Fluid Dynamics And Turbo Machines.
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Part C.
Module-1.
Lecture-3.
Differential Analysis.

Good morning and welcome to the 3rd lecture of the 3rd week of this course on fluid dynamics and Turbo machines. In the last 2 lectures during this week we had looked that the Naviar-Stokes equation, we derived the Naviar-Stokes equation and we have, we will look at the application, some of the applications of Naviar-Stokes equation during the tutorial session of this week. Right now we will start with our 3rd 3rd lecture where we start with the fluid rotation. This is a part of the fluid motion which we have so far not dealt with, so we will start with fluid rotation. So, let us see what is the meaning of fluid rotation. Let us go to the slides now.

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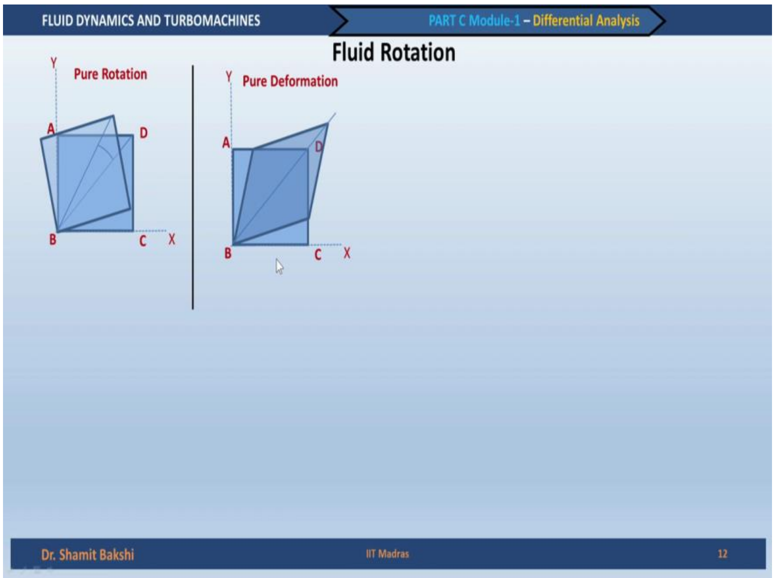

So if you see first of all we take an example of pure rotation. So, what is meant by pure rotation? So we consider this fluid element like we had seen before and these are the axes displayed here and then let us see if this fluid element just undergo a pure rotation then how what will be the position of the fluid element after a time Delta T, after a duration Delta T. So this is basically a pure rotation. Why do you call it a pure rotation because if you look at the fluid element its shape have not changed, it was square shaped before and now also it is square in shape and the size also has not changed. So neither the shape nor the size of the element has changed. What has happened at time T plus Delta T, that is added time Delta T after the first position of the fluid element is that it has rotated about the Z axis.

So this element has rotated about the Z axis. If we draw a diagonal of this element represented as BD in the first position and we take an angle, this is the angle through which this element has rotated. Of course for this particular case, this angle is also same the angle subtended by any of the edges of this fluid element with its original position represents the fluid rotation, magnitude of rotation. Next we let us look at what is pure deformation. So as we have seen before that the fluid element undergoes protection as well as deformation. Rotation we are not seen before, deformation we have seen before and we have seen how we can relate the shear stress or the stress in general with the deformation, with the rate of deformation for a fluid element. Now we are dealing with rotation but in, so let us relook at the deformation part again.

So pure deformation of a fluid element if we see again, this is the initial position at time T of the same fluid element and now in the deformed state let us see how it looks like. This element will look like something which is shown in the 2nd position. So what is the difference here, see the difference here is that the element in the 2nd position has not undergone any rotation. What is meant by that? If we draw a diagonal of the element in the first position and the 2nd position, they are collinear. What is actually meant by that, it means that suppose you consider the original fluid element that is ABCD and you consider the deformed element, let us look at the part of the fluid element showed in this region, so what has happened is this part of the fluid element has actually gone to a new position at time T plus Delta T.

So it has probably moved from here to here, so actually this part of the fluid element has rotated in a clockwise direction. So this part of the fluid element had undergone a rotation. But remember for every part of the fluid element which has undergone a clockwise rotation, if we look at the lower age now, this part has gone through a anticlockwise rotation. So overall if we consider the total fluid element it has not undergone any rotation. That is why the diagonal of the origin drawn from the original element, of course the diagonal which passes through the axis of rotation, if we draw that, then we see that for the initial position, the initial fluid element and the final fluid element, they lie on each other. So, overall the fluid element as not undergone any rotation. This is the meaning of pure deformation.

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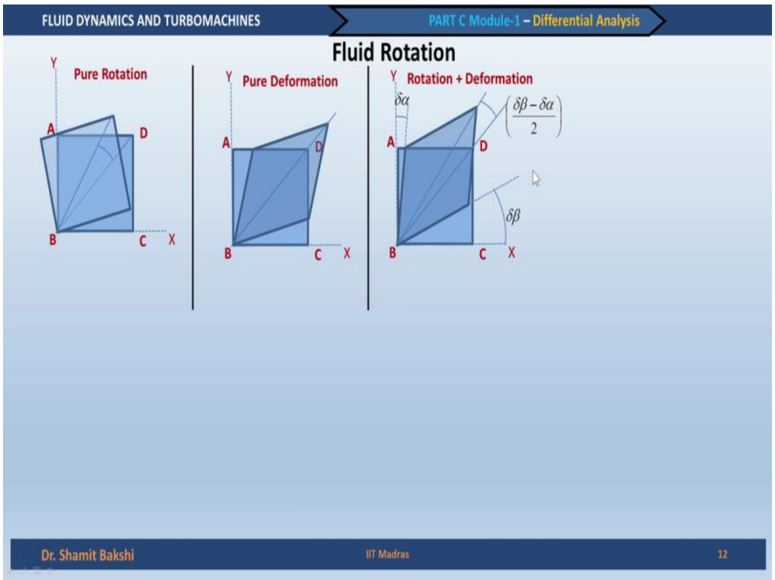

