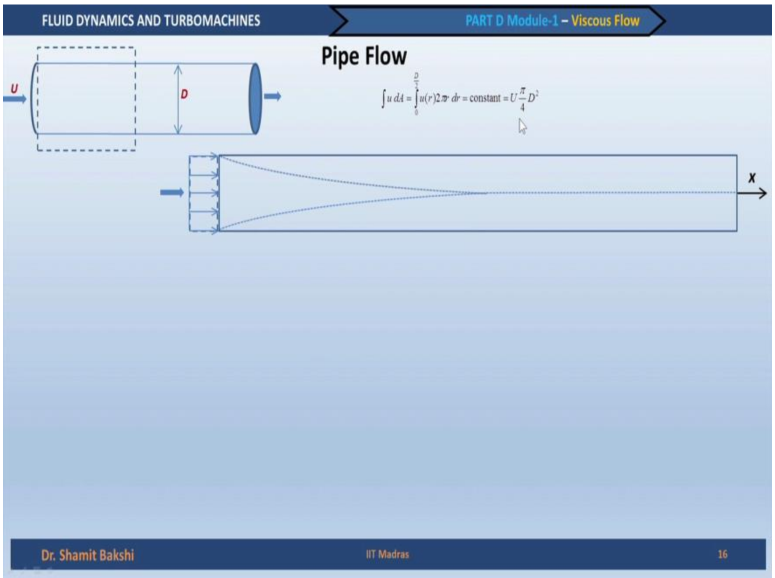
Fluid Dynamics And Turbo Machines.
Professor Dr Shamit Bakshi.
Department Of Mechanical Engineering.
Indian Institute Of Technology Madras.
Part D.
Module-1.
Lecture-4.
Viscous Flow.

Good morning and welcome to the 4th lecture in the 4th week of this course. So in the last 3 lectures we have already introduced and covered some topic on viscous flow. The viscous, the last 3 lectures were about viscous flow on top of a flat surface, so flow over a flat surface, flow over a plate. Then we dealt with dealt with different aspects of that viscous flow and then we moved on to viscous flow on curved surfaces also, on cylindrical surfaces and how it changes, in, how the boundary layer growth and the boundary layer characteristic changes with a pressure gradient.

(Refer Slide Time: 1:47)



So we saw the effect of pressure gradient on the boundary layer, that is the viscous region where the viscous region forces are important in the flow over a surface. We also looked at the drag coefficients and how to correlate drag coefficient with the Reynolds number with respect to both laminar and turbulent flows. The 4th lecture concerns with a very important aspect of viscous flow which is the pipe flow. This has a lot of engineering application. The last topic was more related to aerodynamics, so our 4th lecture, the topic is pipe flow.

So we look at how the viscous flow or the boundary layer characteristics within an internal flow changes as compared to an external flow which we have dealt with in the last 3 lectures. So in the pie, this has a lot of engineering applications actually because we need to use pipes for transportation of flow rates, from, through different lengths, through different, it is household as well as very important engineering applications. So what we are mainly concerned with here is to find out the losses, the flow losses like how much power is required to pump a fluid from one place to another.

So naturally this has a lot of engineering application like transportation of liquid from one place through pipes and ducts from one place to another, of course pumping of water in household supply of water supply system. So a lot of applications are there, there is no end to applications and there is, the understanding of pipe flow is also quite developed in the area of fluid dynamics, we will look at that and this constitutes of lot of theories and large amount of experimental data because the pipe flow calculation, for designing of a pipe which can take certain amount of flow or the designing of a pump, more importantly the power of a pump, deciding the power of a pump which can transfer the fluid from one place to another at a given flow rate.

So a lot of correlations are also available. So we will start with, we will give an introduction to this topic because of its engineering importance. So let us look at the slide now. So let us consider a flow through a pipe, this pipe of course continues and there is a velocity U with which it approaches and D is the diameter of the pipe. And capital R, it is not shown here but capital R is actually the radius of the pipe. So this is a particular symbol, these symbols we will use this particular lecture.

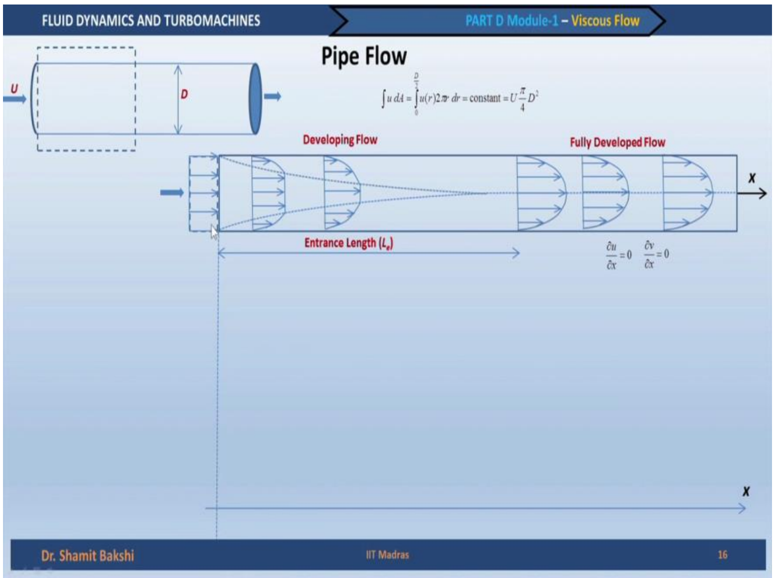
Now let us look at the initial region within the pipe. So let us take a section through the central plane of the pipe, how does it look like, so let us look at it in little more details. So we have taken it out and we are looking at a sectional view into the fluid. So the incident flow into the pipe, again like in the previous example of external flow also, the incident flow is uniform. That means the flow velocity does not change in the direction perpendicular to the direction of the flow. So it is uniform and as it enters into the pipe of course viscous forces becomes important because the pipe surface has to satisfy the no-slip condition.

So you have a boundary layer growing on the pipe and the boundary layer not only grows on one of the surface, in all the surface surrounding the fluid. So we have a viscous boundary layer growing on in the pipe from all the surrounding surfaces. That means as the flow comes in, because of the growth of the boundary layer you have some region which is viscous which is inside the boundary layer and there is an inviscid core or inviscid region at the central part of the pipe.

Of course as you, as the flow continues into the pipe in the direction of the flow so we see that the boundary layer actually merges and as it merges, so this has severe, as it merges the velocity profile also changes. So from this perspective let us look at the velocity profile, the development of the velocity profile as the flow moves through the pipe. Before going into the velocity profile it is important to look at the continuity equation because this is the meaning, this is the thing which decides the velocity profile inside the pipe.

So it only says that integral of U dA because we have considered it as an incompressible flow. So velocity multiplied by area should be, integral of that should be constant. So wherever you take a section, there this quantity is constant. This is basically the flow rate. Okay, so so many litres per, so for example if the entry is at 2 litres per minute or anything, then it has to remain 2 litres per minute at any section because the flow is incompressible and also steady. So everywhere you have that kind of the same flow rate.

