Fluid Dynamics And Turbo Machines.
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Part A.
Module-2.
Lecture-3.
Dimensional Analysis.

Good afternoon, I welcome you all for today's discussion on dimensional analysis. In the last class we ended the discussion on the classification of Turbo machines by talking about the classification based on the direction of flow. Imagine yourself to be turbomachinery person, an engineer working in real-life situation, a customer comes to you and says that he or she has some operating conditions and you have to provide a pump or a turbine. Should you go for radial flow, mixed flow or axial flow or should you start from the scratch and start designing it?

More often than not what the practising engineer will do, you will do as a practising engineer is to get a type of Turbo machine which is suitable for that application and then fine tune it, which means you should know a priory based on the operating conditions given whether to go for radial, mixed and axial. And the way of doing it is by using some nondimensional number, which brings us to the discussion on dimensionally analysis. First, whenever you take up something, the obvious question that comes to mind is why should we do this.

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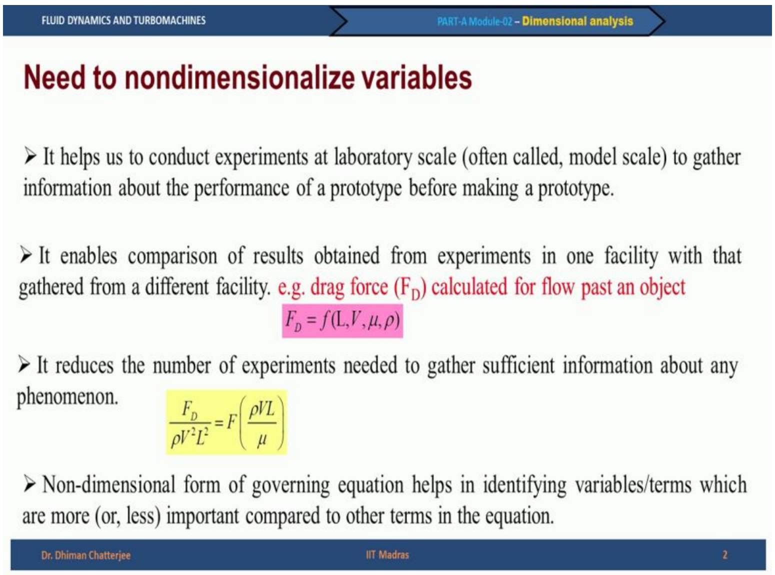

So when we talk about dimensional analysis or non-dimensionalizing, we need to ask ourselves what is the need to do non-dimensionalizing of the variables, why cannot we be happy with dimensional variable? To get to know let us see what are the reasons in brief. So it helps us to conduct experiments at laboratory scale called the model scale and to gather information about the performance of a prototype before making a prototype. Now let us take an example to understand what do you mean by laboratory scale.

Let us say you have come up with a very novel design of an aeroplane which you want to build but when you are trying to build that aeroplane you do not build a big size prototype which involves several manufacturing, designing and intellectual inputs which makes it very expensive and then find it is not working that well or it satisfies your requirement. There is an easier way of doing it and that is you make a miniaturisation of that prototype of that aeroplane, test it in a laboratory, often called the wind tunnel tests and then see whether the new design is superior or inferior to the existing design, right.

So you can make a model which is smaller in size but do not have an impression that the models are always smaller in size. For example, if we talk about an injector nozzle which is very small and used in IC engines, we would like to do a laboratory scale experiment when the model will be scaled up or the model will actually be larger than the prototype. So we do say sizes which are convenient to us and to perform experiments in the laboratory. The next question that comes to my mind is how do I extrapolate the results from my laboratory to the prototype.

A related question that comes to my mind is that if I am doing an experiment on a very complicated phenomenon, it is quite likely that I am not the only one doing this experiment. Maybe you are doing this experiment in some other facility. So how do we exchange information, how do I say that how my results are different or same as yours? Let us take an example to make it more clear. Let us say that I want to find out the drag force experienced by an object as there is a fluid flow past it and I know that the geometric parameters that I can vary is the size of the object, I can vary the speed at which the fluid flow takes place, I can also vary the fluid properties like density and viscosity.

So now if I have to do an experiment at my facility, let us say I do the experiment in my facility where I am using air at room temperature which is 30 degrees centigrade. You may be doing this experiment in air with some other, in some other place at some other temperature. So what will happen, the fluid properties density and viscosity will not be the same. So is it possible for us to directly compare the drag force, no. Then we have to take into consideration the contributions made by the changes in these new properties.

I can even do the experiment in water, in a facility called water tunnel. Then the density becomes several times different, viscosity is also very different. So instead of this drag force if we had done the experiment and made it and expressed in some other form such that the changes are automatically taken care of, it is much more beneficial for comparing. Another advantage comes in the, it reduces the number of experiments needed to gather sufficient information about any phenomenon. Let us take the same example that I gave you, that is the drag force experienced by an object.

Now if I want to gather enough data, let us say I will vary each of these 2 parameters, the size given by the characteristic length L, velocity given by the velocity scale V, the viscosity, dynamic viscosity mu and the density rho. So what I should do, I will repeat this experiment several times. What I can do, I can say that I will use 10 sizes, 10 different sizes and for each of these different sizes I will use 10 different velocities and for each combination of a different size of a specific size and a velocity, I will use 10 different fluids with 10 different viscosities and 10 different densities.

So if I have to do this complete set of experiments I need 10 to the power 4 experiments to do. And experiments of this nature will require time, is expensive, whereas if I could have done, I am not showing you how to do it, I will talk about it later, if I could have done, I could have normalised it and say that the drag force divided by rho V square L square is some function of a parameter rho VL by mu. You look at it that by expressing in this way, I have used all the variables FD, L, V, mu and rho. But instead of having separate parameters in the form of length, velocity, etc. now we have one single parameter which is given by rho VL by mu.

And yes you are right, you have already been taught in fluid dynamics that rho VL, where L is the characteristic length by mu is nothing but the Reynolds number. So that means this 10 to the power 4 experiments could have been reduced to doing 10 different Reynolds number experiments. And hence we get a very significant advantage both in the time in which the experiment needs to be done as well as the cost. And finally one subtle point that whenever we have some governing equations like you have studied in fluid dynamics, the governing mass and momentum conservation equation, let us say momentum conservation equation.

If we non-dimensionalize it, then we can find out which variables which terms are more important or less important compared to the other terms in the equation. What is the advantage, if the terms are less important, we can drop those terms and simplify the equation and solve it easily. So these are the different reasons for which a theoretician and an experimentalist would like to non-dimensionalize variables. And this is not just restricted to fluid mechanics, it is just not restricted to Turbo machines but it is for any application.

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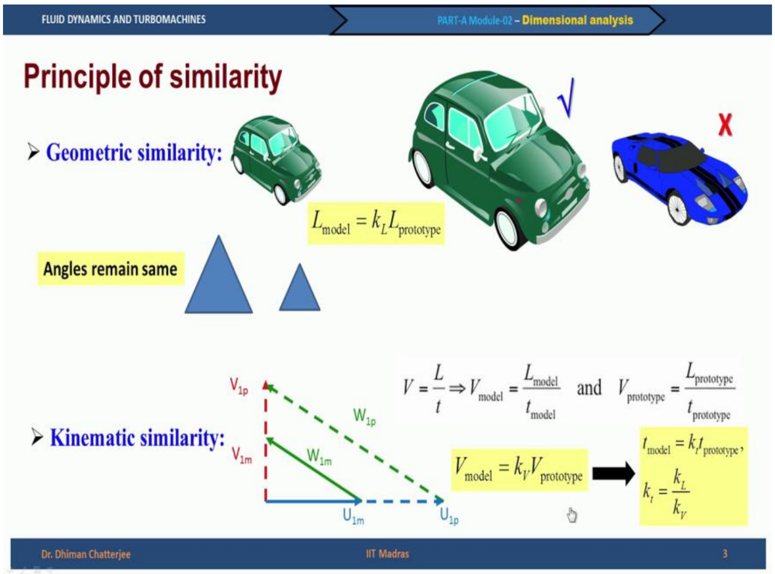

So we, next we come to the principles of similarity. See, as an English saying goes, I cannot compare apples with oranges, that is not possible. What does it mean, I can only compare similar item. So in engineering we talk about different types of similarities, the simplest and the first type of similarity is called the geometric similarity. Let us say I have a car and I want to find out the drag force experienced by the car so that I can offer a new design and hence there will be less fuel consumption. So I want to study geometric similar car… So can I take a car of this type which is a racing car? No. I should take a car which is identical but only scaling up or down.

