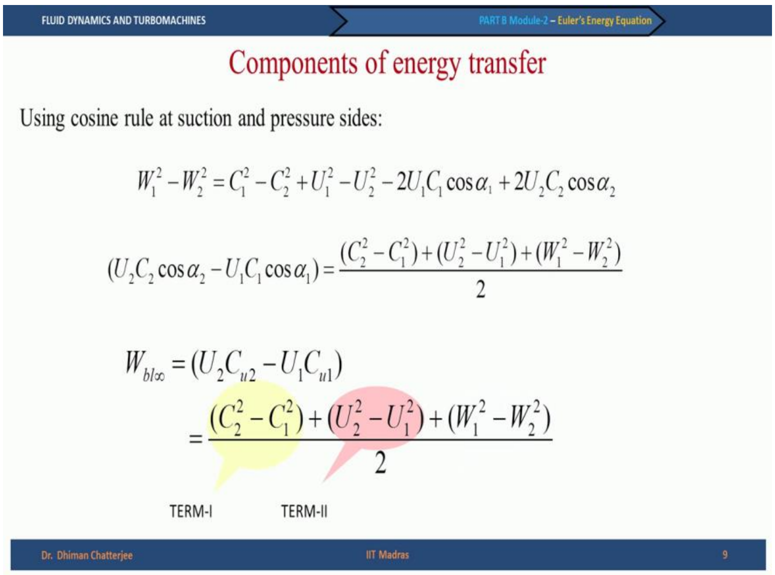
So if we make the rpm or the rotational speed higher, then there is a problem of the permissible stress of the material. So you see that proper choice of material is stemming from the requirement of higher speed or lower speed and the speed requirement comes from the specific work requirement related with the density. So we have to keep this in mind while designing a Turbo machine. And because of these high stresses we find that the shrouds of the air compressors are usually made out of steel plates with increasing thickness towards the hub. But Shroud in a pump is usually made of cast iron because as I told you the speed will be very different.

(Refer Slide Time: 11:07)



Next we can discuss this U2 CU 2 minus U CU 1, the energy transferred in a slightly different way. This is already obtained earlier, I am just reproducing it and we write the velocity triangle, it typical velocity triangle and hence I am not writing C1, U1, etc., I am just writing C, U, W to represent a generic velocity triangle. And if you recollect we have already defined how angles beta and alpha should be defined. And now if we apply cosine rules for the velocity triangles we get that W square equal to C square plus U square minus twice UC cos alpha.

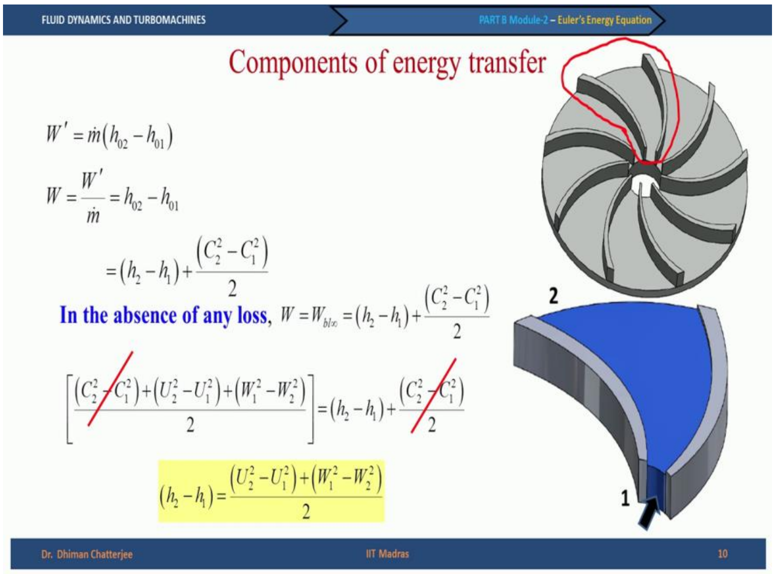
(Refer Slide Time: 11:50)



And we can rewrite it in the form W1 square minus W2 square, and if I write W1 square minus W2 square for the 2 sides the suction and the pressure sides, then we get that W1 square minus W2 square is nothing but C1 square minus C2 square plus U1 square minus U2 square minus twice U1 C1 cos alpha 1+ twice U2 C2 cos Alpha 2. And if we rewrite this, we get U2 C to cos alpha 2 minus U1 C1 cos Alpha 1 is equal to C2 square minus C1 square plus U2 square plus U1 square plus please note it is not W2 square minus W1 square, it is W1 square minus W2 square.