(Refer Slide Time: 6:55)



Now we can talk in terms of flow rate, not mass flow rate but volume flow rate, so we are talking in terms of volume flow rate. In case of a compressible flow you have to talk in terms of mass flow rate. Anyway we are talking about incompressible flow only, so the volume flow rate is constant and at the inlet if you see, you have a flow rate U multiplied by area pie by 4 D square, which is constant. So what should be the velocity profile once it comes inside? So this will be the velocity profile as it comes inside the flow.

So what happens here, we see that in the viscous region of course it has a boundary layer kind of structure, velocity profile is like a boundary layer kind of structure. In the middle region it continues to be an uniform flow and again in this viscous region it has a boundary layer kind of structure. So basically the flow decelerates as it comes, the flow near the solid surface decelerates, the velocity is zero near the solid surface, naturally the presence of the wall retards the flow.

But what happens more interestingly is that the central part of the flow now has to compensate for the lowering of velocity near the wall. Near the wall the velocity is reduced, so at the central post on the velocity has to increase because the flow rate, the volume flow rate should remain constant. So if this part of the velocity, if you consider the uniform flow velocity profile, if this part has suffered a reduction in or retardation in velocity, due to the presence of the wall, then the central part velocity should increase so that the volume flow rate is same as that of the inlet.

So that is why the continuity equation is important to consider here. So if you go further, what happens? The viscous the viscous region increases, the viscous region because the boundary layer is growing, as the boundary layer grows the viscous region is increasing, thickness in the volume or whatever it is, so it is increasing. So as it increases, then the inviscid region now has to compensate for a large region of viscous flow in terms of the flow rate. So the central portion has now even more velocity.

So as we see this has a more velocity, the central portion of the pipe has more velocity, the fluid in the central portion of the pipe has more velocity than the inlet. And as we go forward this increases further and further because the viscous effect gradually penetrates towards the core of the flow. So you have the centreline velocity, if you measure the velocity along the centreline of the pipe, it will gradually increase. Now what happens as the boundary layer merges, at this point let say the flow becomes totally viscous, there is no inviscid region or inviscid portion which was visible at this region in this region where the flow was entering.

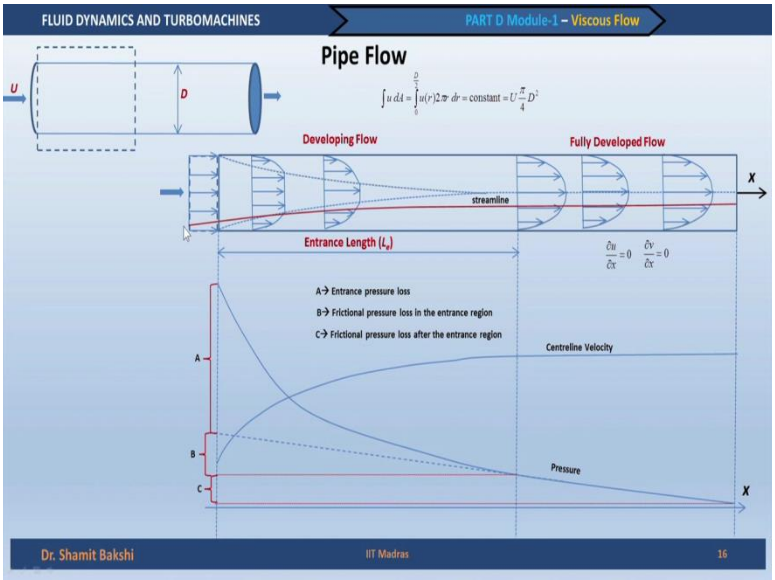
So and after some point of time or after some point of time in terms of the fluid element, fluid particle but after travelling a certain distance from the merging of the boundary layer we get a velocity profile which eventually does not change as it travels along the X direction. So if we have a velocity in a further downstream location, that means that the location latter than the initial location which we are considering, so at a downstream location the velocity profile no further it will change, it will not change any further.

If you can take at different stations, different X values you will see the velocity profile as well as the magnitude, the nature of the profile as well as the magnitude remains the same. This is called the fully developed region and this region is called the developing flow. So this here, see the velocity profile as it moves into the pipe, this velocity profile constantly changes with the inviscid core velocity increasing as we go to more further downstream, whereas here the velocity will not change with as we move along the X direction. So this is known as fully developed flow, this length is known as the entrance length.

So this is the length through which the flow has to travel to get a fully developed profile and in the fully developed profile Dell U and Dell X is equal to 0, that means velocity in the X direction, so U means the velocity in the X direction. So velocity in the X direction will not change along the X direction any further, same happens to the V velocity. So the V velocity also does not change along the X direction, so it almost, it remains constant and this region is known as fully developed region.

The implication of this kind of velocity profile is quite important. Let us see or the development of the flow, this is very important to consider with respect to the flow losses which the flow suffers as it goes, moves through the pipe. So let us look at now we have drawn along with it the X axis and we will plot the centreline velocity, let us plot the centreline velocity.

(Refer Slide Time: 11:46)



So if you plot the centreline velocity, okay so this, it will be characteristically different if we look at the entrance region and that is why we draw this line. And then if we plot it, the centreline velocity looks something like this. So it constantly increases, goes to a maximum velocity and remains constant because after the flow has developed, there is no change in the velocity. It is fully developed means there is no change in no further change in velocity as it moves through the pipe.

But in the entrance region there is a constant increase in the velocity. Now, central region is actually inviscid, the central portion which we were looking at where the velocity is increasing is actually an inviscid region, so what we can do is we can actually apply Bernoulli's equation there. Okay, it is an inviscid flow, so we can apply Bernoulli’s equation there. And let us say this pipe is straight, like what is shown here, so the height at the initial portion and the final portion is also same, height of the pipe from the reference plane frame which is fixed to the ground, it is the same, so if we apply Bernoulli’s equation, it means is the velocity increases the pressure will drop in this region, so the pressure drops.

So the pressure drops in this region very significantly because the velocity increases. Now if you see there is a significant pressure drop in the entrance length, in the entrance region and after that the pressure actually keeps on falling. Even though the central velocity, the centreline velocity is constant, the pressure keeps on falling. So this fall in pressure after the flow has fully developed is of course expected because this is due to friction, this is coming due to friction. So and if you observe it carefully, you will see this drop in velocity is actually linear.

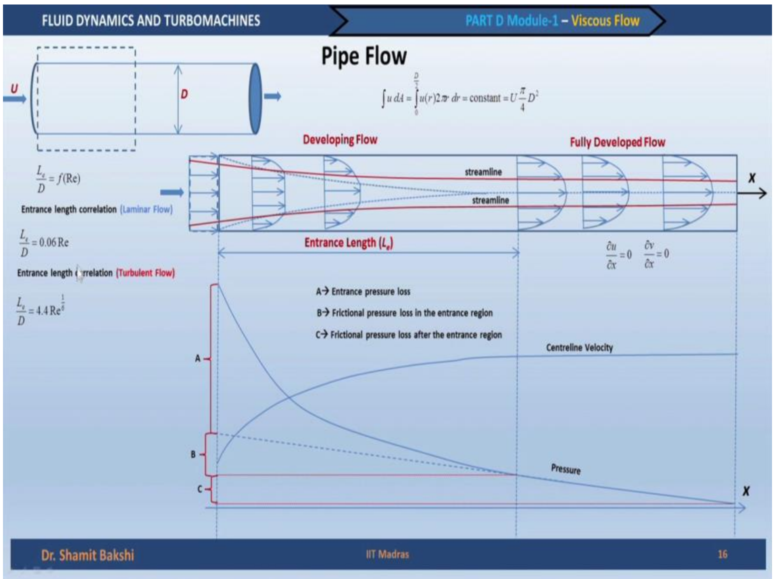
That means this is a straight line, this part of the curve is a straight line. So this is straight line and if we extrapolate that straight line to this region into the entrance region then we can have a good estimate of what was the frictional pressure loss in the entrance length. So in the entrance length you see there are 2 reasons for losses, one is due to the change in the velocity profile or during the development of the flow and there is friction loss. So now we can actually demarcate these 2 losses, the first part after extrapolating this pressure linear pressure curve, we will show later in the next slide that pressure variation, the frictional pressure drop is actually linear for laminar flow.