Now in a real fluid element, it undergoes rotation as well as deformation at the same time. So let us look at that particular situation. So that particular situation, that means rotation plus deformation, again we say that ABCD is the initial fluid element, then the final position of the fluid element now if something like this which is shown here. So this position is different from the position taken or the location of the fluid element shown in pure deformation in the sense that now if you draw a diagonal of the to the fluid element in its final position, you will see that it, it subtends a finite angle with their diagonal of the fluid element in the initial position. So that means now there are some parts of the fluid element which has undergone some net rotation in the sense that you do not have a anticlockwise rotation of a part of the fluid element for every clockwise rotation.

For the figure which is shown here, as the net rotation is anticlockwise, so it means it has a net anticlockwise rotation. That means this BD has gone through a final position after rotating in anticlockwise direction in the case of the final fluid element, final position of the fluid element. So essentially this is what we mean by fluid rotation, with respect to a fluid particle or a fluid element. Now let us see like we did for the case of deformation, if we can find out the value of this rotation, how much is the rotation of the fluid element. To find that out let us say that this angle which is subtended by the lower edge of this element, the final position of the lower edge of this element with the initial position and Delta Alpha is the side edge, it has rotated in a clockwise direction, whereas this has rotated in an anticlockwise direction.

If both these 2 were same, then the net rotation would have been zero but they are not same and we are representing a general case where both rotation and deformation are present, in that case if we can find out the value of this angle then we can find out what is the magnitude of fluid rotation. So we do not want to go through the details of the geometric procedure for fighting of this angle, this is can do as a home work, this is not very difficult to do. If we will do this we will get that this angle is the angle of rotation of the fluid element is given as Delta beta minus Delta Alpha by 2. So for a limiting case when these 2 angles are same we can see the net rotation is zero that means it represents the case of rape your deformation.

Now can we find out this rotation in terms of velocity because when we are dealing with the fluid flow it is important that we represent everything finally in terms of velocity or velocity gradient. This should not be difficult because while dealing with deformation in our previous classes, previous lectures, we have already found out how we can find out the value of Delta beta and Delta Alpha in terms of the velocity gradient. If we remember then we can say that the deformation was obtained as Delta Alpha plus Delta beta and the rate of deformation was often as Delta Alpha plus Delta beta by Delta T. In the limit of Delta T tending to 0, it takes a value of Dell U by Dell Y plus Dell V by Dell X. So now let us see what is the value of this fluid rotation in terms of velocity.

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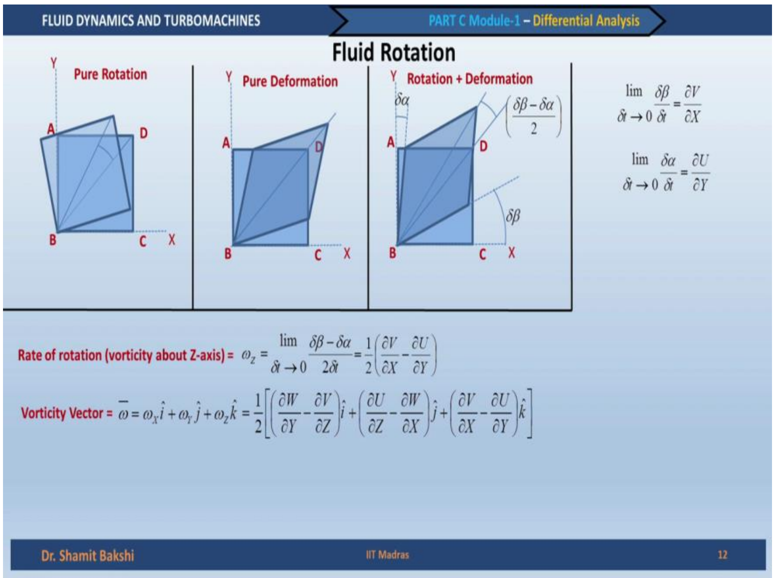

So if you remember the relations which were derived before, we know we can find out the rate of rotation. The rate of rotation of the fluid element is also known as the vorticity of the fluid at a particular point. And the vorticity about Z about the Z axis is what we are talking about here we have considered a two-dimensional flow, the rotation is only possible about an axis perpendicular to the X and Y axis. So this vorticity is given as Delta T tending to 0, the magnitude of rotation that means Delta beta minus Delta Alpha by 2 divided by Delta T. So this is basically the expression for the Z component of vorticity. So as we are talking about the Z component, so we must realise that this rotation could be about any axis. So it is a vector quantity, vorticity is a vector quantity and you can have rotation about X and Y axis also. In case of a two-dimensional flow the only component which is non-zero in the XY plane is omega Z, the rotation about the Z axis perpendicular to the XY plane.