This is to be borne in mind, that whereas C2 square minus C1 square and U2 square minus U1 square, here it comes to be W1 square minus W2 square and then whole divided by 2. So what is my left-hand expression U2 C2 cos alpha 2 or U1 C1 cos Alpha 1, it is a thing but, it is U2 CU2 minus U1 CU1. And hence we can write that W BL infinity is equal to C2 square minus C1 square plus U2 minus U1 square plus W1 square minus W2 square whole divided by 2. And we need to understand what are the contributions of these terms 1, 2 and 3. But before we go into that I would like to extend this discussion on energy transfer further.

(Refer Slide Time: 13:31)

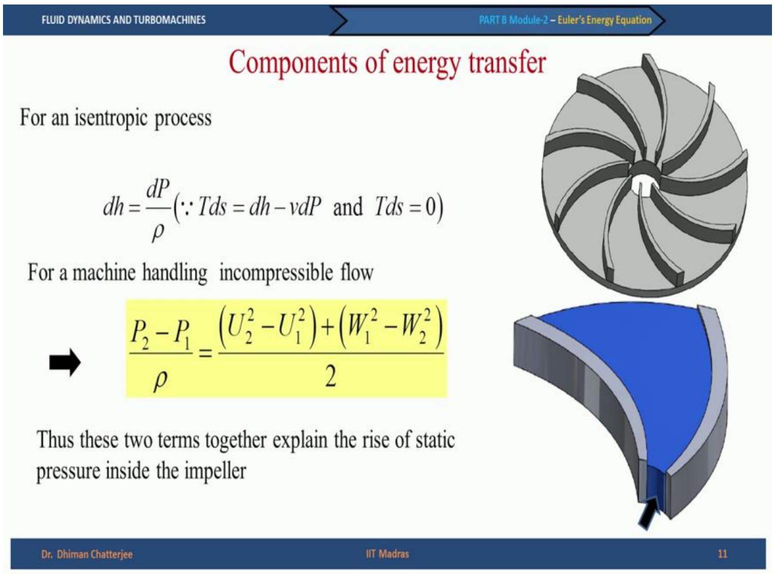


So we can say that we have an impeller which we have discussed so far, you are very familiar by now about this impeller and I am taking only 1 Blade passage, just as a representative Blade passage because in vane congruent flow all blade passages are identical, the flow is identical in all the blade passages and I draw it separately. And let us assume for the time being that we are talking about a pump, then what happens, the flow comes from a smaller radius here which is given as 1, the suction side and it goes out from the outer radius which is 2, the pressure side.

And we can write that W Prime, from the first law of thermodynamics we know that W Prime is equal to M dot h 02 minus h01 and blade specific work and the specific works in this case will be identical because we have talked about the idealised conditions, there is no loss and so we can write first specific work is W Prime per unit mass flow rate which is h 02 minus h01 and you already know that h02 or h01 represents the stagnation enthalpies at the stations 1 and 2 and hence we can write it as h2 minus h1 plus C2 square minus C1 square by 2. And to reaffirm that it is no, idealised case with no losses, so we can say that W equal to W BL infinity equal to h2 minus h1 plus C2 square minus C1 square by 2.