And hence we can say that the characteristic length of a model is related with the characteristic length of a prototype by a constant, it differs by a constant factor. We have to note that the angles remain same. The moment you change angle the shape actually changes and it is no longer geometrically similar. For example, these 2 triangles, let us say they are equilateral triangles, each angle is 60 degree, they are only differing in size but they are both equilateral triangles.

Now if you make one of the angles instead of 60 degree to 30 degree and adjust the other angles, the shape will be different, we cannot talk about geometric similarity. So by geometric similarity I mean that the length scales but the angles remain unchanged. Next we can talk about kinematic similarity. Kinematic similarity is the similarity that we get in motion, which means that the velocities will scale. I have drawn some vector triangles or velocity triangles, this is essentially from Turbo machine applications, you will appreciate it better.

But right now if I say that M subscript M stands for model and subscript P stands for prototype, then we have a velocity W1M, V1 M and U 1M and the corresponding velocities W1P, V1 P and U1P these 2 triangles are similar, the angles are same. So we can say that its velocity is nothing but length per unit time, in dimensional analysis I must say that this point that we are not worried about exact thing, we are talking about the scaling so we say the length per unit time, length is a characteristic length and T is a characteristic time.

So then we say that the V of the model, the velocity in the model is nothing but the length of the model by the time and V in the prototype is length of the prototype by that time. And thus we say that if velocity of the model is KV times the velocity of the prototype, we can say that kinematic similarity leads to a temporal or time similarities and we can say that I am scaling is T model is Kt times the T prototype where Kt is of course related by KL and KV, this is the 3rd category of similarity and perhaps the most stringent one, that is called dynamic similarity.

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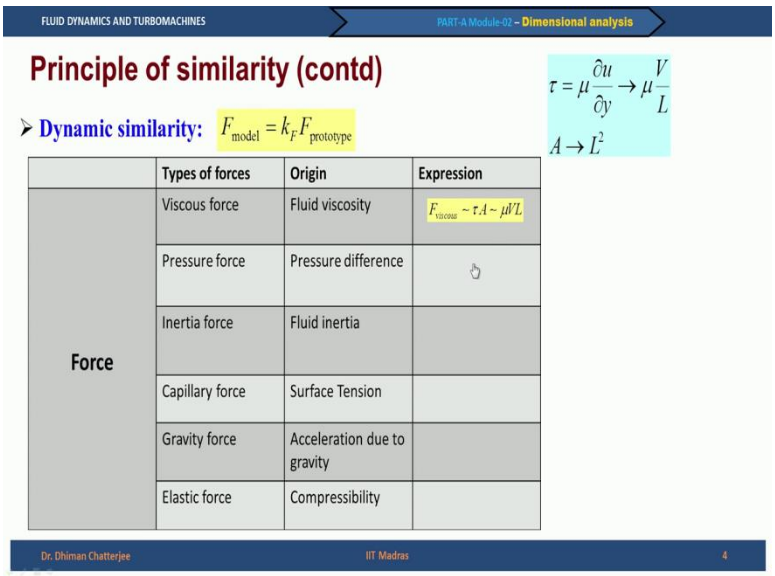

Dynamics as you know deals with not just the motion but the causes of the motion. So in this case we are talking about similarity of the forces between the model and the prototype. In case of fluid mechanics we encountered different types of forces like viscous forces due to the fluid viscosity, pressure force because of the pressure difference, inertial force because of inertia, capillary force due to surface tension, the gravity force because of the acceleration due to gravity, elastic force because of fluid compressibility.

Let us look at each of these expressions from the dimensional argument. If I say the viscous force, then we know from fluid dynamics, we have already studied that viscous force is nothing but shear stress multiplied by the area and then this is given as mu times V times L. V is the velocity and L is the characteristic length, how do I get this expression? We can say that now is equal to mu Dell U Dell Y, you already know this. So in dimensional arguments I will say that it goes as mu times V by L, V is the characteristic velocity, L is the characteristic length.

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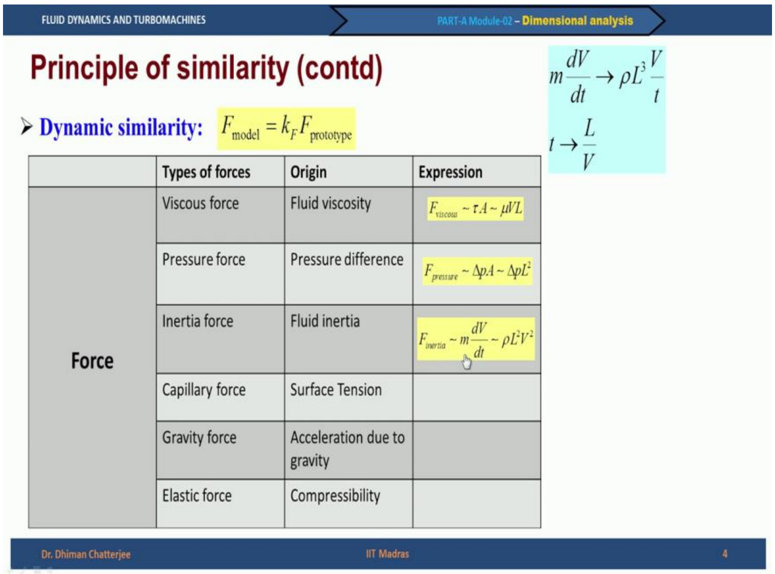

For example in case of a pipe, we talk about the diameter of the pipe as the characteristics length and area goes with L square. So now if I multiply tao which area, and then we get mu VL. Similarly we can talk about pressure difference. In case of pressure difference were talking about the pressure difference Delta P multiplied by the area to get the force and hence it is Delta P times L square. Fluid inertia, when we write an equation, let us say we are talking about Lagrangian tracking, we can write the fluid resultant acceleration because of the summation of different forces. So many times we will come across the terms like M dV dt, so now that goes as rho L square and multiplied by V square in this fashion.

M is nothing but row L cube, L cube being related with the volume and rho times L cube gives you the mass, dV dt will be simply written as V by t and t is the time is nothing but as we have discussed already length by velocity and hence if we couple all of this we get that the fluid force inertia terms is rho L square V square.

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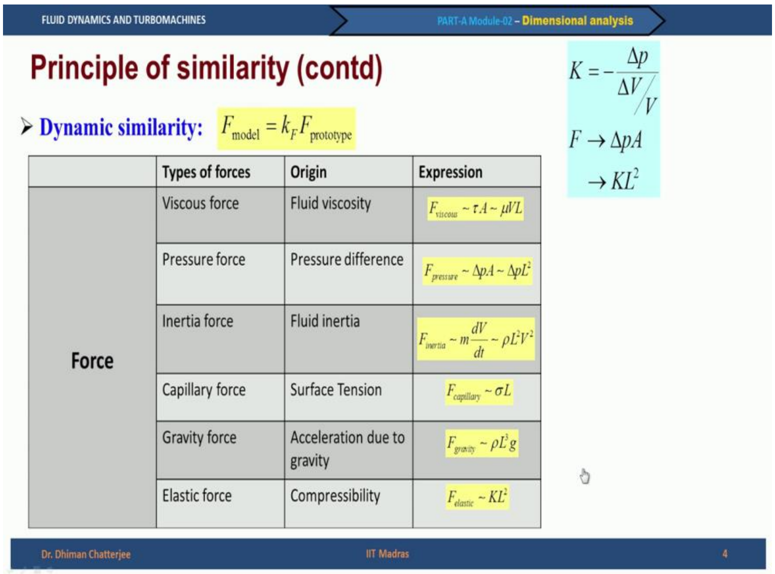

Surface tension will become, contribute to the capillary force in the fashion that F capillary goes with Sigma times L where Sigma is the surface tension term. The gravity force which is acceleration due to gravity is given as rho L cube times g, this is because of the mass is rho L cube as we have talked times g the acceleration due to gravity. And finally we are talking about the elastic force because of fluid compressibility which is K or kappa, many times it is used K, many a times it is kappa which is the bulk modulus or modulus of elasticity times L square. How do I get it, kappa is nothing but - Delta P by Delta V by V and hence the force will be related with we know, Delta P times A and hence were talking from the dimensional argument as KL square.