So now if we, this expression can be obtained by, if we remember that we obtained this value of limit Delta T tending to Delta beta by Delta T as Dell V by Dell X and the limit of Delta T tending to 0 Delta Alpha by Delta T is Dell U by Dell Y and this was by summing them we got the value of the deformation of the fluid element. Now to find out the value of the rate of rotation we can plug-in these values of these limits into this expression, if we do that then we get something like this, half of Dell V by Dell X minus Dell U by Dell Y. So this is basically the vorticity about Z axis. Now as I was telling, if we consider a three-dimensional flow, this becomes a vector quantity and you have 3 components of vorticity, that means omega bar because it is a vector quantity now and you have X component, Y component and Z component.

The way we have written or obtained the value of the vorticity about Z axis, we can write the expression for vorticity about X and Y axis and the net result will be something like this. So if you carefully observe this expression you see that the Z component of velocity is plugged in from here to this, of course the half is taken outside this bracket because it appears in each component of the vorticity vector. So by comparing, we are not going to derive the this X and Y component of vorticities here but just by comparing with this expression, a similar expression can be easily written like this for the Y component it is Dell U by Dell Z minus Dell W by Dell X and similarly for the X component. Now if you look at this expression carefully, this is of course a vector quantity and if you see this can be written in a very simple form using our understanding, using vector calculus.

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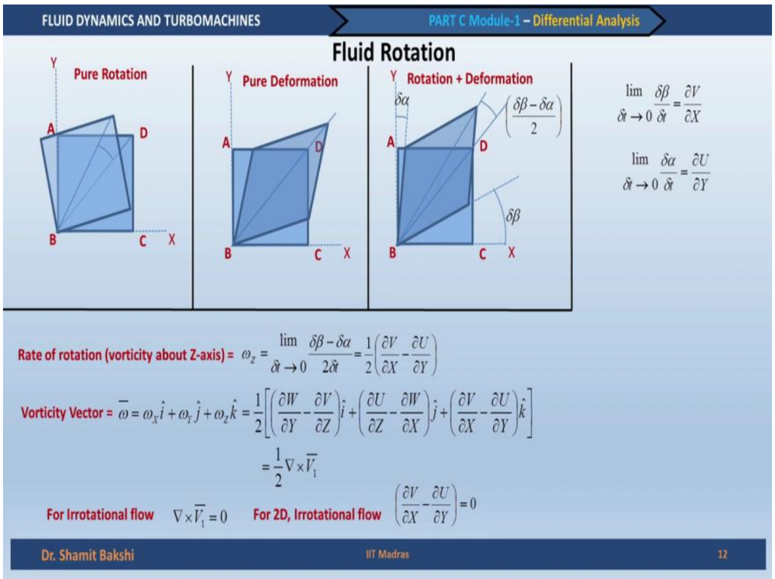

So this can be simply written as half of Dell cross V1 bar, please remember V1 bar is the velocity vector which we have defined in the beginning of this chapter. Now this, so in other words basically vorticity is half of the curl of the velocity vector. This also tells us that for a irrotational flow, that means in a flow where there is no rotation of the fluid element, in that case this vector, this curl of the velocity will become zero. So just coming back to our two-dimensional case, it only means that omega Z will be zero. So for a 2-D a rotational flow we have Dell V by Dell X minus Dell U by Dell Y is equal to 0. We will use this expression afterwards in the next few slides.

This if you look at carefully has given as a procedure for finding out whether a velocity field, just by knowing a velocity field the velocity field and finding the gradients of velocity field we can tell whether the velocity, whether the flow is rotational or irrotational. So it means that if you introduce a particle in the flow whether that particle will rotate about its own axis, rotating about its own axis is the very important factor because if you see the fluid element also, it is actually in a rotational case, it is rotating about its own axis. Okay, so now let us see go to the next slide.

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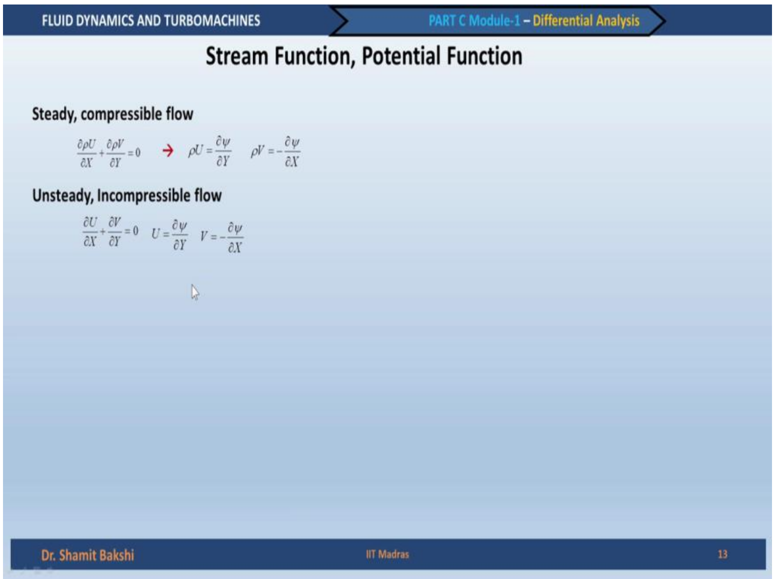

In this slide we also introduce certain new terminologies which are very related to our present discussion that means the differential analysis of the fluid flow. So we introduce this stream function and potential function. Based upon our first slide of this lecture and the derivations done in the last 2 lectures we know that for a steady compressible flow from our previous lecture that for a steady compressible flow the continuity equation is given like this, like Dell Dell X of rho U plus Dell Dell Y of rho V is equal to 0. For a 3-D case we are always dealing with today, so I have just written the 2-D case here, for a 3-D case you will have Dell Dell Z of rho W along with this expression. So now let us see if we in defining stream function we actually try to define a function in such a way that that single function will automatically replace or automatically satisfied this equation.