So and of course so far what we are discussing is only a laminar flow because the the velocity profile itself is a representative of that because it changes gradually towards the maximum velocity at the Centre if you start from the wall. In the case of a turbulent flow this will be, the velocity profile will be quite different, it will rapidly change near the wall and then almost remain constant, so it will have, it will be flat profile. So coming back the velocity profile here changes slowly and for this kind of flow the frictional loss, the frictional pressure loss is actually linear. So it linearly varies with distance.

So by extrapolating we can estimate this part, that is the frictional pressure loss in the entrance region. And A is the entrance pressure loss. This is not due to friction, this is just due to the entrance. Loss due to the fact that the flow is accelerated when it through the developing region. So of course we can mark the frictional pressure loss after the entrance region here, also see which signifies this loss, so this is basically the pressure loss after the entrance region. Of course in this particular example this frictional pressure loss looks like small but this is because of the fact that the length is also very small.

When we consider the actual pipe flows, this length will be much much more than the entrance lines. So to get an idea about that, let us see if we can get the value of the entrance length, how much is the entrance length for a given flow. So we will go to that, basically what, in a real example where the length in the after the entrance length is quite significant, this pressure loss will be more significant. So basically it depends on the length of the pipe. Now, before going into the entrance length let us plot the streamlines within the pipe.

(Refer Slide Time: 16:42)



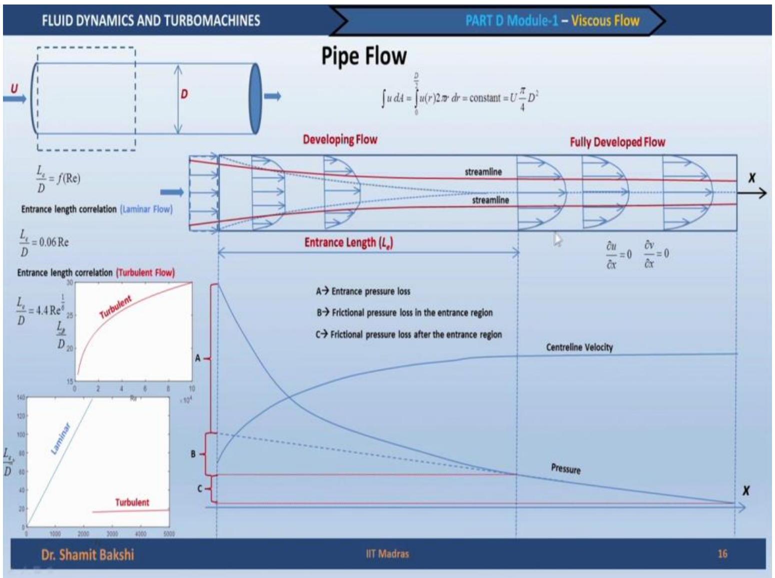
So this red curve actually shows the streamline. So this is also demonstrating the fact which we discussed in the last few lectures that in the boundary layer, when you have a boundary layer flow like this, then the streamlines actually bend. So it bends away from the boundary layer. So it comes as a uniform flow and it gradually bends, it moves away from the wall. If we draw a similar streamline on the other side we will see it that also bends. So basically as the flow takes place through the pipe, the streamlines from both sides, they come closer.

What it means is that basically the flow is trying to avoid the wall and move towards the centre. Why do we say that because we know that streamlines means that this is a boundary of the flow, nothing can penetrate, the flow cannot penetrate the streamline because there is no velocity component perpendicular to the streamline, so it cannot penetrate the streamline. That means between the 2 streamlines the flow rate is always constant, the same flow will happen.

What it actually shows here, the plot of the streamline shows here is that whatever flow was taking place through this area, through a large area at the entrance, in the developed length, the same flow is taking place through a smaller region which is expected because as the flow moves through the pipe, it tries to follow the central region which has less viscous effects and moves away from the boundary layer. That is what it is demonstrating, that is what the streamline diagram within the pipe flow in the entrance begin is demonstrating.

So now we got that, some idea about the flow development within the pipe, let us go into the estimate of the entrance length. So actually if you look at this entrance length there are correlation is available and using dimensionally analysis you can show that it is actually a function of the Reynolds number. So if you look at a typical value, so entrance length correlation for laminar flow, it will be something like this, so it is given as 0.06 times of Reynolds number. For a turbulent flow it is different and this is the correlation which is used for estimating the entrance length for a turbulent flow.

(Refer Slide Time: 19:12)



Now let us look at, by using this correlation let us look at the actual numbers, how much are these values? So if we plot that, if we plot entrance length, the non-dimensionalized with diameter for a laminar flow we see of course the entrance length increases as the Reynolds number, so this X axis or the horizontal axis is actually Reynolds number. So as the Reynolds number increases the entrance length also increases and the transition Reynolds number for a laminar flow is 2300, around 2300 and at that value of 2300 the entrance length is 140 times that of the diameter of the pipe.

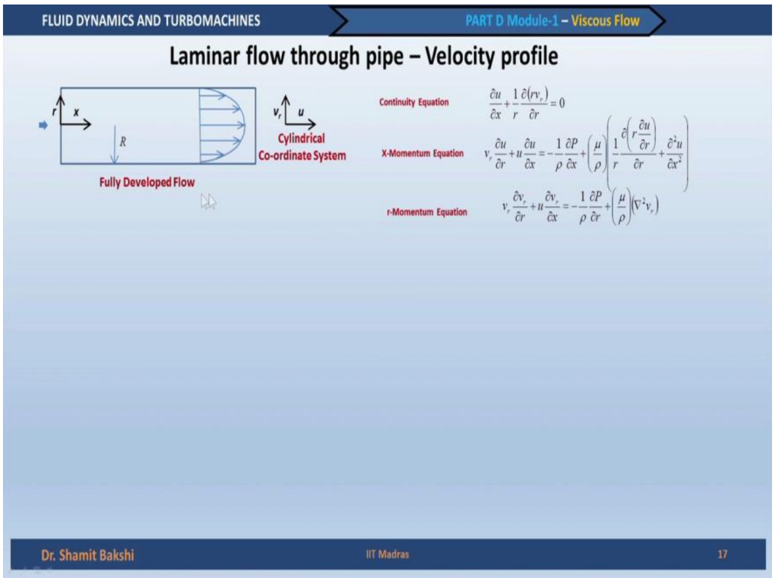
So that is basically an estimate, that gives an estimate of the entrance length. But it is quite different if this curve is plotted, the turbulent portion is plotted, we see there is a sudden jump, there is a sudden fall in the entrance length. This is because the turbulent flows, there the mixing is more and this results in a quick merging of the boundary layer. So this phenomena of merging is present even in a turbulent flow but it takes place much faster than in the case of a laminar flow and as a result of that the turbulent flow if you see here we can see it in a little more details using a different axis.