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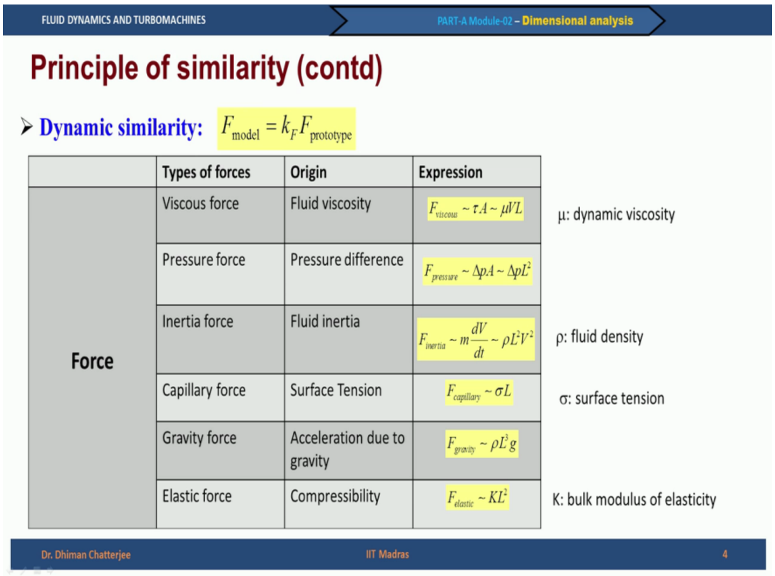

Here we have to note that we are talking about different fluid properties which are mu is a dynamic viscosity, rho is the fluid density, Sigma is surface tension term and K or kappa is the bulk modulus of elasticity. Many times, particularly where, in fluid flows where there are multiple forces present, we have to talk about which force or which of the forces are more important or more dominant than others. So we try to find out the ratio of the forces.

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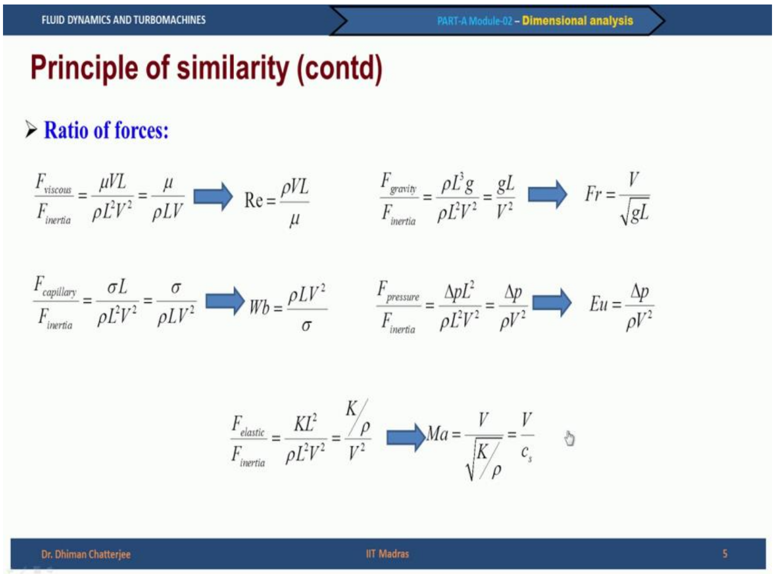

When we talk about the ratio of the forces we can say that for example in applied flow I want to find out how important is viscous force related to the inertial forces. Then I can find out the ratio of the 2 forces and I get mu by rho LV, this is of course more popularly and widely known as Reynolds number which is rho LV by mu. So if we have a flow in which Reynolds number is much much greater than 1, then what can we say, that the numerator is much much larger than the denominator, which means the inertial effect dominates in comparison to the viscous effects, whereas if Reynolds number is much much less the 1, a flow we get called the Stokes flow, in that case we have the viscous effect to be dominant.

Similar analysis can be done for the ratio for the gravity force to the inertial force, in this case we connect the rho L cube g by rho L square V square and we get g L by V square. This is given in terms of fluid number, fluid number most commonly used is V by root gL, it is sometimes also talked about in terms of V square by g L but I will stick to V by root g L, that is the most commonly used form of the fluid number. Now you have to note that this L in Reynolds number or the L that is given in fluid number, these are not just the length, these are the characteristic lengths, which means in case of a pipe flow, in the definition of Reynolds number which just talked about this L becomes the diameter of the pipe.

In case of the fluid number, the commonly seen is in an open channel flow we talk about the depth of submergence. We can talk about the importance of surface tension effect with respect to the inertial force and this gives rise to another non-dimensional number widely used called Webber number which is nothing but the ratio of the inertial forces to the surface tension forces. We can talk about pressure force to the inertial force which gives rise to Euler number which is Delta P by rho V square.

And we can also talk about the elastic forces to the inertial forces and if we readjust the term KL square by rho L square V square, we readjust the term, we get in the numerator K by rho by V square, this leads to the Mach number definition which is V by square root of K by rho and we know that square root of K by rho is related with the sound in the medium CS and then V by CS is the Mach number definition which you have already come across in fluid dynamics.

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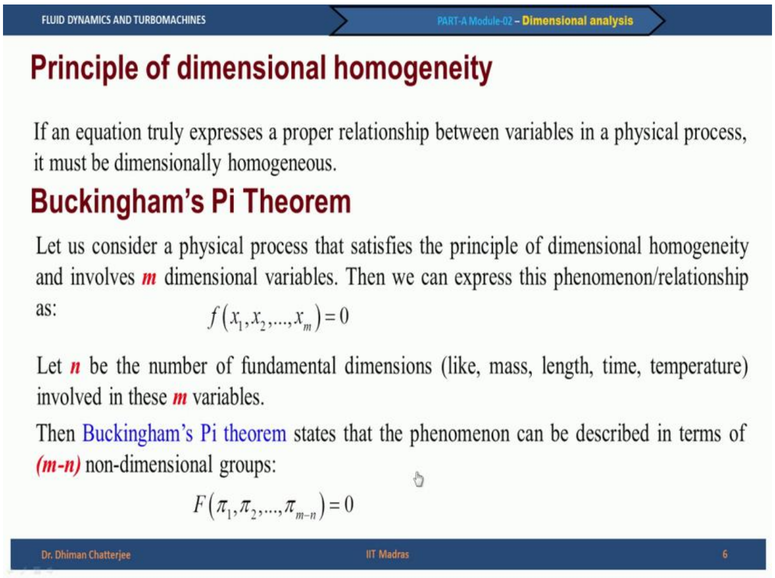

The next part that you have to keep in mind is the principal of dimensional homogeneity. It states that if an equation truly expresses a proper relationship between variables in a physical process, it must be dimensionally homogeneous. I want to digress here a little from today's topic of dimensional analysis. Whenever you are doing any work, if you are coming up with an equation which you are deriving from the first principles and we have got it, please check that with all the terms have the same dimensional relationship, that is same dimensions, it should be dimensionally homogeneity.