And let us now put in the expression of W BL infinity in terms of the velocities. If we do that then we can write that C2 square minus C1 square plus U2 square minus U1 square plus W1 square minus W2 square square whole divided by 2 is nothing but h2 minus h1 plus U2 square minus C1 square whole divided by 2. So that means C 2 square minus C1 square term gets cancelled from both sides and we are ending up with H2 minus H1 be nothing but U 2 square minus U1 square minus W1 square minus W 2 square by 2.

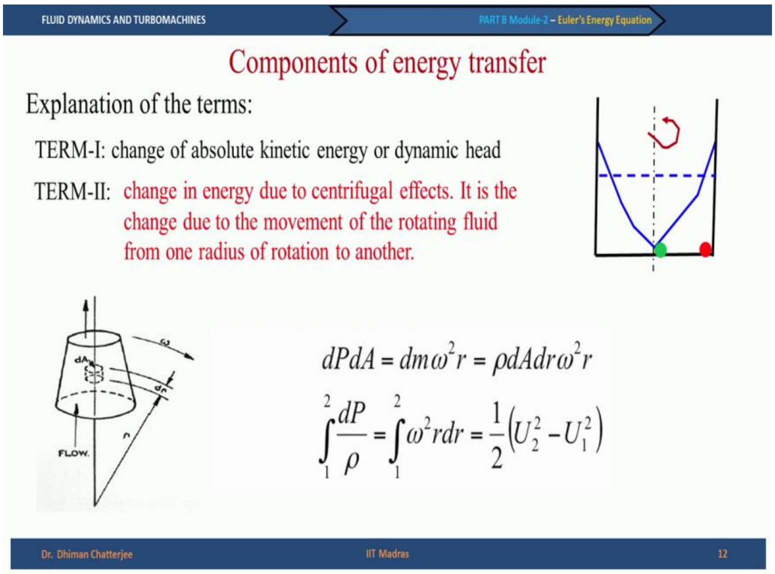
(Refer Slide Time: 16:11)



What does it mean, it says that the change of static enthalpy can be expressed in terms of the velocity components U and W at the inlet and at the outlet, essentially the difference in the squares. So if we now continue this discussion of instead of static enthalpy change, if we want to talk in terms of pressure, so how do we proceed from here? We can say that for an isentropic process because we have already assumed this as an idealised case, we can write that dh equal to dP by rho because we know that T ds equal to dh minus V DP and T ds is 0, so V is nothing but the specific volume and it has been replaced by density.

For an incompressible flow handling machines, what we get it is density is constant and we can say that P2 minus p1 by rho which is nothing but the enthalpy change is equal to U2 square minus U1 square plus W1 square minus W2 square whole divided by 2. So what we have arrived at it is that out of the 3 components we have written earlier for work transfer that is C2 square minus C1 square, the first term, then the other 2 terms are U2 square minus U1 square and W1 square minus W 2 square. We find that the 2nd and the 3rd terms together contribute to the static pressure change and this is very important.

(Refer Slide Time: 17:34)



We need to understand this portion a little more clearly. So let us look at the term wise contribution of the first, 2nd and 3rd term now and when we look at the first term, it says the change of absolute kinetic energy or the dynamic head, that is very clear because we are talking about C2 square minus C1 square. Now you may say that W 2 square minus W1 square is also change of kinetic energy, but what about U2 square minus U1 square? But we will show you that C2 square minus C1 square is essentially different from W 2 square minus W1 square even if I say that letter U1 equal to U2.

Why? Because the 2nd term that is W1 square minus W 2 square contributes to the static pressure change or the static enthalpy change. So let us look at the term 2 which we are talking about is the blade peripheral velocity change. But before we go into that, let me digress a little bit. Let me recall a basic studies which we do in fluid mechanics where we talk about a beaker which is filled with water. Let us say that this is a beaker which is filled with water up to the lines shown by dash and let us say that we are interested in knowing the pressure at 2 points which are at the same depth.

And these points are given by let us say a red ball and green ball. So when the beaker has a flat free surface, that is this is water, about it is air, then we know the depths being same, the pressures are identical. What happens when I start rotating this beaker? I rotate it about its axis and you know that the free surface will become parabolic and now you see that the mere rotation of the beaker gives rise to a pressure difference between the 2 points which were previously when the beaker was at rest had identical pressure.