So this means that if we define a function rho U as Dell psi by Dell Y and if we define another function and we if we define rho V in terms of the same function as minus Dell psi by Dell X, so if you see the function has been chosen carefully so that if you plug-in this function into the continuity equation it will get automatically satisfied. That means Dell Dell, in the X part of this expression it will become Dell Dell X of Dell psi by Dell Y on the other hand in this case it will become minus Dell Dell Y of Dell psi by Dell X. So they will cancel each other and it will automatically get satisfied, the continuity equation will automatically get satisfied. The advantage is now, by doing this what you have done is actually you have reduced the number of variables in this formulation of the fluid flow. That means you have replaced both U and V by single function psi or derivative of a single function psi. And while doing that you have made sure that it satisfied some of the equations automatically by definition itself.

So this is a very useful technique of in mathematics to reduce the number of variables. But of course the price you have to pay is in the fact that the order of the equation increases because now U is replaced with a derivative term. So the order of the equation, the eventual equation, momentum equation, continuity is automatically satisfied but the momentum equation will increase. We will not go to the momentum equation now. Let us look at how this will look like, how this function looks like in the case of a unsteady incompressible flow. So for a steady incompressible flow which will be the case, which will be dealt with in both of the examples which are discussed in this course, we know that the form of the continuity equation or mass conservation equation is like this Dell U by Dell X plus Dell V by Dell Y is equal to 0. Now taking example from here we can define the stream function for this kind of a flow in a little different way and make sure that it satisfies the continuity equation automatically.

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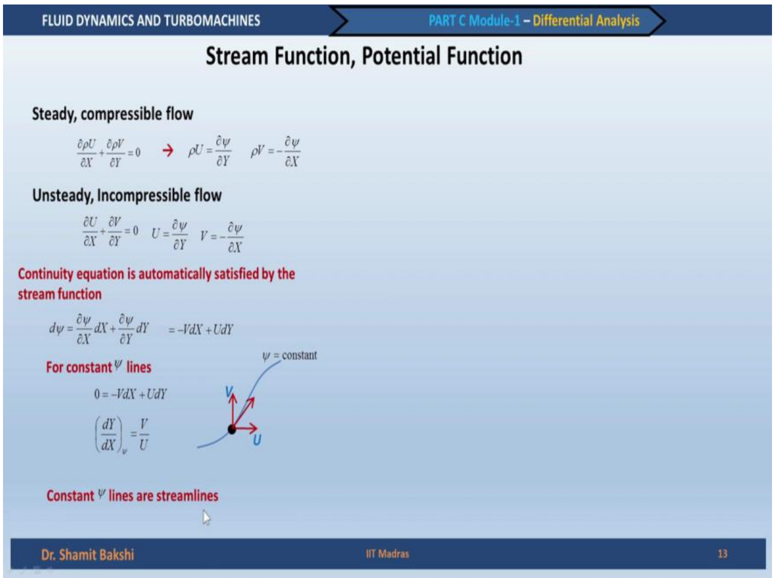

How it can be done, it is now very simple, instead of writing rho U as Dell psi Dell Y, we can write U as Dell psi by Dell Y and V as minus Dell psi by Dell X. If we do that, you can easily see that we plug-in this U these values of U and V into the continuity equation, they get automatically satisfied. So that was the objective of this of this definition of this, of defining they function. This function psi is basically the stream function. So the stream function is a function which automatically satisfies the continuity equation. Now it has, it is not only that it only satisfies this equation, that is basically the motivation by from where this function is defined but if we look at the expression little carefully we can see it has more important physical significance in the sense that lets find out the value of d psi by applying the rules of partial derivative, partial derivatives we can write d psi where psi is a function of both, of 2 variables, and 2 independent variables X and Y.

We can write this as Dell psi by Dell X multiplied by dx and Dell psi by Dell Y multiplied by dy so now if we plug-in Dell psi by Dell X as minus V from here and Dell psi by Dell Y as U from here, then what do we get? We get this as minus V dx plus U dy. This is quite simple but this gives an important information. So what is that, now let us say we have a constant psi line or we have any constant psi line where psi is constant. If you go along this line then psi is constant. That means if you go along this line then d psi is zero. If d psi is zero, that means minus V plus U dy is zero. So in other words it means that dy by dx for a constant psi line is V by U. Or this is basically, if you look at this carefully dy by dx for psi is equal to constant, it means the slope of the constant psi line at any particular point.

What this derivation tells us or what this expression tells us is that the slope of a constant psi line is equal to the ratio of the V component, that means Y component of velocity divided by and the X component of velocity. If we look at this, in this figure it means that, let us say this is a constant psi line and we have a velocity vector like this, so okay, we do not look at the velocity vector to begin with, let us look at the components of the vector, so this is the U component, this is the V component of the velocity vector then this V by U is basically the tan of the angle made by the velocity vector with the x-axis. The tangent or the tan of angle, tan of Theta where Theta is basically the angle subtended by the velocity vector with the X axis.

So what it means in other words is that, so if we draw a velocity vector to this psi is equal to constant line, then it will be tangent to the psi is equal to constant line. Which means that this psi is equal to constant line is nothing but the streamline. So in the 2-D flow as we demonstrated here the psi is equal to constant line are actually streamlines. By this expression we have shown that the slope of the psi is equal to constant line is equal to the tan of this angle made by the velocity vector with the X axis. So that means the velocity vector is actually tangent to the psi is equal to constant line. So these are actually streamlines. So the stream function is very present streamline in a two-dimensional flow. This cannot be extended to that three-dimensional flow but this is, even this is very quite useful, constant psi lines are streamlines.

