

Experimental Stress Analysis – An Overview
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Lecture – 2.1
Visual Appreciation of
Field Information – Part - 4

Let us continue the discussion on overview of Experimental Stress Analysis.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Typical Results for Various Problems

The problems considered are

- ★ Beam under four point bending
 - Closed form solution by Strength of Materials is possible
- ★ Cantiliver Beam
 - Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
 - Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
 - $\nabla^4 w = \frac{P}{2\pi D}$ obtainable from theory of elasticity
- ★ Spanner tightening a nut
 - Due to complex nature of the geometry only a numerical solution is possible

In these cases relevant experimental contours (recorded or simulated) are shown to appreciate the nature of fringe contours.

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We have discussed in the last class the problems considered are beam under four point bending, cantilever beam and the point to note here is; when I do an analytical method I am able to get the stress field, I am able to get the strain field, I am also able to get the displacement field.

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The slide is titled "EXPERIMENTAL STRESS ANALYSIS" and "Overview of Experimental Stress Analysis". The main heading is "Disc under diametral compression – Analytical solution".

- The famous assumption of plane sections remain plane before and after loading is not possible and one has to resort to the method of theory of elasticity to solve the problem.
- Closed form solution for stress field is as follows

The diagram shows a circular disc with a vertical diameter. Two downward-pointing arrows labeled 'P' are positioned at the top and bottom of the diameter, representing diametral compression. A coordinate system is centered at the origin of the disc, with the x-axis pointing to the right and the y-axis pointing upwards. A radius 'R' is shown as a line from the origin to the right edge of the disc.

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Now, the next problem is we move on to disc under diametral compression and that is what is shown here, I have the disc and the center of the disc is taken as the origin, I have x and y coordinates and R is a radius of the disc, I have diametral load P which is acting on it and we will call this diameter as either capital D or small d. And for this problem, you have closed form solution from theory of elasticity. You will not be able to approach and solve the problem from strength of materials, but you will be able to solve only from theory of elasticity because plane sections do not remain plane before and after loading. You also had that problem in the case of a cantilever, when you have a shear the planes do not remain plane; they have a warping.

Fortunately, there is no coupling between normal stress and shear stress so you could live with flexure formula that is why you call that as engineering analysis, the moment you come to circular disk, you have to depend on theory of elasticity.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = -\frac{2P}{\pi t} \begin{Bmatrix} \frac{(R-y)x^2}{r_1^4} + \frac{(R+y)x^2}{r_2^4} - \frac{1}{D} \\ \frac{(R-y)^3}{r_1^4} + \frac{(R+y)^3}{r_2^4} - \frac{1}{D} \\ \frac{(R+y)^2 x}{r_2^4} - \frac{(R-y)^2 x}{r_1^4} \end{Bmatrix}$$

$r_1^2 = x^2 + (R-y)^2$ and $r_2^2 = x^2 + (R+y)^2$, R denotes the radius of the disc, D represents its diameter, t is the thickness of the disc and P is the compressive load applied.

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Fortunately, theory of elasticity provides the solution and here again it is given in a convenient form σ_x , σ_y τ_{xy} and you have this as minus $2P$ by πt and I would request all of you to take down this equations. Though these equations are very long, very valuable information when we develop photo elasticity we could directly use these equations for our interpretation.

And you have σ_x , R minus y , x square divided by r_1 power 4 and r_1 is defined as r_1 square equal to x square plus R minus by whole square and you have the second term is R plus y x square divided by r_2 power 4 and r_2 square is defined as x square plus R plus y square. We have already seen R denotes the radius, D represents diameter, t is a thickness and P is the compressive load applied.

And you have the expressions are the slightly difference for σ_y and τ_{xy} and I would like you to have these equation written down and obviously, the stress as what we see now, they are much complex than what we had see in the case of a cantilever beam or the four point bending problem and naturally you would not be able to calculate σ_1 minus σ_2 right away in the class and then provide the counters, so we have to depend on computer graphics to do that job.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The strain field is as follows

$$\begin{cases} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{cases} = \frac{-2P}{\pi Et} \begin{bmatrix} \alpha \frac{x^2 - \nu\alpha^2}{(x^2 + \alpha^2)^2} + \beta \frac{x^2 - \nu\beta^2}{(x^2 + \beta^2)^2} - \frac{1-\nu}{D} \\ \alpha \frac{\alpha^2 - \nu x^2}{(x^2 + \alpha^2)^2} + \beta \frac{\beta^2 - \nu x^2}{(x^2 + \beta^2)^2} - \frac{1-\nu}{D} \\ 2(1+\nu) \left[\frac{\beta^2 x}{(x^2 + \beta^2)^2} - \frac{\alpha^2 x}{(x^2 + \alpha^2)^2} \right] \end{bmatrix}$$

$\alpha = R - y; \beta = R + y;$
 $r_1^2 = x^2 + (R - y)^2 = x^2 + \alpha^2$
 $r_2^2 = x^2 + (R + y)^2 = x^2 + \beta^2$

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When stress field is so complex, strain field is going to be much complex than this and this is how the expression look like and I have epsilon x, epsilon y, gamma x y; that is equal to minus 2 p by phi Et and here again I would like you to make a decent sketch of the; I mean you take down the notes of these expressions. I have this as alpha into x square minus nu times alpha square, divided by x square plus a square whole square and alpha is defined as alpha equal to R minus y, beta equal to R plus y and you have; alpha and beta are also related to r 1 square and r 2 square as follows.