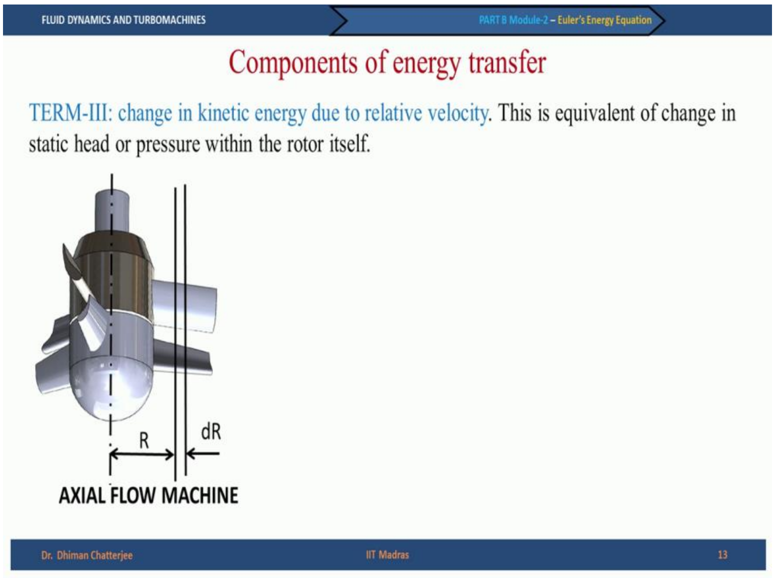
So what gives rise to this pressure? It is the centrifugal effect. So now we are talking about not just a stagnant liquid, we are talking about the fluid which is flowing from one radius inside the blade impeller to another radius and we are talking about the energy change. To see it, let us see that we have a flow which takes place and we have a fluid element which is at a radius R and which is having a small and rotation is omega. And then we can say that the change in pressure is equal to half U2 square minus U1 square, that is half and multiplied by within bracket U2 square minus U1 square.

So this talks about the centrifugal effects. Let us pause here for a minute. I had earlier told you or just now also showed you that in case of a pump the fluid enters at a smaller radius and exits at an outer radius which is larger. And in case of turbine, the reverse direction was shown in the last class. That time I could not explain you why we are taking this, is it just a convention or there is a flow physics. Now you will appreciate that in case of a turbine what are we doing, we are taking energy from the fluid and producing power.

So in case of a turbine the U2 which is at a higher diameter and is greater than U1 which is at a smaller diameter, so what happens? We are talking about a conversion. In case of a pump, the reverse happens, in case of a pump, we are talking about the pressure building, we want to raise the pressure. So in that case this U2 square minus U1 square will also be positive if the flow takes place from the inlet to the outlet. So what happens is that by the help of the geometry and by choosing the flow direction we are taking advantage of the blade rotations.

If you look into the history of technology you will see that earlier days the turbines were made, instead of making inward flow, the turbines were also made in outward flow but that practice has now been given up and we are now talking about and inward flow radial turbines.

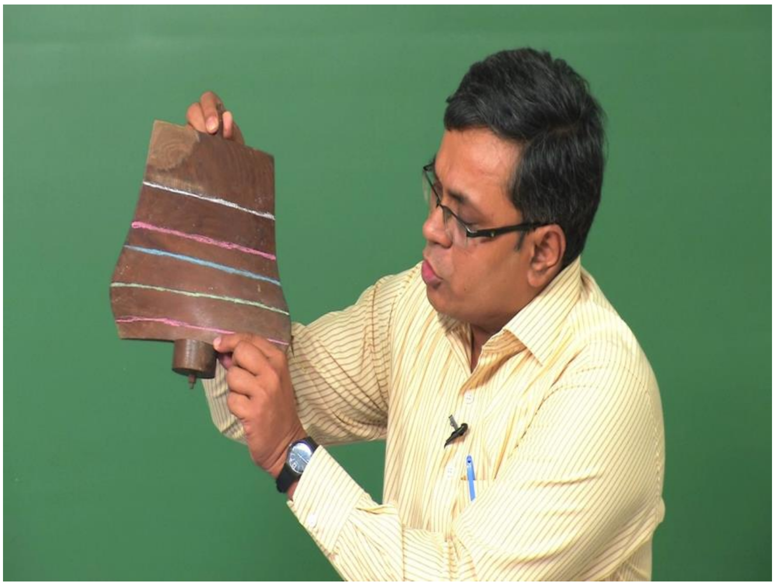
(Refer Slide Time: 21:54)