And once we satisfy it, then we know that the relationship is properly established. This gives rise to the very famous theorem known as Buckingham's pie theorem. Let us consider a physical process that satisfies the principal of dimensional homogeneity and involves m dimensional variables. Then we can express this phenomenon relationship as some function f of X1, X2 to Xm variables which is equal to 0. Now we can also say that these m dimensional variables have n number of fundamental dimensions like mass, length, time, temperature, etc.

So we are talking about m dimensional variables involved in a physical process on which involves n fundamental dimensions. Then Buckingham's pie theorem states that the phenomenon can be described in terms of m - n non-dimensional groups, that is the previous functional relationship of small f of X1, X2 to Xm equal to 0 reduces to another function, let us say F of pie 1, pie 2 to pie m - n equal to 0. Where each of these pie terms are nothing but non-dimensional groups. How do we proceed here? I will take you through this with the help of 2 examples which I hope will clarify our understanding on this topic.

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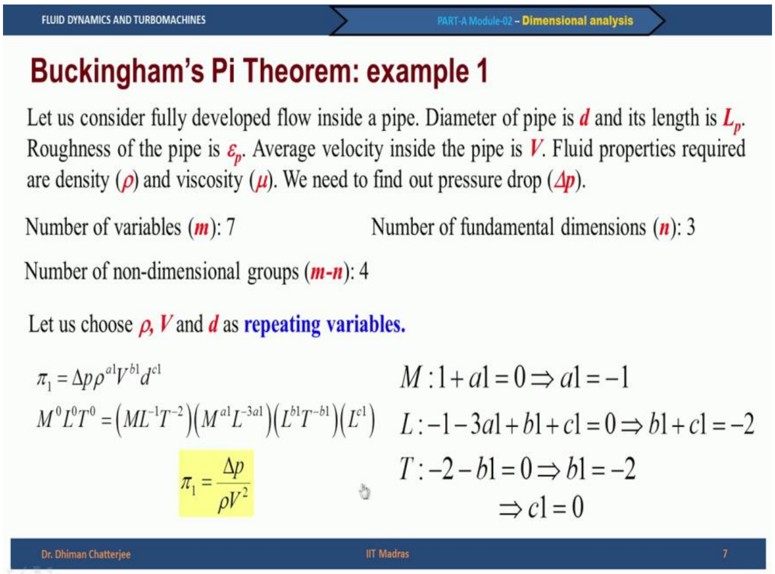

So the first example is about a pipe flow. Let us consider a fully developed flow inside a pipe, the diameter of the pipe is d small d and its length is given as LP, P for the pipe. The roughness of the pipe wall he has Epsilon P, average velocity inside the pipe is V, fluid property that we required to solve this problem are density, rho and dynamic viscosity or simply speaking viscosity mu. We need to find out what is the pressure drop that takes place because of the flow over a length of LP. So we know if we count these red notation symbols which are given, then there and our variables which are m equal to 7.

And we also have the number of fundamental dimensions n equal to 3. How, let us quickly check it. For example diameter has the length, then velocity has length and time and density has mass and length. So among these variables we have all the 3 fundamental dimensions. Then Buckingham's pie theorem states that we should have n - m, that is 7-3 or 4 nondimensional groups. We will come these groups as pie 1, pie 2 to pie 4. So let us say we form the first group pie 1, to do that we have to choose the repeating variables.

The choice of repeating variables is based on the fact that it should involve all the fundamental dimensions, in this case all 3 fundamental dimensions and it should not be independent variable. For example, pressure or pressure drop in this case is what we want to find out, it depends on the pipe dimensions, it depends on the velocity, it depends on the fluid properties, so that is a dependent variable. We are not going to use these as a repeating variable. So if I write pie 1 as Delta P as my variable choice along with then we can write rho to the power A1, V to the power B1 and D to the power C1.

These indices are actually taken 1 stands for this pie 1 and A, B, C are assigned. What we are trying to find out is the values of A1, B1 and C1 and we know that pie being a nondimensional number, it does not have any MLT and pressure is nothing but force per unit area and hence we can establish it as mass but unit length per unit time square. So how do you say that? For example the common unit of pressure is Pascal, Pascal is nothing but Newton per metre square and what is Newton, nothing but kg metre per second square. So we get kg per metre per seconds square and this is established as ML to the -1 T to the power -2.

Then density is kg per cubic meter but we do not know what power goes to make it satisfied, so we write M to the power A1 multiplied by L to the power -3 A1. Similarly we have velocity term and the diameter term. Now what we are going to do is we are trying to find out a set of equations connecting the indices for the powers of M, L and T separately. When we do it for M, we see that 1+ A1 equal to 0 which gives me A1 equal to -1.

L gives me the -1 for the from the first delta P term, then -3 A1 + B1 + C1 = 0 which gives me B1 + C1 equal to -2. And P gives me -2 and - B1 equal to 0 which means B1 equal to -2. Thus we have C1 to be equal to 0 because B1 + C1 equal to -2 and hence we can write pie 1 as Delta P by rho V square. Please note that this Delta P is the pressure drop because of fluid viscosity of fluid friction.

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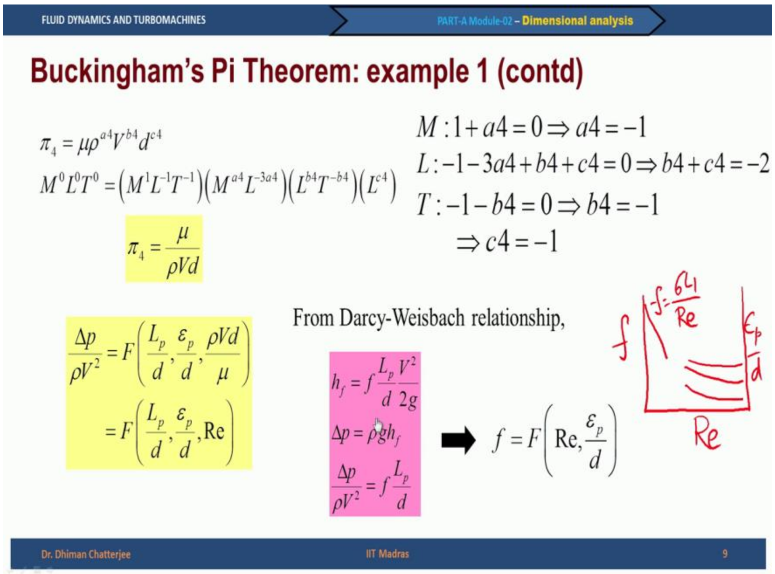

Then we can do the 2nd variable by 2 where I choose the length of the pipe as LP and then LP we write rho to the power A2, V to the power B2 and D to the power C 2. We follow the same argument and then we can write for M, that is zero, there is no other term here + A2 equal to 0 which means A2 equal to 0. For length we have 1-3 A2 + B2 + C2 equal to 0 which gives me B2 + C2 equal to -1 and finally for T also we get 0 - B2 equal to 0 which means B2 equal to 0, and then we get C2 equal to -1.

+ we get pie 2 as LP by D. Of course you could have got this pie 2 directly by Inspection since length is involved, so the corresponding term would have been directly diameter. Pie 3 relates with pipe wall friction Epsilon P and I am not going to do it, it will directly give you, similar to pie 2 as Epsilon P by D and pie 4 involves the viscosity, the last term and we can set it up and we get that for M, it is 1+ A4 equal to 0 which means A4 equal to -1, for L it is -1 -3 A4 + B4 + C 4 equal to 0 which means B 4+ C 4 equal to -2 and for T it is -1 - B 4 equal to 0 which means B 4 equal to -1 which gives C4 equal to -1.