So if you see here, for a turbulent flow which is, which starts from around Reynolds number 4000, so for that kind of a flow the entrance length is about let us say 15 times to 30 times the diameter of the pipe, much less than the that of a laminar flow. So this is the this is the region where this first entrance loss due to the acceleration of the flow is present. Afterwards you have the pressure loss due to friction. Now one thing we mentioned here the transition to turbulence in the case of a laminar flow, so the laminar flow of course the Reynolds number in case of a pipe flow like we defined before is find as rho into velocity, the density into velocity into diameter of the pipe divided by mu, the viscosity, the dynamic viscosity.

Now this value of 2300 is for a smooth pipe, this could change if you have a rough pipe and it is also possible that you use a very smooth pipe and use a lot of control mechanism so that the transition does not take place, you can delay the transition also. So that is also possible. But for most of the cases we do not use so much of control parameters, so 2300 is a number which is acceptable by the engineering community as a value for the transition to turbulence.

And like we saw in the case of drag coefficient, there is always a sharp change in the behaviour of the flow when the transition takes place, like in the case of the flow past a cylinder, the same is true for the flow past a sphere also, that there is a sudden change in the drag coefficient. Similarly we can see the entrance length also, there is a sudden change when the transition takes place to turbulent. So now let us look at if we can analyse the developed flow using our approach, the fluid dynamic approach, differential approach which we have introduced in the last week, during the lectures of the last week.

(Refer Slide Time: 22:47)



So for that we go to the next slide. So let us see if we now consider only the fully developed flow because the developing flow is a very complicated, it has viscous flow at the near the wall and an inviscid flow at the Centre. So and that is only limited to the entrance length. For long pipes and in which case the flow is mostly turbulent because the velocities are high, so for those cases the entrance length is only as we saw in the last slide 30 times the diameter of the pipe. So if you, the flow losses in that region is not so significant losses.

In fact the entrance loss is designated as a minor loss. The major loss in the pipe flow is basically the frictional losses and that mainly happen in the fully developed region. So this, let us look at the fully developed flow from using the differential approach which we have introduced during the last week. For looking at pipe flow we used, introduced a cylindrical coordinate system because this is useful because the cylindrical coordinate system represents the geometry of the pipe more accurately.

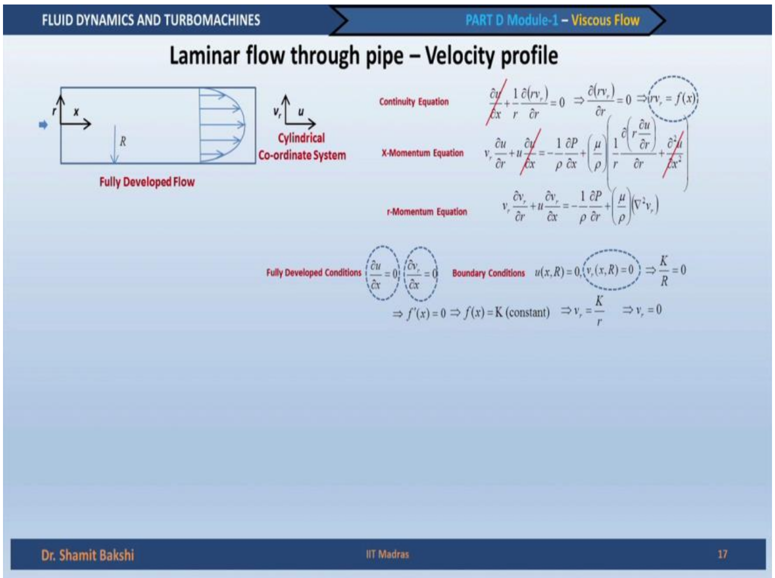
For a circular pipe, circular cross-section pipe, it exactly reproduces or represents the geometry of the object. So this is always better because in fact when we deal with a sphere, it is better to use spherical coordinate system. So we used XR coordinate system rather than the XY coordinate system which we have dealt with so far, XY coordinate system is basically the Cartesian coordinate system. So as we do that, so this is basically a cylindrical coordinate system and the velocities in the X direction is, the velocity in the X direction is U and that in the R direction is given as VR.

So in the cylindrical coordinate system in the vector form the velocity the governing equations are same, like we say for example incompressible flow it will be dell dot V is equal to 0 as the continuity equation but when you the dell operator in a Cartesian coordinate system is different, that is Dell Dell X I hat plus Dell Dell YJ hat in the case of a Cartesian system. In case of a cylindrical system it will be different, so the equations will be different and we have not derived that but we will use it for this particular case.

So the continuity equation is something like this and the X momentum equation is something like this. So if you look at here you have will in the let us say X momentum equation you have the convective term like it appeared in the case of the Cartesian system like VR and Dell Dell R of U and U of Dell Dell X of U. So this is for the U velocity, so this particular operator is operating on the velocity U. Dell P by Dell X term is also there and this is basically the viscous term which is and this one is Dell square of U.

So if you write Dell square of U in cylindrical coordinate system, it will look something like this. The R momentum equation now looks something like this, so you have the VR, you, you know V R Dell Dell R of VR and U Dell Dell X of VR. So we have not expanded this term here because we will see quickly afterward that this term will all vanish. Okay, so we have not expanded this Dell square here. So let us see what else conditions do we have. These are the governing equations but I such these governing equations looks very complicated.

(Refer Slide Time: 26:51)



So what more condition we, do we have? The next condition which we have is this is a fully developed flow, so that means the velocities does not change along the X direction. So fully developed condition says Dell U by Dell X is equal to 0 and Dell VR by Dell X is equal to 0. The radial component of velocity with respect to X is also zero, sorry the gradient of the radial component of the velocity is also zero. So we have to utilise all these conditions to simplify these equations.

Now boundary conditions also we have to impose, so they are simple, U at X, R is equal to 0, R is the radius of this pipe measured from the centreline of the pipe. So because this is, because of the no-slip condition and similarly VR, the perpendicular velocity on the wall is also zero at X, R. Now let us look at this condition Dell U by Dell X is equal to 0. This fully developed condition immediately reduces and makes a lot of simplification because this term goes out, this term also goes out and this term also goes out.