The 3rd component is the change in kinetic energy due to the relative velocity. And I have already shown this is equivalent to the static head or the pressure within the rotor. So let us look at it for an axial flow machine. The reason I choose axial flow machine is because I can offset U1 and U2 because U1 equal to U2 and hence U1 square minus U2 square is equal to 0. So if you recollect now the only term which contributes to the pressure change is a relative velocity.

And when we talk about this axial flow machine, what we are saying that this is an axis of rotation and if we remember the cylindrical development we carried out in the last class, we took any radius R and drew the cylindrical development. Now I am interested in what is the volume flow that is taking place in a small radius dR of the blade thickness. Let us look at the axial flow blade little more carefully.

(Refer Slide Time: 23:00)

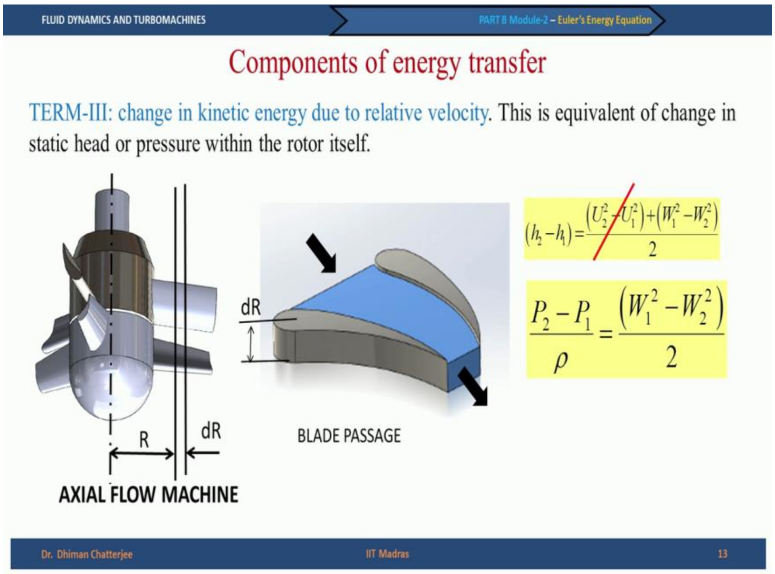


So let us look at it, if you look at it, there is a pin which is going right through it and this is the wooden blade, a model to show you how the blades look like. It has blade is twisted and hence we will tell you the why it is twisted more later on when we talk about the turbines in the case of in the next week, we will talk about the turbines but right now let us assume this is a axial flow machine and the blade profile is shown, the blade is twisted. We cannot make the blade directly from the hub, this is the portion which is the hub to the teeth.

So what we actually do is, if you can visualize these lines, the lines are now made coloured for ease of visualisation, you can see that we are making these blades in small planes , so we will take that, this is the hub radius, this is the tip radius, we divide it from the hub radius to the tip radius which is called the blade height into several small strips and we make the blades in the small strips and then stack it about this stacking axis.

So now you imagine that in the diagram that I have shown, I am talking about any radius, let us say this blue line and we are talking about a dR, which is a small region and it is not just one blade if you remember in the diagram that we have shown in the solid model, we are talking about 3 blades. So we are talking about the volume flow that is taking place inside this regime.