And hence we can write pie 4 as mu by rho VD or the reciprocal of it rho VD by mu. See if we have a nondimensional group as pie 1 then one over pie 1 is also a nondimensional group or pie 1 raised to the power any constant is a nondimensional group, pie 1 multiplied by any of the other pies is also nondimensional group. We should see which way is best for us to establish. Summing all these things we can say that Delta P by rho V square is nothing but the function F of LP by D Epsilon P by D and rho VD by mu, of course the last term is Reynolds number.

Now from fluid dynamics you have studied the pressure drop inside the pipe and you are familiar with Darcy-Weisbach's relationship. What does Darcy-Weisbach's relationship give you? The friction of head drop HF is nothing but F LP by D V square by 2g and since Delta P is nothing but rho g HF, we can write the Delta P by rho V square is nothing but F times Dell P by D. Compare it with the nondimensional groups we got, Delta P by rho V square is a function of LP by D along with Epsilon by P by D and Reynolds number. So these 2 can be compared only if we know that friction factor is a function of Reynolds number and Epsilon P by D.

So if you come across Moody diagram you will know that Moody diagram essentially has a characteristic like this. Epsilon P by D by the way is called the relative roughness. So this is for your laminar flow, then there is a transition and then there are flows which are in the turbulent flows. So you see that in case of laminar flow of course the roughness is not important and in case of laminar flow you get F equal to 64 by RE but in case of a turbulent flow F is a function of both relative roughness Epsilon P by D and RE. This is called the Moody diagram or Moody chart. So this is what we get even from the dimensional argument.

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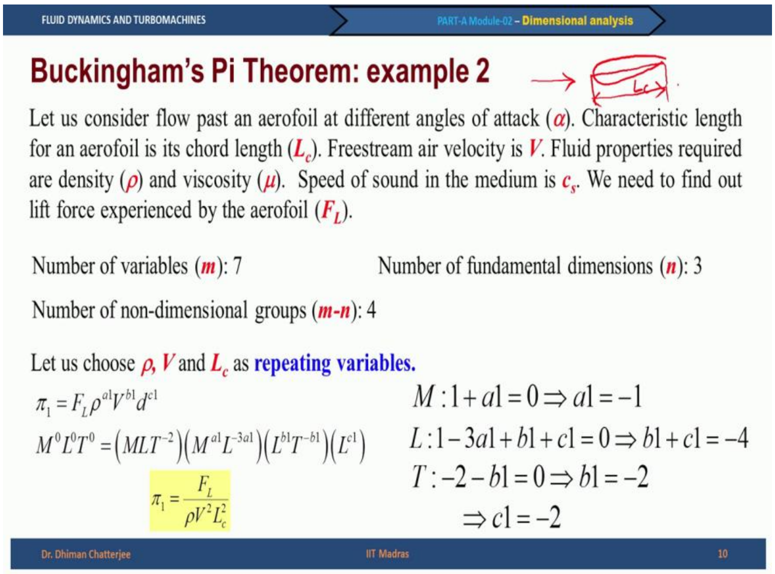

So you see how powerful it is, without solving it we get an expression which matches with our detailed derivations. The 2nd example is for an external flow, in this case we talk about an aerofoil at different angles of attack. The characteristics length of an aerofoil is its chord length LC, that is if I say I have an aerofoil, so you can see that this is the aerofoil, the fluid flow takes place in this direction, then we are talking about an angle of attack which is Alpha which is the angle between the free stream direction and the cord and the cord is given by LC. So the free stream air velocity is V, the fluid properties required are density rho and viscosity mu.

The speed of sound in the medium is CS and we need to find out the lift force experienced by the aerofoil. So the number of variables m is 7 here again, number of fundamental dimensions can be shown to be 3 and hence Buckingham's pie theorem says that there are m - n or 4 nondimensional groups. So let us choose rho V and LC, the cord length as repeating variables and we can follow the relationship like we did last time, we can say pie 1 as FL rho to the power A1, V to the power B1 and D to the power C1. And hence we get following the similar arguments as we have done so far for M, we get A1 equal to -1, for L we get B1 less C1 equal to -4 and for T we get B1 equal to -2. Which means C1 equal to -2 and hence we get that pie 1 is nothing but FL by rho V square LC square. This pie 1 is called the lift coefficient.

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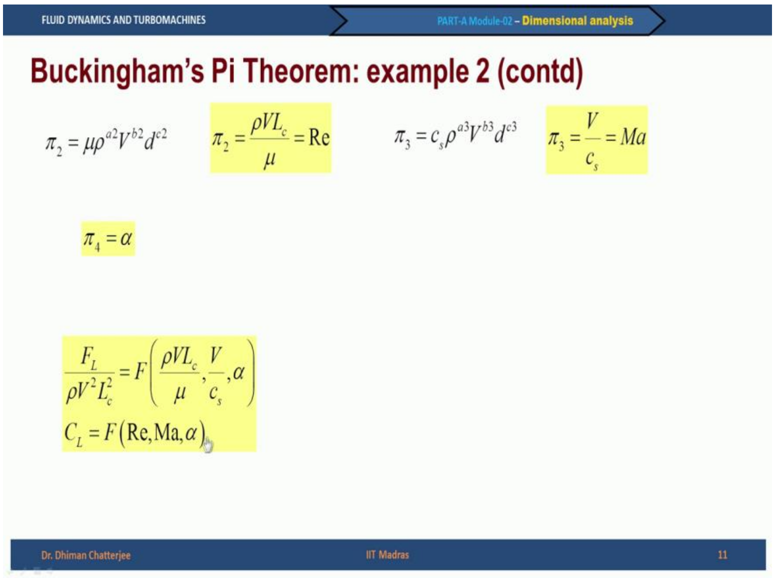

We can similarly do pie 2 with viscosity which we have already done, I am not doing it once again and we get pie 2 is nothing but Reynolds number, strictly speaking you will get mu by rho V LC but as I told you that you can always write and in, reciprocal of the nondimensional group as a nondimensional group itself and hence pie 2 is nothing but Reynolds number. We can talk about pie 3 as CS and here you can do it by Inspection also because CS is velocity and hence you have a velocity term, so it is quite obvious that pie 3 will be nothing but V by CS which is Mach number, we have already defined and finally pie 4 which is Alpha, Alpha is in radians is itself a nondimensional number.

So we can say that the lift coefficient CL is a function of Reynolds number, Mach number and Alpha. So you see that we can get the essential relationships what are the important parameters that affects the variable of our interest, for example lift or the lift coefficient in this example. So if I say that I am talking about a low Mach number, so the Mach number effects are not significant and let say I have fixed the alpha value and I want to compare the experiments that you are doing in your laboratory and I am doing in my laboratory. Then what happens, I find that CL is only a function of Reynolds number.

Now Reynolds number means it has rho V, L and U, so I really do not need to bother about what size of the aerofoil you have used, what is the velocity you have used or what is the fluid medium you have used. As long as you keep Reynolds number same as mine I will get the same lift coefficient, I should get the same lift coefficient as you have got, of course within the experimental uncertainties. So what does it mean that instead of getting, I have told you earlier that 10 to the power 4, there of course we talked about in terms of drag, we talk about lift, we get 10 to the power 4 variables if I have to match your experiments.

But when we talk in terms of lift coefficient or the drag coefficient in the earlier case, we are talking about a function of Reynolds number only. Of course in the limits of low Mach number and for a given alpha.