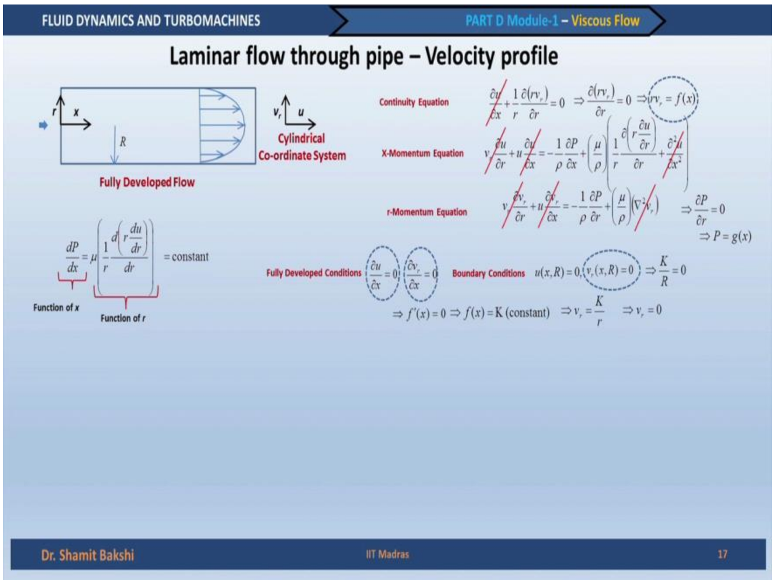
So it immediately gives us a lot of simplification and it also tells us that U is only, mathematically speaking, it is only a function of R and not a function of X. This differential equation, because this condition is also a differential equation. So Dell U by Dell X is equal to 0 means U is not constant, U is a function of r in the radius. And R means small r here, capital R is basically constant which is the radius of the pipe. Let us see with this simplification how does this equation look like.

So this looks like Dell Dell R of R VR is equal to 0. So what does it mean, it means that R VR is a function of X, like we said here U is a function of R. R VR looks like it is a function of X, so what do we, how do we proceed now? So using this fact that R VR is a function of X we can come back to this equation, the other fully developed condition. What we are getting from here is Dell VR by Dell X is equal to 0. So R VR is a function of X but Dell VR by Dell X is 0, so what does it mean?

It means immediately that F prime X is equal to 0 because if you plug-in VR is equal to FX by R here, it means F prime X is equal to 0 where F prime X is basically Dell F by Dell X or dF by dX because this is a total derivative now. So F prime X is 0 means what, FX is constant, that is what it means. So essentially what we get here is the velocity profile of the R velocity profile. So it means that VR is basically K by R. So by utilising this fully developed condition without solving, going into the momentum equation we get the velocity profile in the R direction.

But look at this profile, this has to satisfy this condition also that the R velocity at the wall is zero. So what does it mean, it means, so if you, if it has to satisfy this condition it means capital K by R, K is a constant is zero, that means K is equal to 0, if R is not infinity of course. So K is zero, so VR is zero, so simply by using, so this is, this shows that by utilising this condition and it is important actually we utilise the fully developed conditions and the boundary conditions to arrive at the solution.

(Refer Slide Time: 30:35)



So by doing this systematically you see that you can arrive at the, it very simply, at a very important simplification that the radial component of velocity becomes zero. Now if the radial component of velocity becomes zero, okay, we cannot directly write it as zero, we have to come through this route and once it is zero it becomes, it makes our equations very simple because a lot of terms will disappear. See this goes out, this entire equation almost everything goes out except this Dell P by Dell R because this is VR is zero and VR is zero everywhere according to this condition.

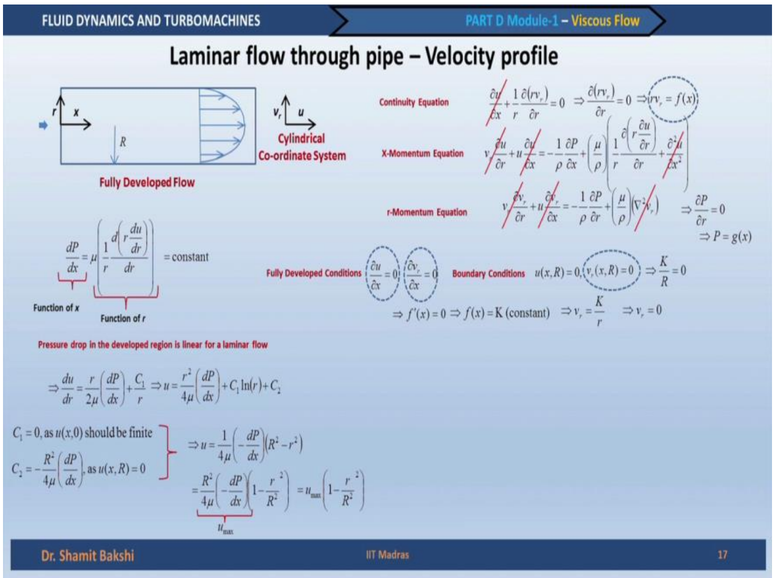
So Dell VR by Dell X will also be zero. Dell square VR because VR is zero everywhere, so Dell square of that will also be zero. So what our R momentum equation tells us is that Dell P by Dell R is equal to 0. So Dell P by Dell R is 0, it means B is a function of X, okay it is independent of R, this relation we had also derived previously if you remember in tutorial problem in the last week. Similar case for a flat flow kind of condition, not for a pipe flow.

So pressure is a function of X, so pressure see velocity is not a function of X, velocity is only a function of R in a fully developed flow but pressure is not a function of R, it is only a function of X in a fully developed flow. And this is, this comes we will see soon that this dependence is due to the fact that there is a pressure loss due to friction. So now after looking at this that pressure is only a function of X, we can now rewrite this equation. So here all the 3 terms has have gone, we are left out with pressure gradient term and the first part of the viscous term.

It can be written as an ordinary differential equation because it has 2 variables only, one is pressure which is a function of X and which we have a derivative with respect to X and another is U which is a, which is purely a function of R. So this becomes an ordinary differential equation. Now if you further consider this, the left-hand side is a function of X, the right-hand side is a function of R and they are equal. So what does it mean? These 2 can only be equal if they are constant.

So this simplifies and gives us very important conclusion that DP by DX is actually constant for a pipe flow and this is true for, this is of course true for laminar flow because turbulent flows you cannot drop out the unsteady term, unsteady because it is highly unsteady, turbulent flow is highly unsteady, so we cannot drop out the unsteady term. Okay, so for a laminar flow we get that, so pressure falls linearly along the direction of the flow. We in the plot in the last slide we had used utilised this to get the pipe the pressure loss in the entrance region, now we have proved it using the Naviar-Stokes equation.

(Refer Slide Time: 33:08)



Okay, that is what it says, the pressure drop in the developed region is linear for a laminar flow. Now we can continue with the right-hand side and if we integrate it we get this thing, this du by dr as this, this can be easily integrated and C1 is basically, so dp by dx is a constant but we write it as dp by dx and C1 is the first integration constant arriving from the first integration and the by integrating this equation again, so this also can be, this is easily integrable and you can get this equation which is which involves the 2^{nd} integration constant C2.

So this is basically the velocity profile. Now you can see that you have another boundary condition left and you have to find 2 constants C1 and C2, so how to resolve that, how to get C1 and C2 when only one condition is given? That is the velocity at the wall, U velocity at the wall is zero, V velocity at the wall is zero is already utilised here. So to get that solution we look at look carefully at this form of this solution. So if you look at this, look at this term, the C1 Ln R so if you look at this term, this C1 has to be zero. Why, because U X, 0 should be finite.