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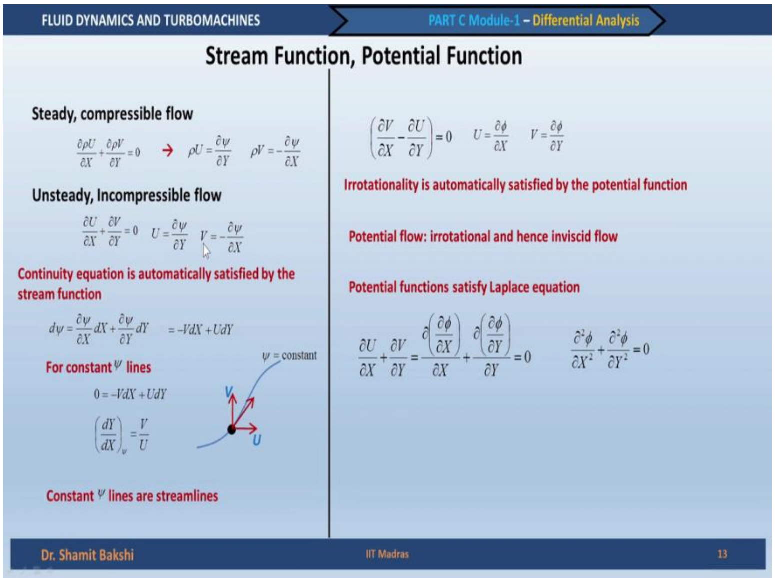

Now there is another function which has been introduced written in the top of the slide which is known as potential function, let us see what is that function. In the last slide itself we saw that irrotationality condition is Dell V by Dell X minus Dell U by Dell Y is equal to 0. This is basically omega Z, omega Z is equal to 0. If we apply this or if we define a function now, let us say we name it as phi in such a way that it automatically satisfies this is a irrotationality condition like we have defined the stream function in such a way that it automatically defines, it automatically satisfies the continuity equation, now we defined a function in such a way that it automatically satisfies the irrotationality criteria that is omega Z is equal to 0.

If we do that, then U becomes, we can define U as Dell phi by Dell X any V as Dell phi by Dell Y, Phi being the potential function. You can see by the definition of the function itself if we plug-in the values of U and V, it gets automatically satisfied because by plugging it here you get Dell 2 Phi Dell X Dell Y, by plugging this thing here you get Dell 2 Phi Dell X Dell Y and if you subtract them it becomes zero. So a irrotational flow can be represented using this potential function. So irrotationality is automatically satisfied by potential functions, by potential function that means you can only defined a function when the flow is irrotational. That means irrotationality essentially means that the flow is inviscid because if the viscous effects are important in the flow, it will cause rotation of the flow of the fluid element.

So now another thing we can observe here from the definition of the potential function, the way the potential function is defined, if we plug-in the this expression for velocities into the continuity equation now, so we plug-in these values of velocities into the continuity equation then it will form an equation which is in the form of the well-known Laplace equation. That means, let us, it is demonstrated here, you have U plug-in, U is equal to Dell phi by Dell X and V is equal to Dell phi by Dell Y, if you do that, then you get the equation in the form of Laplace equation. It means that the potential functions always satisfy the Laplace equation. Why it always satisfy Laplace equation because any flow has to satisfy the continuity criteria. Of course this is for an incompressible flow, so it has to satisfy, if it is a 2-D incompressible flow, it has to satisfy equation, Laplace equation in this form.

So the potential function will always satisfy the Laplace equation. And if we can define a potential function, it means that the flow is irrotational. So essentially potential functions are true or are they exist for inviscid or irrotational flows. Stream functions, let us see if we do this kind of thing for the stream function or be ask the question whether the stream function should satisfy the Laplace equation or not, the answer directly comes, if we plug-in the value of the U and V velocity in terms of stream function into the irrotational, the irrotationality criteria, if we do that, we see that it comes in the form of, if we plug-in U and V into this equation, we see that the stream function, if the flow is irrotational will also satisfy the Laplace equation. So it is not mandatory for a stream function to satisfy Laplace equation because you can define a stream function in a rotational flow also, it has to be just unsteady flow or if this definition is correct, it has to be 2-D unsteady incompressible flow for this definition. For 2-D 3-D compressible flow it has to satisfy this kind of a definition.

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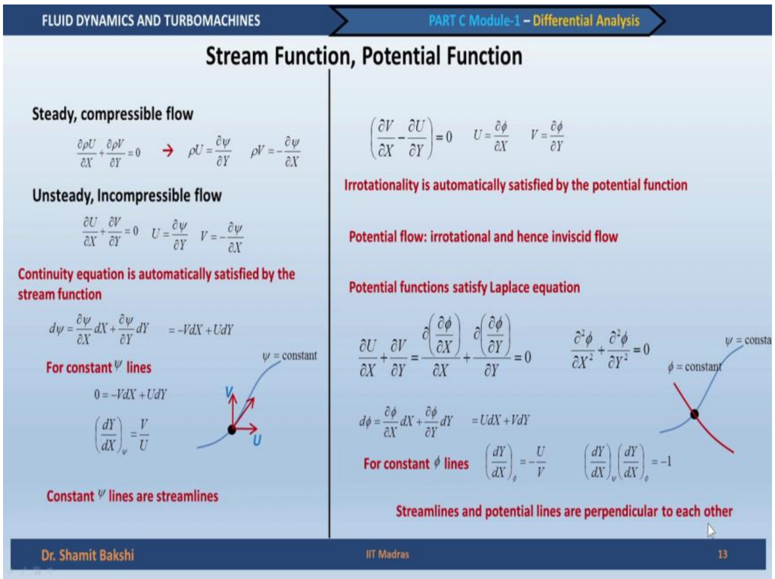