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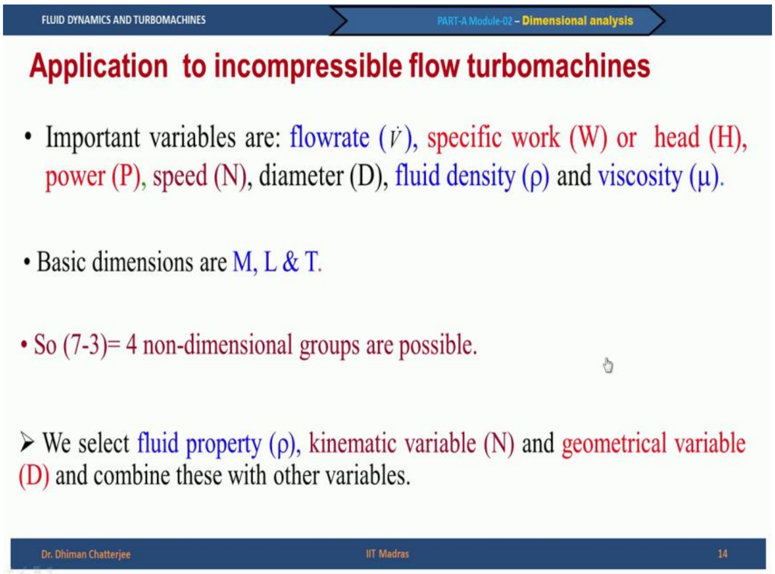

Now how does it relate with incompressible flow Turbo machines? So I will give an example for the similar dimensional analysis for incompressible flow Turbo machines. The important variables in terms of Turbo machines are the flow rate, the specific work already we have defined, what is the definition of specific work? Specific work is the difference in the useful energy per unit mass of course across the Turbo machine. And sometimes as I told you that for, particularly for the Hydro Turbo machines the specific work is related with the head by a constant g acceleration due to gravity.

Then we can have the power N is a rotational speed of the blades, D is the typical characteristic diameter of the blades, fluid density rho and viscosity mu. We have basic dimensions M, L and T and we have 7 variables. So which means here also we get 4 nondimensional groups and we select the fluid properties rho, kinematic variable N which is the rotational speed and geometric variable D and combine these with the other variables to get the nondimensional parameters.

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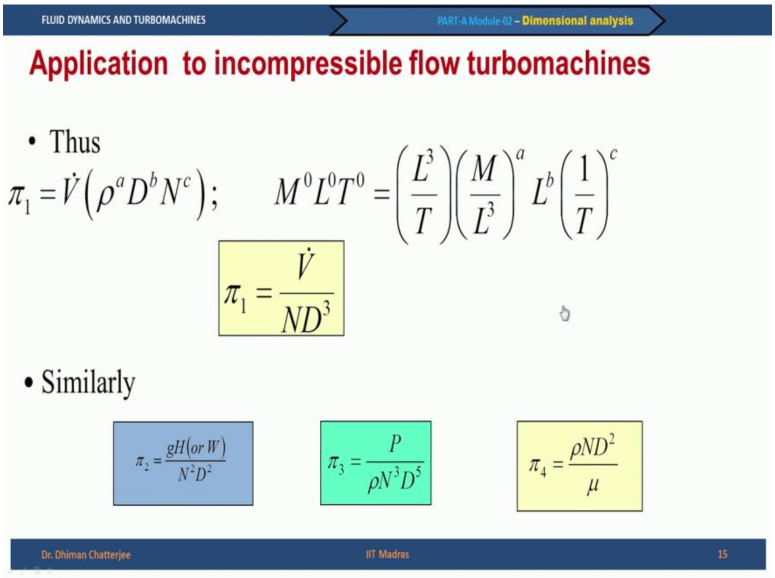

I will not go into the details but if I write like this pie 1, pie 1 is nothing but volume flow rate V dot is the volume flow rate, please do not get confused with velocity, V dot is the volume flow rate times rho to the power A, D to the power B and N to the power C, following the same argument we have done for the previous 2 examples, we can find that pie 1 is V dot I ND cube. Similarly we can find out pie 2 as W by N square D square or alternately we can write it in terms of g H by N Square D square. We can write pie 3 as P by rho N cube D to the power 5 and pie 4 as rho ND square by mu. Let us look at these terms once again.

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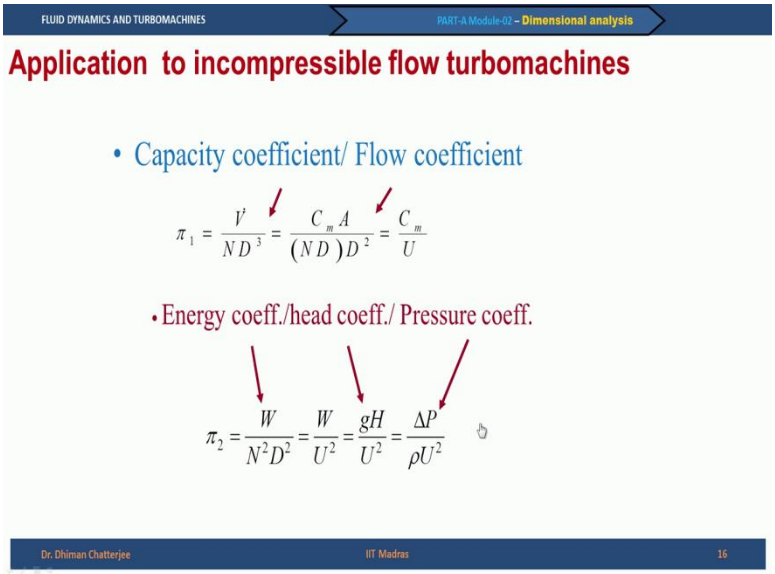

So the first term pie 1 is called the capacity of the flow coefficient. It is V dot by ND cube, now what is volume flow rate? It is nothing but the characteristics velocity times the area. So we can write V dot as Cm times A, I will talk about this Cm when we talk about velocity triangles later on. And ND cube can be written as ND times D square. Now if we have a blade which is rotating at an rpm of N and has a diameter D, then ND is proportional to the blade peripheral velocity and we can write area in terms of D square and hence we get pie 1 as can be written either in terms of volume flow which is called the capacity coefficient V dot by ND cube or in terms of the velocities as called Cm by U.

The energy coefficient or the head and efficient or pressure coefficient can be related with pie 2, pie 2 is called W, it is given by W by N Square D square and ND is related with U, just now I am told, so you get pie 2 as W by U square. And it can be related with the head, this is called head coefficient often g H by U square or it can be related with the pressure rise or decrease and hence we can call it by 2 as pressure coefficient as well.

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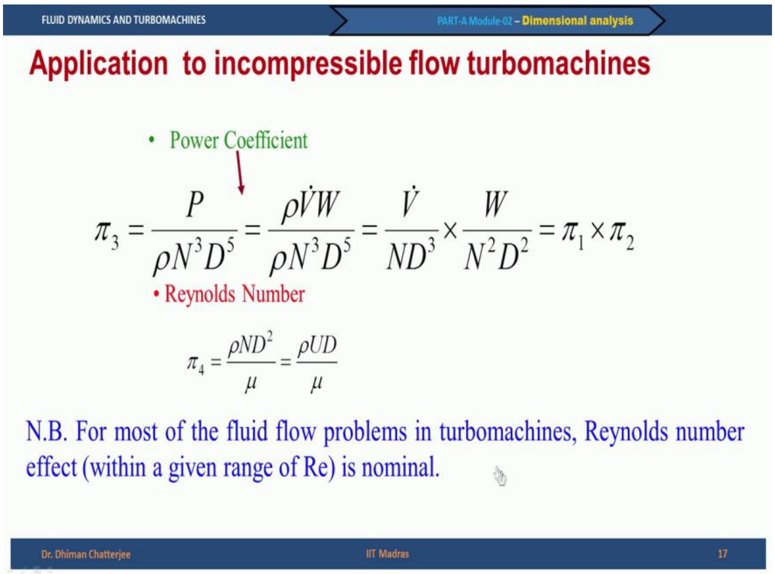

The 3rd one pie 3 can be obtained either directly as we have done or it can be obtained in terms of products of the pie 1 and pie 2. I have already told you that any nondimensional group like pie can be multiplied by another nondimensional group to give you a nondimensional group. So here I will show you that example. So we got directly that by 3 is nothing but P by rho N cube D to the power 5 and we know that we can establish this in terms of V by ND cube and W by N square D square which will give me pie 1 multiplied by pie 2. And hence it is called the power coefficient.