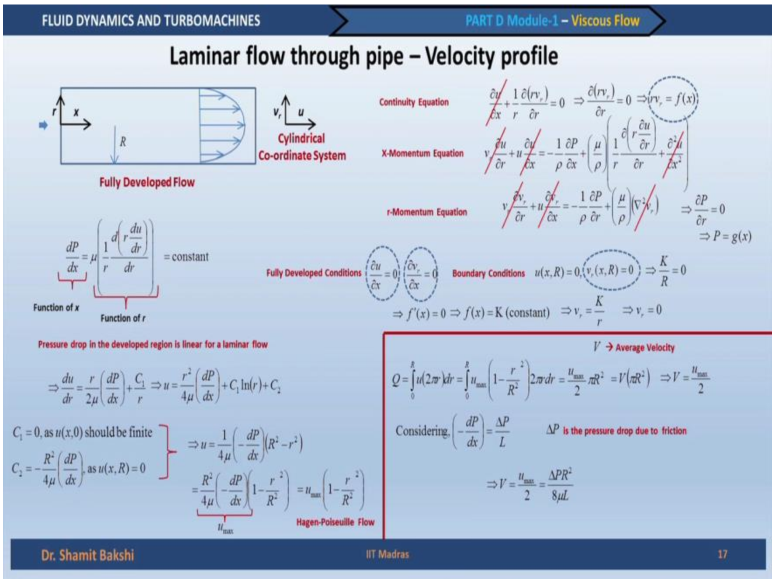
If you put U, if you try to find out U at R is equal to 0, so R is equal to 0 is basically R start from here, so R is equal to 0 is the centreline of the pipe, so R is equal to 0 if you put zero here, Ln of zero is minus infinity, E to the power minus infinity is zero, so this is minus infinity, so you cannot have infinitely large velocity. So this actually has to vanish. C1 should be zero, so without using boundary condition, just by observing the equation itself we can say this and what happens to C2, C2 can be obtained from the other boundary condition.

So if you plug-in the boundary condition that U X, R is equal to 0, you get the value of C2 as this. So now just by plugging in the value of C2 into this solution C1 and C2 both into this solution we get the velocity profile. So we can get an exact solution, this is one of the rare case in which we can get a solution analytical solution for a laminar flow, at least for a laminar flow, for turbulent flow you cannot get an analytical solution like this but definitely get for a laminar flow a exact solution of this form.

Okay, so now this is a very useful equation, so and we can rewrite this equation by rearranging this term in this form and there is a reason for writing it in this form because by writing it in this form you can see that the prefix here, it is like a coefficient, so you can write this as a U Max, so maximum velocity multiplied by this. Why is this U Max because at R is equal to 0, the velocity should be this and that is the maximum velocity.

At R is equal to 0, U should be U Max, that is clear from the velocity profile, so basically you can represent this velocity now as U Max is equal, U Max into 1 minus R square by R square and we have got through this analysis a magnitude of U Max. This is a very useful and it will be useful in the estimation of the pipe pressure loss due to friction in the developed region of the pipe.

(Refer Slide Time: 36:49)



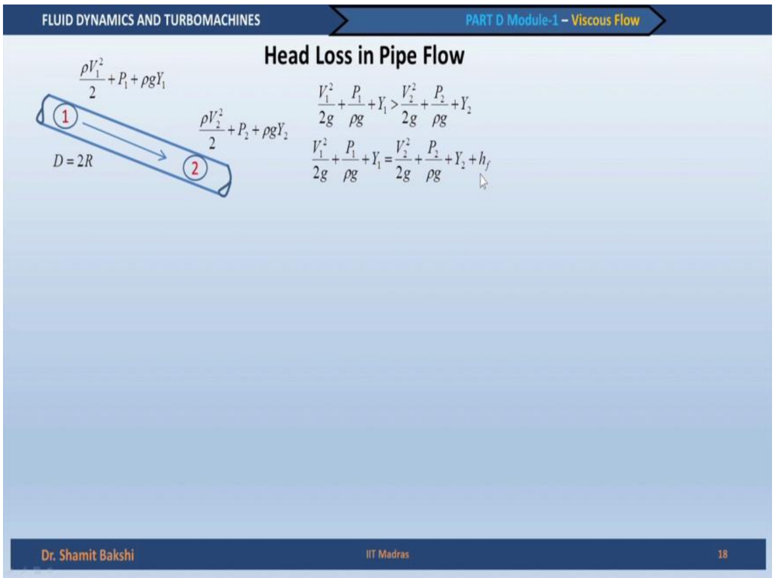
This kind of flow is called Hagen-Poiseulle flow, Hagen and Poiseulle, they did a lot of experiments on finding a correlation which we for arriving at correlation for frictional losses. Of course this derivation came afterwards, this was done in the 19th century, this derivation came afterwards of course. But this just to commemorate the contribution to the pipe flow literature, it is named after these fluid dynamists. So now what we can see is what the important conclusion from all this mathematics is that we can find out the flow rate through this pipe using this velocity profile, we can integrate this as U 2 pie r dr between 0 to r and then get the flow rate as U Max by 2 into pie R square.

So if you integrate this, you get it as U Max by 2 into pie R square. And this is important way of writing this equation because the first one if you see, it is like a velocity multiplied by the total area. So this is like an average velocity, U Max by 2 is an average velocity. So we name this average velocity as V, for the present lecture, for the pipe slow lecture we define V is the average velocity. Now this, it means that the average velocity is actually U Max by 2. Now we know U max, we know an exact expression for Umax for laminar flow, so we can use, we can also get an exact expression for average velocity.

So the average velocity and we can do that by considering minus dp by dx is Delta P by L, so what is Delta P, Delta P is the pressure loss due to friction. So if you take first station as 1 and the 2nd station as 2, so Delta P is P1 minus P2, so P2 is less than P1, so P1 minus P2 because there is a frictional loss, so P1 minus P2 is basically the Delta P. And of course P1 minus P2 is positive because there is a pressure loss but this gradient is negative because the pressure, if you move along the X direction then the pressure is reducing, so the gradient is negative, that is expressed here.

Now if you plug-in this value of dp by dx here, you can get a expression for, so Delta P is dropped to do, pressure drop you to friction and we can write the average velocity in terms of pressure term, this is a very important equation. This is equation which was experimentally obtained by Hagen by his experiments. Of course the details give us the reason for this kind of variation for the average velocity. Now we will see how we can use this for our actual finding out lost through a, due to friction for a pipe flow.

(Refer Slide Time: 39:50)



So this is also called head loss in a pipe. So we will see how to estimate this head loss. This head actually represents the height, how it represents we will come to that. So let us say we have an inclined pipe like this and we have one location 1 and another location 2. The diameter of this pipe is 2R, so either we use either diameter as a scale or R as the radius of the pipe at the scale. So in the station 1 the Bernoulli constant will be like this, we introduced this before rho V square by 2+ P1 plus rho G Y1.

So this is the Bernoulli constant at the station 1, at the station 2 it will be like this where V is basically the average velocity, see that is why we got the expression for average velocity in the last slide because that is what is useful when we try to estimate friction for a actual file, actual pipe. We cannot use the total velocity profile, it is too complicated. So a simple way is to estimate the average velocity. That can be obtained by knowing the area of the pipe, the cross-sectional area of the pipe and the flow rate.