So coming back to this, so it will only satisfy the Laplace equation if the flow is irrotational. But the potential functions are always satisfy the Laplace equation because continuity satisfying continuity equation is a mandatory requirement. Now like we did here, let us see what does the constant phi lines signify. So we write again d Phi in this form, if we do that, we can plug-in the value of Dell phi by Dell X as U and Dell phi by Dell Y as V so it is U dx plus V dy. Now for constant Phi lines d Phi is zero, that means dy by dx for constant Phi lines, is dy by dx for constant Phi lines is equal to minus U by V. So this is also important because if you see this slope, this is a slope of a constant Phi line like this was a slope for a constant psi lines.

What we can see from these 2 expressions immediately is if we multiply these 2 expressions, what we get is minus 1, that is dy by dx for psi, constant psi, dy by dx constant Phi is equal to minus1. In a physical terms, it means that the slope of the constant psi line and the slope of the constant Phi lines, their product is minus1, of course we know from our coordinate geometry, from our knowledge of coordinate geometry but this is true if the 2 lines are perpendicular to each other at the point of their intersection. So this is a psi is equal to constant line and let say we consider a constant Phi line through this point, then Phi is equal to constant will be perpendicular to the psi is equal to constant line. So we also say that streamlines and potential lines are perpendicular to each other. So in this slide were actually introduced these 2 important functions in fluid mechanics which is useful to deal with the equations which we derived before. Because by the definition of this function, certain criteria, certain mathematical criteria corresponding to some physical criteria is automatically satisfied.

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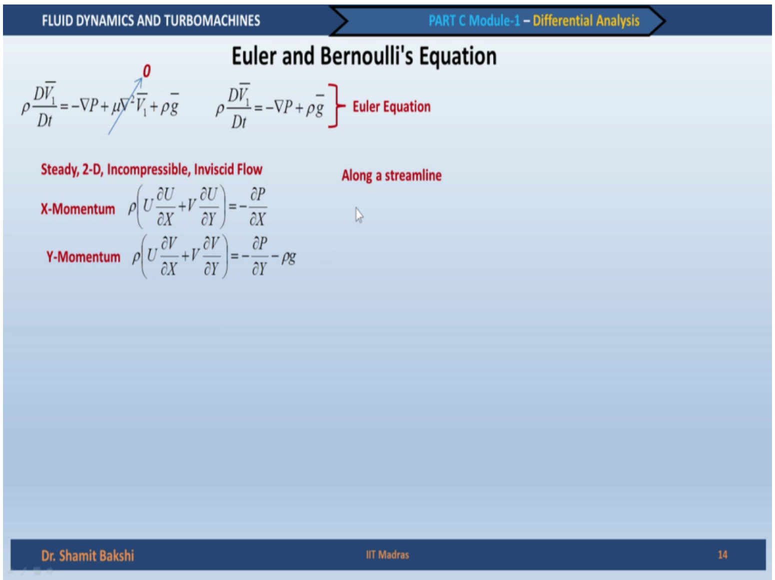

Another aspect of the equations or the differential equations which we have derived in the beginning of this week is the special form of these equations in a little simplified conditions. So the first one is, we have seen that this is basically in a vector form, this is basically the Naviar-Stokes equation, you have the acceleration term which we saw that you have the presence of local and the convective accelerations here, the pressure gradient, the viscous term and the body force terms in terms of the body weight. So, if we remove this term from this equation, what we get is the Euler's equation. So this is basically the Euler equation. Of course this equation was derived before this Naviar-Stokes equation came, so this has separately an existence.

So this was derived much before this equation actually came, they inviscid form of the momentum conservation equation. So this is Euler equation, so now if we consider a steady two-dimensional incompressible inviscid flow, then we can write the X momentum equation which is basically a special form of this equation that means 2-D and steady form. This is already inviscid, 2-D and steady form of this equation, so it will look like this, the X momentum equation to not have any gravity term, it has only the flow acceleration term only to do the convective component and the pressure gradient term, all these 2 terms are only present here. The Y momentum equation as is in this form, the only extra apart here is the gravity term. So you have the flow acceleration term, the convective acceleration term, pressure gradient term and the gravity term.

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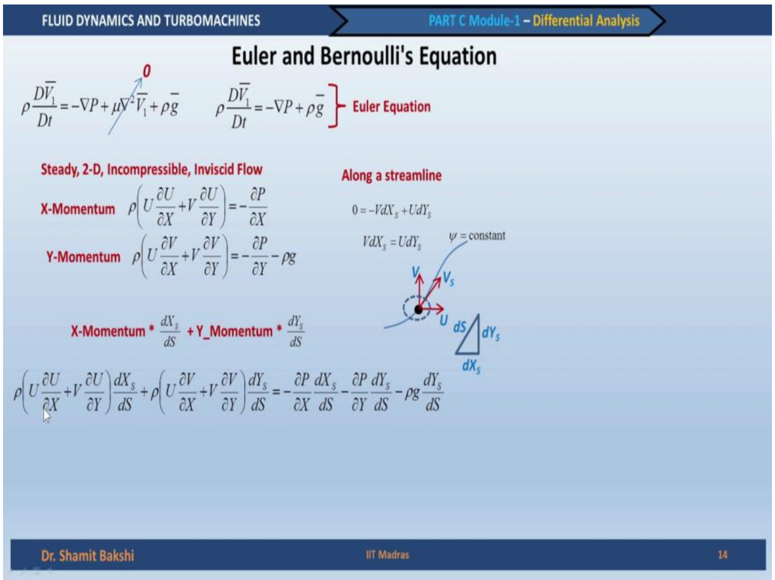