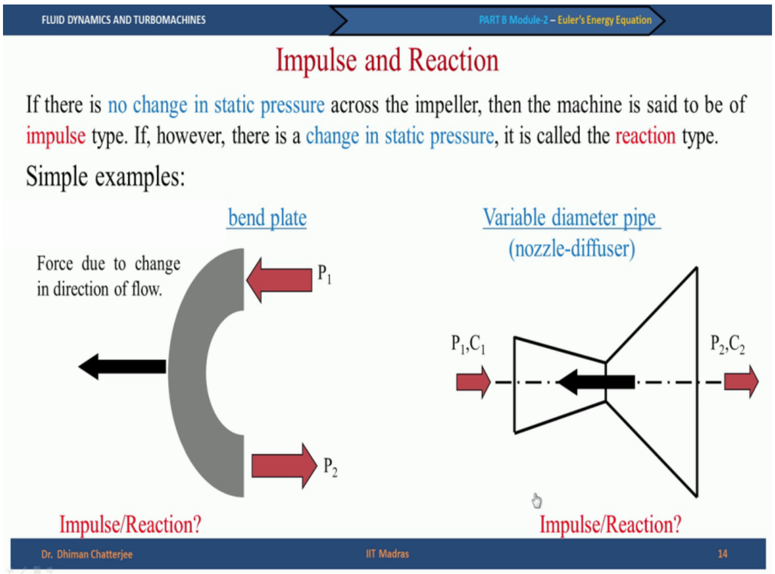
(Refer Slide Time: 24:35)



If I now look at it in the cylindrical development but instead of a planar view we are taking a height at delta R or dR, then we see the blade passage. In this case you can see that the fluid enters and leaves and this is the height dR we are talking about. So in this case what happens, U1 equal to U2 and hence what is left is W1 and W 2. From the mass conservation we can say easily that the velocity at the outlet given by this arrow is more than the velocity at the inlet, okay and hence there is a change of W1 and W 2 because this is a relative velocity.

And we can say that in the absence of U2 minus U1 square, any contribution from there, the static pressure change or the static enthalpy change is due to the relative velocity, magnitudes change. Okay. So this also contributes to the pressure rise or the pressure fall in case of turbine.

(Refer Slide Time: 25:36)

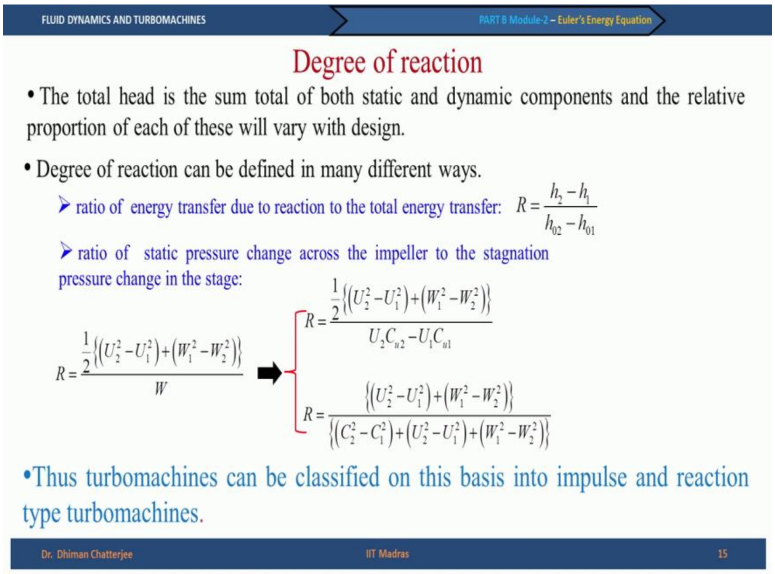


Now we are at a position to define these 2 terms called Impulse and reaction. But before we talk in terms of Turbo machines, let us talk in a simple way so that we all can have a common understanding of these terms. We will say that is there is no change in static pressure across the impeller when the machine is said to be of impulse type. If however this is the change in static pressure, it is called the reaction type. So let us take some simple examples where we can understand these concepts with the help of the fluid mechanics knowledge we have gathered.