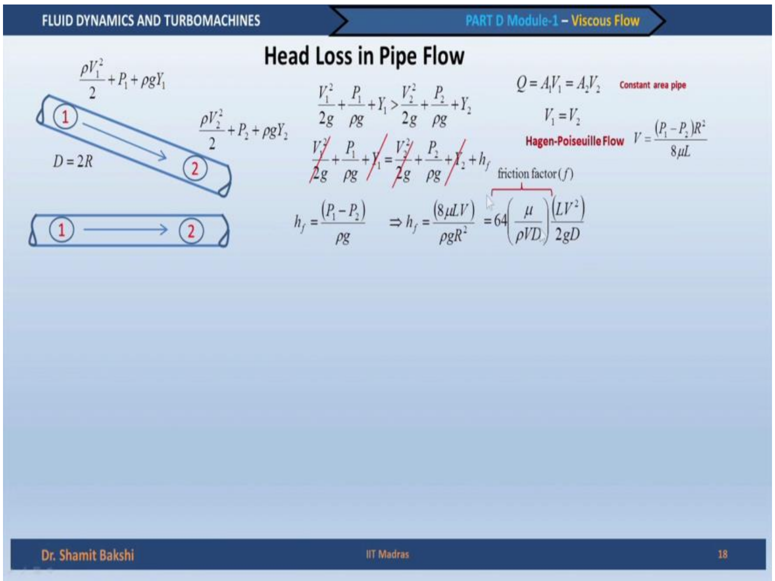
So this is basically the Bernoulli constant at the first station and the 2nd station. Now because this pipe is, pipe flow is a frictional flow, friction has a very important role to play here, these 2 are not constants, so these values are different in the pipe. How are they different, of course this will be more than this because there is a pressure loss as the flow takes place from 1 to 2. But it is better to represent this in this term. So if you, what it says is the Bernoulli constant in station 1 is more than the Bernoulli constant in the station 2.

Now we have written it in a little different form, see we have written it by dividing the entire expression with rho G. So what it does is by representing, by dividing it by rho G we get something like the total energy per unit weight. This was generally the kind of approach which was used by the experimentalist who did experiments on pipe friction losses. So this is, this practice has been returned all throughout and we write even Bernoulli's equation in this form, that means this is basically each is a representative of energy.

Of course you can arrive at this Bernoulli's equation also starting from the steady flow energy equation, that is first law of thermodynamics applied to a control volume. So that can be also done, so without considering the, there is no worker, there is no heat transfer here so if you apply those conditions and get, apply the steady flow energy equation you will get arrive at the same equation. But the important thing here is that this one is more than this, so that means there is an energy loss, energy per unit weight that is more at the first station than the 2nd station of course because of the friction.

But we can convert this into an equation, this inequality into an equation by introducing this as a extra term on the right-hand side. So this is like the frictional head loss, head loss due to friction. And all this energy per unit weight is actually having, you can check on your own that it actually has a unit of height. And that is why we say head loss or this is represented in terms of height in length scale. So by introducing this head loss, we can now write this as an equation and let us see if we can estimate this head loss in the pipe because this is the friction which has to be overcome when you want to transfer any liquid through any pipe and that will give you an estimate of the pump power required to pump the fluid through the pipe.

(Refer Slide Time: 44:03)

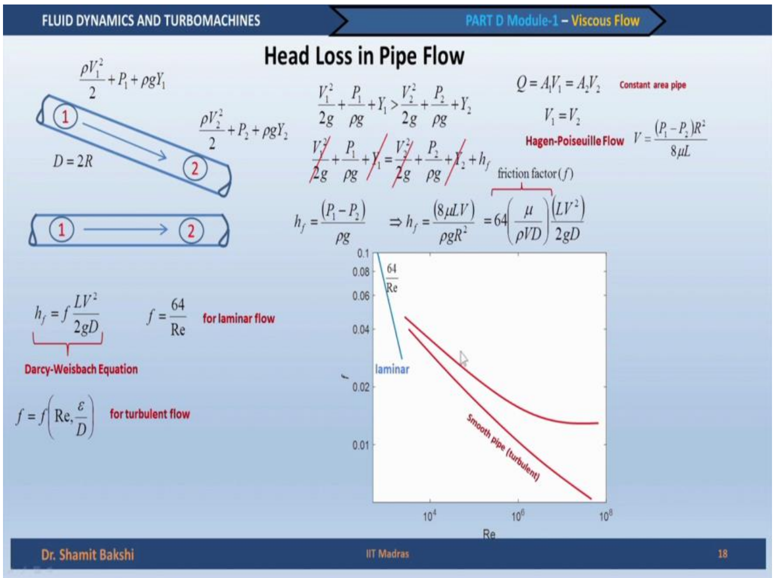


So let us see if we can get an estimate of this head loss due to friction. Before going into that, of course we can use the continuity equation applicable for this flow which is A1 V2 is equal to A2 V2, the flow rate is the same, so as this is, that area duct, V1 is equal to V2. Okay so this is constant area pipe or constant area duct, so V1 is equal to V2. So what happens is these 2 terms goes out and if we consider a horizontal pipe, that means we are not pumping something down which will help us, which will help in pumping or pumping up, then you have to do work to pumping up, pumping it upward, we neglect the effect of gravity in that way and then just see what is the only, what is the contribution of the friction, pipe friction.

So if we consider the horizontal pipe we can remove this Y1 and Y2 and we are left out with HF. HF is equal to P1 minus P2 by rho G. Now this is basically the head loss due to friction. Now if you remember in our Poiseulle flow example in the last slide what we got as an expression for P1 minus P2. We got an analytical expression for P1 minus P2 for the case of a laminar flow, we can utilise that equation, so we can use the Hagen-Poiseulle flow solution which says that the average velocity is P1 minus P2 is R square by 8 mu L.

So you can utilise this expression here and if you do that you get that the friction loss, so this was what you got, so you can now replace P1 minus P2 from this equation and plug it in here and you get an expression for head loss due to friction. This is very important relation for estimating the frictional losses in a pipe flow. This can be rewritten in a little different form by reorganising the terms which are appearing here. So you can take out mu by rho VD from here because this will, this is a familiar term for us rho VD by mu is basically the Reynolds number for the pipe.

(Refer Slide Time: 46:11)



So we can write this now as this first one is actually named as friction factor. And we can write the head loss due to friction as F LV square by 2 g D. This is a very compact way of writing the expression for the friction factor. So basically you see right now what we have done, we have actually for the laminar flow, we have actually analytically derived an expression for the head loss due to friction in a pipe flow. This equation is known as Darcy-Weisbach equation and this came before this derivation. So this was arrived at using dimensional analysis that these parameters will influence the head loss due to friction.

Now what changes in all this description is only the, is mainly the friction factor, friction factor is equal to 64 by RE in the case of a laminar flow. So for a laminar flow we say that the friction factor only depends on the Reynolds number. And the in the in a way like it is inversely related to Reynolds number. High Reynolds number higher flow rate generally speaking higher flow rate higher velocity, so for that you will have a lesser value, lower value of friction factor.