Now let us see what happens along the streamline. So we have defined the streamlines in the previous slide, they are basically psi is equal to constant lines, that means the lines along which the stream function has a constant value. So let us see what form these 2 equations are valid in 2, in the Cartesian coordinate system, in 2 directions of the Cartesian coordinate system. That means one is the horizontal direction and one is the vertical direction like what is represented in the simple situation here. Can we transform this equation in a different form if we move and apply this equation in the streamline coordinate system? That means can we apply this single equation which is applicable as we go along the streamline because we know certain criteria have to be satisfied along the streamline. So along a streamline we can say, we take up this point, so basically this is the length along the streamline and these are the elemental psis along the X and Y direction.

So ds, this is dXs and this is dYs. If we do this, then along the streamline we can always write that like we derived before d psi is equal to this, d psi is equal to 0 and so minus V dXs plus U dYs is equal to 0. So this particular criteria is satisfied along the streamline. Let us see if we can utilise this expression and get an equation along the streamline, valid along the streamline. So to do that what we do is this, we multiply the X momentum equation with dXs by ds and Y momentum equation by dYs by ds. So if we do that, this is done with an objective to get a momentum equation valid along the streamline, along the psi is equal to constant line. So if we do that and utilise this relation, let us see what happens.

Here in this expression, what we have done is we have done this particular operation to this expression, that means we have multiplied the, although it looks a very big expression, what it exactly what it actually means is we have just multiplied this one with dXs by ds and the Y momentum equation with dYs by ds of course on both the left hand and the right-hand side. So after doing that we get this equation. So after getting this equation we can make a few observations, for example if you look at this equation, this is directly plug-in from here, U have this term, U Dell U by Dell X and if you take this actor inside, you have dx by ds. On the other hand you have V Dell U by Dell Y, now if you see, if you club this V and dXs, this V and dXs, it is U into dYs. So why do not you replace this V dXs with U dYs, what do you want to do that because then this equation becomes very simple. We will see what it becomes in the next line.

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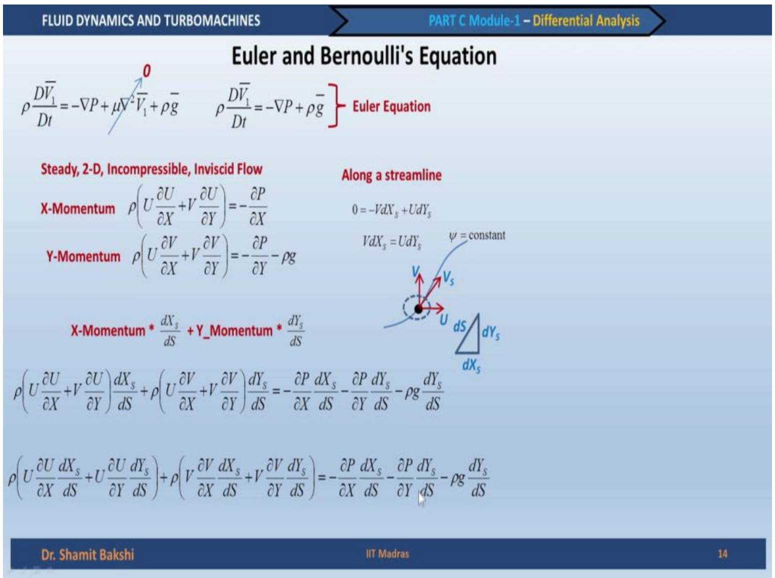

So this expression remains as it is, we are just taken in this factor inside as a multiple of this one, the 2nd one by replacing V dXs with U dYs, what we can get is U Dell U by Dell Y into dYs by ds. This dYs is actually dy but only thing is we want to emphasise the fact that this is along the streamline, otherwise it is just dy. So if we do that, now if you look at this full expression, this has, if you take U outside, you can take U outside the bracket, you can write this as rho U, this is Dell U by Dell X into dx by ds Dell U by Dell Y into dy ds. This is the chain rule of partial differential equations and then if we apply that rule, then we can write this as, simply as dU by ds, that is what we wanted to get, that is what that means we wanted to get this equation rewritten in the streamline coordinate system.

So like we did it here, the same thing has been done for the Y component also, that means here what we have done is, the first expression, the first expression in the Y component that is Y dYs has been replaced by V dXs, so if we do that, again you get this expression as dV by dx into dx by ds, dXs by ds. If you take rho V out you get dV by ds here. On the right-hand side it is already in the form of the chain rule that is dV by ds, dx by ds plus or this minus if you take outside, this will be plus dV by dy dy by dx, so this is already in the chain rule form so that you can write it in the streamline coordinate system, that is you can write it as dP by ds. So we utilise that factor here and then we can write this equation in the streamline coordinate system and this rho U dU by ds plus rho V dV by ds is equal to minus dP by ds minus rho g dYs by ds.

So as such if you see this equation looks quite similar to our U or V momentum equation and then that means you can apply this equation along a streamline in this form. So now the advantage is it is a total derivative and you can actually integrate this. For integrating this if you now club these 2 particular velocities, what you will get is essentially d of, so this U dU is actually half of d of U square or d of U square by 2. V dV, just by the rule of derivative it is D of V square by 2. If you club them, it is D of U square plus V square by 2, U square plus V square is basically VS square, what is VS square, VS square is the velocity along the streamline.