And the 4th one is our well-known Reynolds number written slightly differently but we can always get back the Reynolds number. Pie 4 is rho ND square by mu, now ND is related with the blade peripheral velocity and hence I can write rho U times D by mu which is our Reynolds number. In most of the Turbo machine applications the Reynolds number is in the turbulent flow range and the effect of Reynolds number is not very significant.

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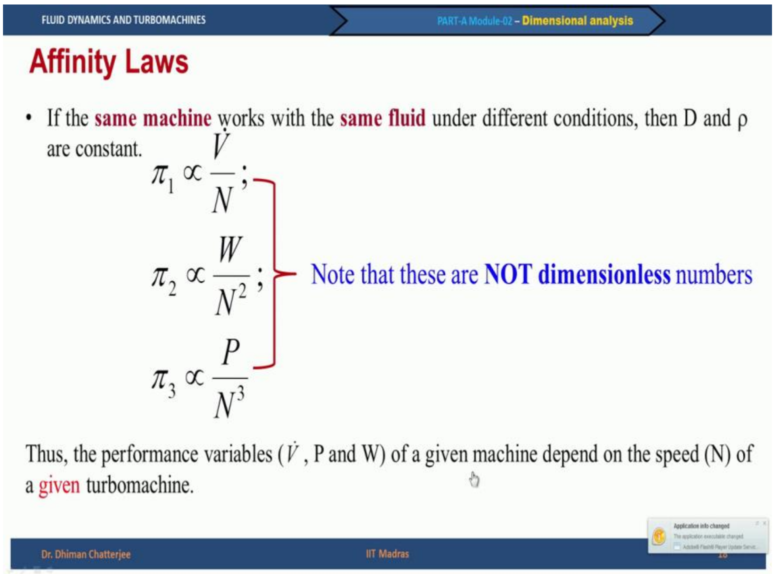

So we will now talk about affinity laws. What are affinity laws? If I say that I have a same Turbo machine which works with the same fluid but under different conditions. Whenever I am saying the same Turbo machine, that means my size is fixed, so diameter D or the characteristic length D is fixed and it works with the same fluid so density, viscosity are same. What happens is if I turn this Turbo machine and run it at a different speed? If I run it at a different speed, then I get pie 1 is proportional to V dot by N, pie 2 is proportional to W by N square and pie 3 is P by N cube.

What does it mean, it means that if I increase the rotational rpm of a Turbo machine, let us say a pump, then the volume flow rate will increase. If the head developed will also increase as N square and power requirement will increase with N cube. But please note that these are not dimensionless numbers. Thus the performance variables V dot, P and W of a given machine depends on the speed at which this Turbo machine is run.

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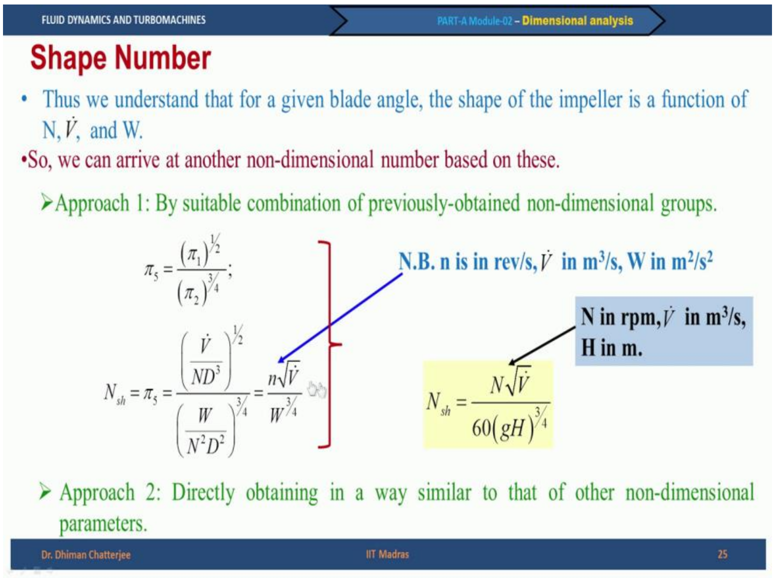

This brings us to a very important concept of shape number. Shape number as the name suggests you it has to do something with the shape of a Turbo machine. So in the next one or 2 slides we are going to take you through this shape number, concept of shape number and how it is related with the shape of a Turbo machines. So we understand that for a given blade angle, see earlier also I have told that angles cannot be changed. So if I talk about an impeller, if its angle is not changed, then the shape of the impeller is a function of the speed, the volume flow rate and the specific work.

So we can derive another nondimensional based on these and we could have also combined the previously obtained nondimensional groups, that is instead of doing it from the scratch using N V dot N W, we can combine the previously obtained nondimensional numbers to get a new nondimensional number, how, let us look at it. So we say that pie 5 can be written as pie 1 to the power half divided by pie 2 to the power three fourth and this gives me the pie 5 which is given a name called the shape number Nsh which is nothing but N V dot by W to the power three fourth. So this quantity which we obtained by manipulation of the other 2 pie groups give me a 3rd or the 5th pie, pie 5 and we get this is called as shape number.

In this case please note that the small n is in revolutions per second as opposed to the capital N which is given in rpm. And V dot is in metre cube per second and W is in metre square per second square. And we say that shape number can be related with the rpm by N by 60 V dot and multiplied divided by g H to the power three fourth. So we have defined the shape number, next we will try to talk about 2 more quantities which are related with shape number and all these 3 together will give you the shape of the Turbo machines. Note that when I write N by 60, the N should be in rpm. Of course we could have derived these shape numbers or the pie group independently by starting from scratch.

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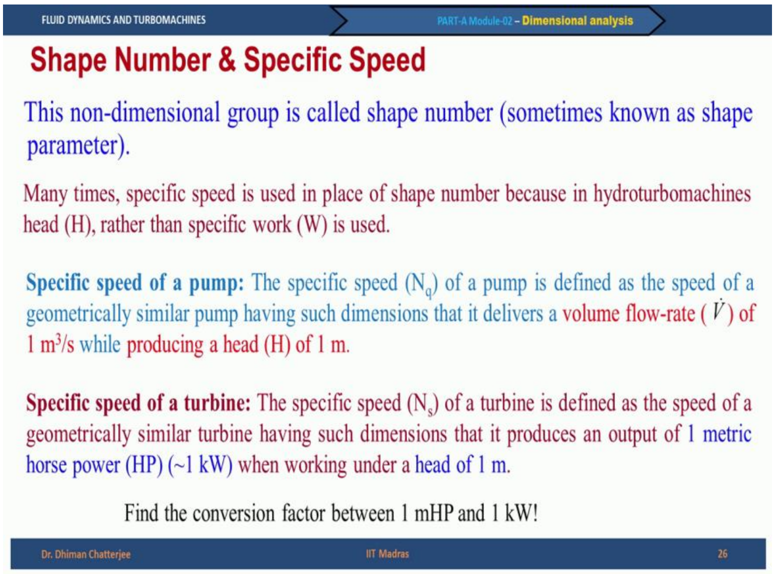

And there is, there are many times particularly in Hydro Turbo machines since g or the acceleration due to gravity is fixed, so we write it in terms, instead of specific work we write it in terms of the head and then we get instead of the shape number we define it in terms of specific speed. The specific speed of a pump is defined as the speed of a geometrically similar pump having such dimensions that it delivers a volume flow rate V dot of 1 metre cube per second while producing a head of 1 metre. And specific speed of turbine is defined as the speed of a geometrically similar turbine having such dimensions that it produces an output of one metric horsepower when working under a head of 1 metre.

Many times I find that there is a confusion among the students about what to write for specific speed, should it involve the volume flow rate and the head or should it involve the power and the head, I suggest a way of remembering it. When you think about a pump, you think what is the most important thing you are looking for. You want to take bath, so the water should have gone to the top of the tank in your building and it should have a sufficient volume to come out. So what you can think it, in case of pump relate it yourself with the volume flow rate that the pump gives because you have to take bath and also you need a sufficient head so that the pump can take the water from the ground to the top of your building.