So this is a plate and the flow takes place along this bend plate and leaves. This entire plate is exposed to atmosphere, so what happens P1 equal to P2 but because of the change in flow direction, there will be a net force. You know that from the conservation of angular momentum. So this force that is experienced by this bend plate is because of the change in the direction of flow. Contrast this with a variable diameter, let us say nozzle diffuser configuration, in which the pressure at the inlet outlets are different because even if I take it as an incompressible flow, the velocities are different, pressures are different and we get in net force coming out because of the change in pressure, though the flow direction is same.

So what happens, in the first case there is a simple change of flow direction, no change in pressure. In the 2nd case there is a change in pressure but no change in flow direction. And hence we will call by our definitions that the first one is an impulse type and the 2nd one is a reaction type.

(Refer Slide Time: 27:15)

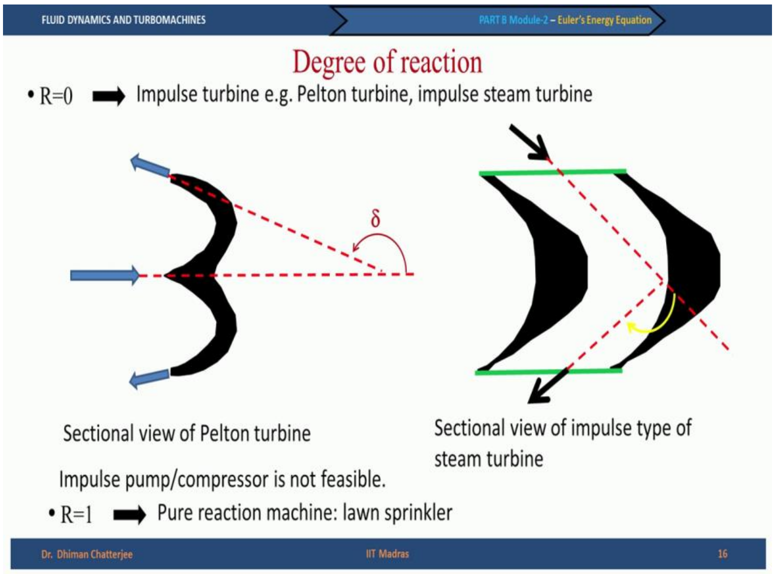


So in any machine in general we can have both a change of pressure as well as a change of directions. And hence we can define a term called degree of reaction which relates the changes in the static pressure to the overall change or the static enthalpies to the overall stagnation enthalpies. And degree of reaction can be defined in many different ways. We can say that this is a ratio of energy transferred due to reaction to the total energy transferred, like R equal to H2 minus H1 whole divided by H02 minus H01.

We can also say the ratio of static pressure change across the impeller to the stagnation pressure change in the stage, we can therefore write as we have discussed so far that R is nothing but U2 square minus U1 square plus W1 square minus W 2 square whole divided by W and it can be expressed in different ways. For example I can replace W as U2 CU 2 minus U1 CU2 and rewrite R or I can write R in terms of the velocity is C1, C2, W1, W2, U1, U2 as given here.

In either way the definition we have assumed that a portion of it is going for the static pressure rise, static enthalpy change which is in the numerator divided by the total energy transferred or the total blade specific work. In this case being an ideal world, so we say it is a specific work. Thus the Turbo machines can be classified on the basis of this into impulse and reaction type Turbo machines.