Of course a head loss because this head loss is proportional to V square and this Reynolds number has rho VD. So head loss is basically linearly proportional to velocity. If you increase velocity, this is our common knowledge also, our understanding also, if we increase the velocity or flow rate through a pipe the frictional losses will increase but the friction factor reduces, when we write in terms of the Darcy-Weisbach equation. So this discussion is not only limited to laminar flow, a similar expression of, although we cannot analytically derive like we did for a laminar flow but the expression is applicable also for a turbulent flow.

Only thing is the turbulent flow friction factor is not only a function of Reynolds number like here, it is a function of Reynolds number and epsilon by D, so what is epsilon, it is called, it is the basically roughness of the pipe. So what is the dimension of roughness of the pipe, divided by non-dimensionalized with diameter, this is how the friction factor changes for a turbulent flow. For a laminar flow it is 64 by RE. Now let us plot this friction factor with respect to Reynolds number.

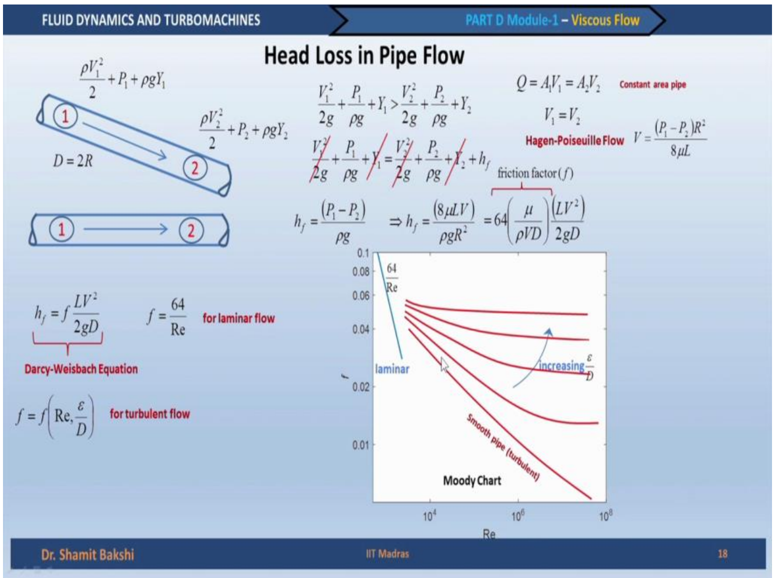
For the laminar part as we derived here, this is 64 by RE and of course you can notice that we have used a log log scale here, so this is a logarithmic, this is not a linear scale, the vertical scale for friction factor, and the horizontal scale for Reynolds number, both are logarithmic scales because we want to plot this for a wide range of Reynolds number and the variation of friction factor in during this wide range of Reynolds number is also a lot. So when we have to plot some, things like that where the variation is too high, then it is easier to plot them in log scale.

So this is the first part, although it looks linear, it is actually not linear, it is looking linear because it is plotted in a log log plot. So this is the first part, the laminar part, the friction factor reduces with increasing Reynolds number like this. If you go to turbulent flow, what happens? So this is 64 by RE of course, so turbulent flow for smooth pipe, you have a variation like this. So before going into the turbulent flow we can look at this region also, so this region is basically a transition region from 2300 to 4000 which is a small region, you have a transition from laminar to turbulent flow.

And as we observe before like in the case of entrance length, like in the case of drag coefficient, we can see here, even in the case of friction factor that is a sudden change in the value of the friction factor when the flow translates from laminar to turbulent. So this is, this shows that the turbulent flow is characteristically very different from the laminar flow. It is not smooth transition, so that is why they have to be dealt with separately. So now of course we can give qualitative explanation for why, so if you see here, actually the friction factor increases if you go to the turbulent flow.

Okay so the reason is one qualitative explanation is that the velocity profile, if you look at the gradients of velocities near the wall will be higher in case of a turbulent flow because of the characteristic of a velocity profile, it is more flat, it is more full profile. So for that the shear stresses will increase because of the higher velocity gradient because shear stresses are proportional, shear stress is proportional to the velocity gradient. So we have higher friction factor also.

(Refer Slide Time: 51:36)



Of course this is in case of turbulent flow for smooth pipe the epsilon by D factor is close to 0, as you increase the epsilon by D factor, it changes. So you have higher value of friction factor because the roughness will increase the friction factor and if you go higher and higher, you get different a different, a higher and higher, sorry if you go higher values of friction factor the roughness at higher values of epsilon by D, you get higher values of friction factor. And as you see here the curve also becomes more and more flat. In fact in this region it becomes very flat.

This means that the, in this region of the flow now friction factor is almost independent of Reynolds number. See Reynolds number is changing but the friction factor remains constant, so it does not depend on Reynolds number for a rough pipe, it mainly depends for a turbulent flow in a rough pipe. If we go for higher roughness, so in this case you see all throughout the entire Reynolds number regime region you see the variation is negligible with respect to Reynolds number.

So only thing for higher roughness pipes, only thing which influences the friction factor is the roughness, these are called rough pipes, so the factor which changes the friction or the friction factor is basically the roughness of the pipe. So and in this case if it is, if the friction factor almost remains constant with Reynolds number, you can see if F is constant in this factor in the Darcy-Weisbach equation, if F is constant, it means the head loss is quadratically or is proportional to V square.

So that means in case of the laminar flow it was with V HF, the head loss, this is not HF, this is basically friction factor, so head loss was proportional, now it is proportional to V square. In between in this region, so this is head loss in this region is proportional to V and in this region it will be between, the exponent of V will be between 1 to 2. So this is basically a comprehensive representation of friction factor with respect to Reynolds number to estimate the head loss in the case of a pipe flow.

Okay so this is basically the increasing, the direction of increasing epsilon by D and now we can utilise this kind of a diagram very well for doing engineering calculation or estimates of what will be the fictional head loss in the case of a pipe flow. And this diagram is also known as Moody chart which is tabulated and plotted by Moody and that is why it is named after him. And this is very useful as a design you know tool for designing in the industry because you can select your pumps, you can select your pipes, you can select your flow rate, you can select, specify the roughness of your pipe by using the Moody chart.

Or if you know the roughness of the pipe you can the head, the pressure head that your pump can generate, you can find out what flow rate is possible to get using a particular, using a particular pump. So it has a lot of application, so most of this, particularly the turbulent flow region is derived from experimental correlation. The laminar flow is actually, even the experiments falls very close to the 64 by RE curve. So this is quite, this is analytical solution but this part is totally based on lot of experimental data and useful for a design engineer to design a different thing which I just mentioned.

So this actually brings us to the end of the 4th lecture and the first module of this course which dealt with the fluid dynamics aspects. The latter part of the course will deal with the Turbo machines aspects, will deal with Turbo machines and the next 4 weeks you will be taught by Dr Dhiman Chatterjee, he will be teaching you on concepts of Turbo machines and I am sure you will find the basics of fluid dynamics which you learned during the first 4 weeks of this course useful in the latter part of this course and also to, this will act as a stepping stone for you to do a advanced level fluid dynamics course.

So this brings us to the first, end of the first module and I thank you for watching this video and wish you all the best in your endeavour for learning fluid dynamics and Turbo machines, thank you.