So specific speed can be related with the volume flow rate with the head. In case of turbine what you are interested in is that for a given head difference, what is the power output. You are not really interested in the volume flow rate as user. So you can say the specific speed should be related with the head as well as the power and in these 2 cases Nq and Ns can be given by the following relationships.

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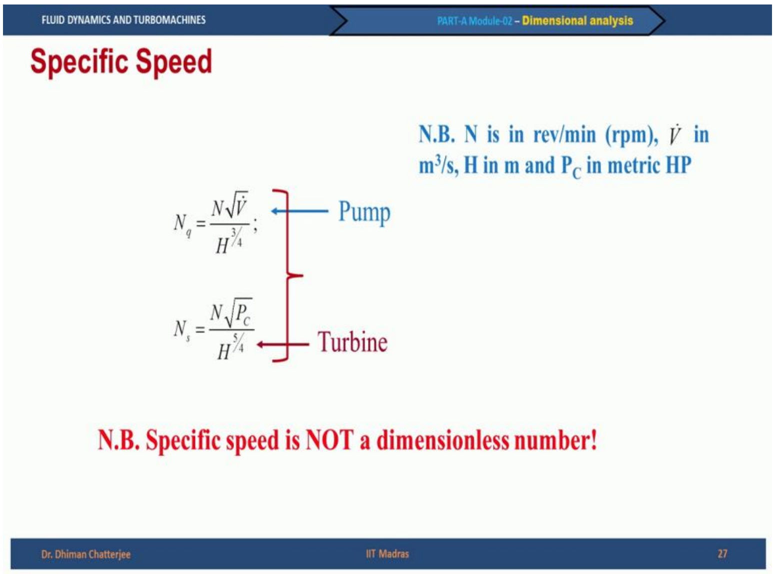

Nq can be given as N square root of V dot by H to the power three fourth and Ns which is for the turbine is given as N square root of Pc, coupling power, diverted by H to the power 5 by 4. But please be careful that for these pumps and turbines, the specific speed relationships are not free from dimensions. So you have to be very careful while using these relationships, particularly if you are a designer of a pump or a turbine. So most commonly used are for the design industries is, of the pumps and turbines is in, is in revolutions per minute rpm V dot is in metre cube per second, H is in metres and Pc is in metric horsepower. And this is not a dimensionless number. So you have to keep the units properly managed.

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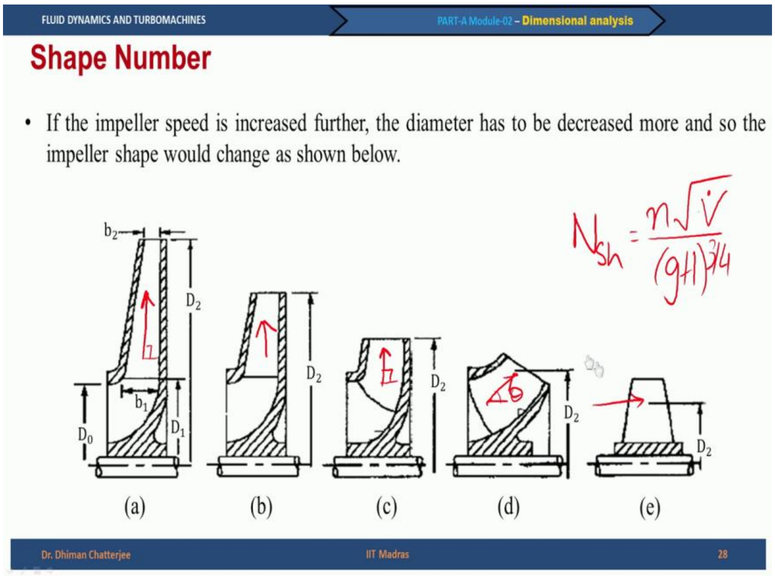

And we can say that the shape number, if the impeller speed is increased further, then what happens, if the speed is increased, then the diameter has to be decreased, why, because we want N times D which is U to be constant because we are talking about the same volume flow rate and what happens as we increase the specific speed, the diameter goes on reducing. First you see at low specific speed the flow is coming here and we get a nearly radial flow, we get a radial flow. You see that the flow is perpendicular to the axis, you increase the speed further, the specific speed increases, you still try to work with the radial flow, you say that I know only radial flow, I will work with radial flow, you try to reduce it.

But you cannot reduce it further because the length of the blade reduces and then what you try to do, you try to make the inlet edge curve so that the effectively you get slightly longer length. But if it increases further, such manipulation is not possible and you end up getting a flow which makes an angle Theta with the axis, in this case of course the axis if I show it is 90 degree, in any one of these cases it is 90 degree and finally we come to a stage that even flow is not possible and we get an axial flow where it is parallel to the axis.

Thus we can see that with change of specific speed, I am saying specific speed, I am not saying individually the speed or **or** any one of these quantities because if you look at the definition of specific speed, it is N root V by H to the power three fourth. So you can increase the shape number by either increasing the impeller speed as I told you here or by increasing the volume flow rate or by reducing the head. So it brings to a very important conclusion for us which we will again revisit when we talk about the pumps and turbines.

See if you say that you are handling a machine in which the volume flow rate is high for a given speed, then you need to have an axial flow pump. If your, on the contrary your head requirement, head to be developed is much high for the same speed, then we can go for a radial flow pump, the same with turbine. In case of low head turbine, we will see that if the H is low, that is in the denominator, then shape number which is given as… So if you have a very low value of H, then what happens is you are essentially get a very large value of shape number.

The same way you will get if you increase the speed or if you increase the volume flow rate. And hence we can say that the overall effect is one of increasing the shape number is the change in the shape and from the radial through the mixed to the axial flow Turbo machine.

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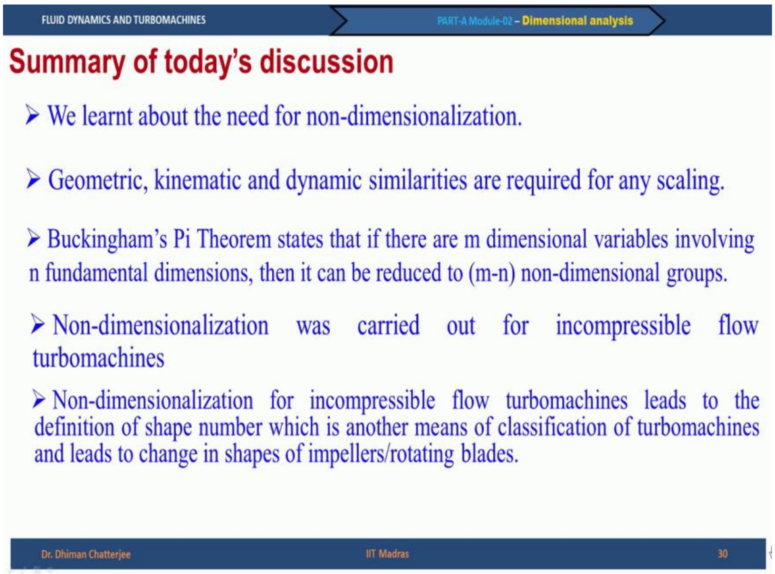

So we come to the summary of today's discussion on the dimensional analysis and its influence on the choice of Turbo machines as follows. We learnt about the need for the nondimensionalisation, we talked about the geometry kinematic and dynamic similarities that are required for any scaling. The Buckingham's pie theorem states that if there are m dimensional variables involving n fundamental dimensions, then it can be reduced to m - n nondimensional groups. This nondimensionalisation was carried out and extended for an incompressible flow Turbo machine.

And this nondimensionalisation for incompressible fluid Turbo machines leads to the definition of shape number which is another means of classification of Turbo machines and leads to change in shape of impellers or the rotating blades. Thank you.

THANK YOU