(Refer Slide Time: 29:01)



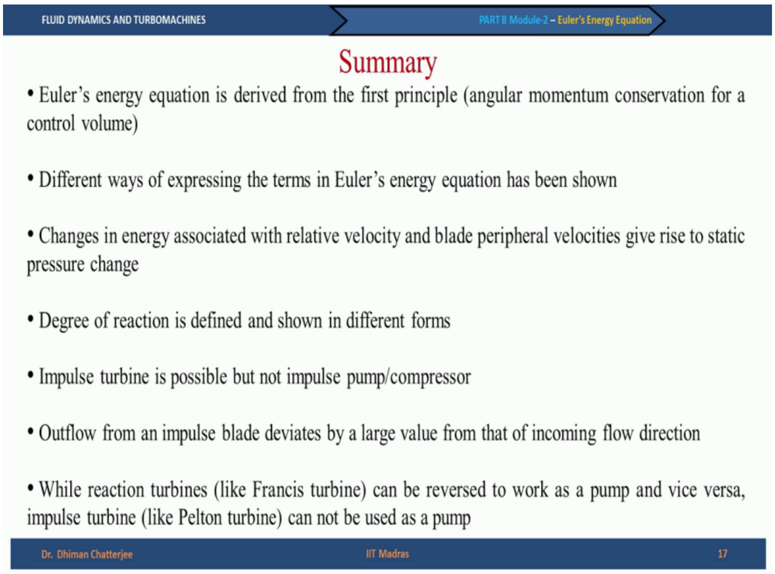
We will take some special cases, we say that R equal to 0, which means there is no change of static pressure in the runner and these examples are the impulse turbines like Pelton turbine or impulse steam turbine. Please note that when I say R equal to 0 or pressure change is not change, I refer to the pressure change is not there in the runner. It does not mean that pressure change cannot take place in any component of a Turbo machine, that is not possible, okay.

So let us look at a section of a Pelton turbine blade, this we will do in detail later on, so right now you take it for granted that this is the blade profile, how you get this blade profile, what is the significance, we will talk when we discuss about Pelton turbine. But let us assume this is a sectional view and the fluid velocity is from the left, it comes, it gets splitted into 2 halves and the flow follows these curvatures and leaves as shown. So in the absence of the blade, what I would have got is the flow should have gone along this straight line.

But now the fluid is living and at this angle, which means there is a deflection angle is Delta which is close to 180 degrees. In case of Pelton turbine we will see later that it is about 165 7 170 degrees. Similarly if I take a steam turbine impulse turbine and the blade, I can show that the velocity of the incoming flow suffers a large change in direction when it leaves. Thus in both cases, impulse turbines actually produce a large change in the direction, just like the bend plate we talked about. This is one of the features of an impulse type of machine.

And we can say that since the objective of the pump or a compressor is to increase pressure, so impulse pump or compressor is not feasible, it is not desirable. We will not design and impulse pump compressor. And if I take R equal to 1, then we know that lawn sprinklers is a very good example of pure reaction machine. And there the velocity that comes in actually makes the sprinkler to rotate and then you get water all around in your lawn.

(Refer Slide Time: 31:20)



So to summarise we can say that Euler's energy equation is derived from the first principles here, from the control volume approach following the angular momentum conservation. Different ways of expressing the terms in Euler's energy equation has been shown. We talked about changes in energy associated with relative velocity and blade peripheral velocities giving rise to the static pressure change. We have defined the degree of reaction and shown it in different forms and we have also talked about an impulse turbine.

In this case I can also say that though impulse turbine is possible and we do have examples but impulse pump or compressor is not possible. And outflow from an impulse blade deviates by a large value from that often incoming flow direction, while reaction turbines can be reversed to work as a pump and vice versa. That is if you take a pump, you can make it rotate in the reverse direction, have the flow get admitted in the reverse direction, you will get a turbine.

Maybe the turbine does not work as well as the pump would be. But such devices are not unheard of. In fact there are applications called pump as turbines or PAT which are used or proposed as low-cost power generation method in remote areas. However impulse turbine like Pelton turbine cannot be used as a pump. In the next lecture we will talk about a very important aspect of efficiency but before we talk about efficiency we will have to find out why do flows differ from the deviates from the vane congruent flow discussed so far.

So in the next class we will start the actual flows, then that will lead us to efficiencies and which will be more realistic and we will later on in the next week when we talk about the pumps and turbines or when we talk about the steam and gas turbines in the final week, we will use these efficiency concepts. Thank you.