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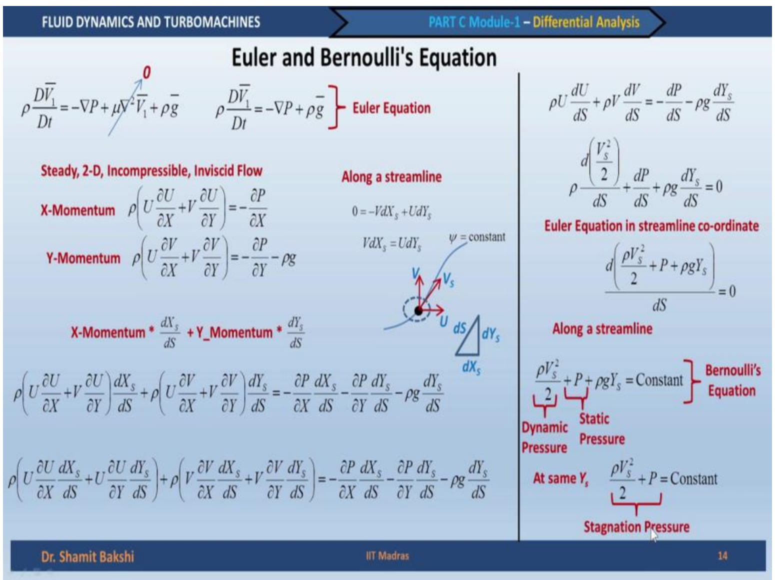

So this is basically the velocity vector, it means the velocity vector because along the perpendicular to the streamline there is no component of velocity. So this is basically the velocity, the total velocity, magnitude of the velocity vector. Okay, so now we have this expression, we have taken the DP by ds also from the right-hand side and we can write like this, this is essentially the same Euler equation but now written in the streamline coordinate system. So this is another form of the, you can now write this, like this DDS of this and finally if you integrate this equation you get a very useful equation which is valid along a streamline that rho VS square by 2+ P plus rho g into YS is equal to constant. That means if you move along the streamline, the sum of all the 3 things will remain constant under the assumptions which I have made.

That means it is a steady 2-D incompressible inviscid flow. So we have to remember this assumption, that is a steady 2-D incompressible inviscid flow and this equation is actually known as the Bernoulli's equation, this has wide application in fluid mechanics as well as many other allied branches of engineering for finding out the pressure, velocities, etc. in different applications. So, let us get little more physical insights into this particular expression or particular equation. If you look at this, the first term is actually the, it emphasises the velocity, so this is called, this can be called as pressure because this a unit of pressure, rho V square has a unit of pressure and of course we can say that this equation has to be dimension dimensionally consistent, so any one term, if it is pressure, the other term also should have unit of pressure. So this is basically dynamic pressure, the 2^{nd} part is called static pressure, so the static pressure essentially means that, the name is given as static pressure because this pressure is called dynamic pressure.

So this is basically the pressure, this is for a flow, the static pressure is also for a flow, not for a static condition but it only emphasises that it is done the pressure due to the non-velocity component of the flow. So the velocity component now, this will be little more clear when you go to the next step, so if we consider the same value of YS between 2 points in the streamline, than what we can write this **as** as rho V square by 2+ P equal to constant. So this actually means that the sum of the dynamic pressure and the static pressure is constant. So this pressure is called a stagnation pressure. That means so what it means is if you have a flow and you stagnate the flow, then what happens is the velocity becomes zero and the pressure increases.

So the and the stagnated flow has a high-pressure, now if you suppose if you follow the streamline and if we find that the flow from a stagnating condition has come to an accelerated condition, that means it has some velocity along the same streamline. It will automatically mean that the pressure along the streamline has reduced. This is a very, this helps us to explain a lot of things in fluid mechanics, that means the static pressure, so the velocity and the pressure, these 2 heads exchange continuously between each other the values continuously between each other along a streamline. So if one becomes I the static pressure becomes high, then the dynamic pressure becomes slow, if the dynamic or if the flow velocity increases, the pressure should reduce, if the pressure, the static pressure increases then the velocity should reduce.

If we consider a converting section the flow goes from a low velocity too high velocity because in the throat of the section the velocity is high, so the pressure is to reduce. So this is basically, this is a simple application of the same Bernoulli's equation. We will see an application of this during the tutorial session also, so this brings us to the end of the 3rd lecture and basically the lecture session of the 3rd week of this course. During this today's lecture, what we have looked at, we started with, we started with the flow rotation, we defined what is flow rotation, we defined the vorticity in the two-dimensional, in the two-dimensional flow, we defined vorticity vector from there we defined the condition for irrotationality, then we defined stream function and potential function.

We saw that the stream function actually represents the streamline in a two-dimensional flow and we saw the stream functions are functions which automatically satisfies the continuity equation. Potential function on the other hand are functions which automatically satisfies irrotationality condition, so they are only defined for an irrotational flow. From there we moved on to do little simplified and very important forms of the momentum equation or the Naviar-Stokes equation, the differential form of that equation is called the Euler equation which is the, which is for the inviscid case and the same inviscid differential equation, if it is integrated in the streamline coordinate system we get the Bernoulli's equation along the streamline. So this is what we have discussed during this lecture, this brings us to the end of this lecture session for the 3rd week, we will do some tutorial problems. Thank you.