You have r 1 square as x square plus alpha square, r 2 square as x square plus beta square and you have this second term here is beta into x square minus nu times beta square divided by x square plus beta square whole square minus 1 minus nu divided by d. So, this is the expression for epsilon x, you have a long expression for epsilon y and you also have a very long expression for gamma x y and though they are long, it is better that you have a copy of this in your note books because this will help you, because they are not readily available in the books that you have access to, have you been able to make a copy of this equations.

You have seen the stress field, strain field both of them are longest and definitely that displacement field is going to be much longer than what you have seen all along.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Disc under diametral compression – Analytical solutioncontd

The displacement field is as follows

$$u = -\frac{2P}{\pi E} \left(\frac{1-\nu}{2} \left\{ \tan^{-1} \left(\frac{x}{R-y} \right) + \tan^{-1} \left(\frac{x}{R+y} \right) \right\} - \frac{(1+\nu)}{2} \left\{ \frac{(R-y)x}{r_1^2} + \frac{(R+y)x}{r_2^2} \right\} - (1-\nu) \frac{x}{d} \right)$$

$$v = -\frac{2P}{\pi E} \left(\frac{1}{2} \ln \left(\frac{x^2 + (R+y)^2}{x^2 + (R-y)^2} \right) - \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R-y)^2} \right) + \frac{(1+\nu)}{2} \left(\frac{x^2}{x^2 + (R+y)^2} \right) - (1-\nu) \frac{y}{d} \right)$$

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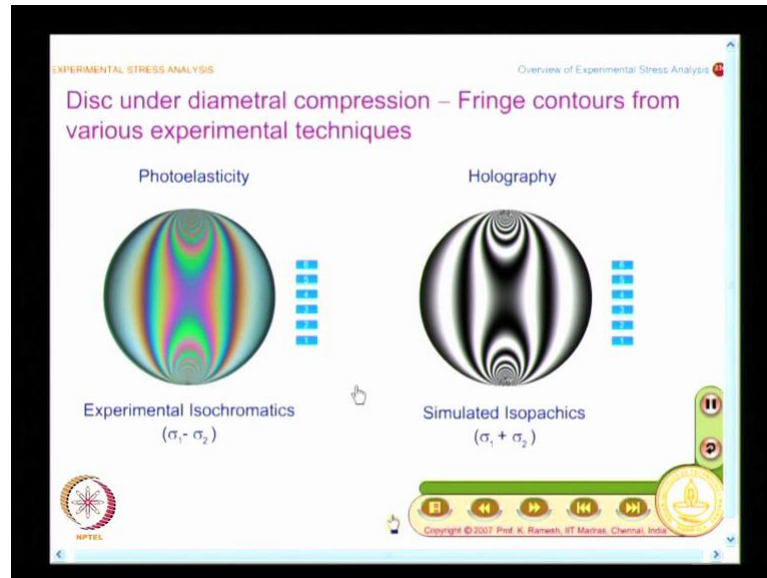
So, it is better please take your time to write it down and you have this as u equal to minus 2 p by phi t e into 1 minus nu divided 2, you have a tan inverse x by r by minus y plus tan inverse x by R plus y, then you have a minus of 1 plus nu times divided by 2 into r minus y x divided by r 1 square, plus R plus y x divided by r 2 square, minus 1 minus nu x by d. On similar lines you have a long expression for the v displacement, you have a term with natural logarithm x square plus R plus y whole square divided by x square plus R minus y whole square and the expression goes like this and obviously, it is very difficult to visualize what could be the nature of the displacement field by looking at this equation.

If we have to plot, you have to go to computer software, plug in these equations, have plotting software to call out the numbers, collect them and then draw the contour.

On the other hand, I take the model, I put it in the appropriate optics, I get stress information, and I get displacement information. The moment you want to go for strain; right now we do not have a technique which will give whole field strain information. You can get from strain gauges and plot them or from displacement information, you can do numerical differentiation and plot the strain, but right now you do not have a whole field experimental technique, which would give you whole field strain data conveniently.

Now, what we will look at is; the circle disc under diametral compression is a bench mark problem in photoelasticity.

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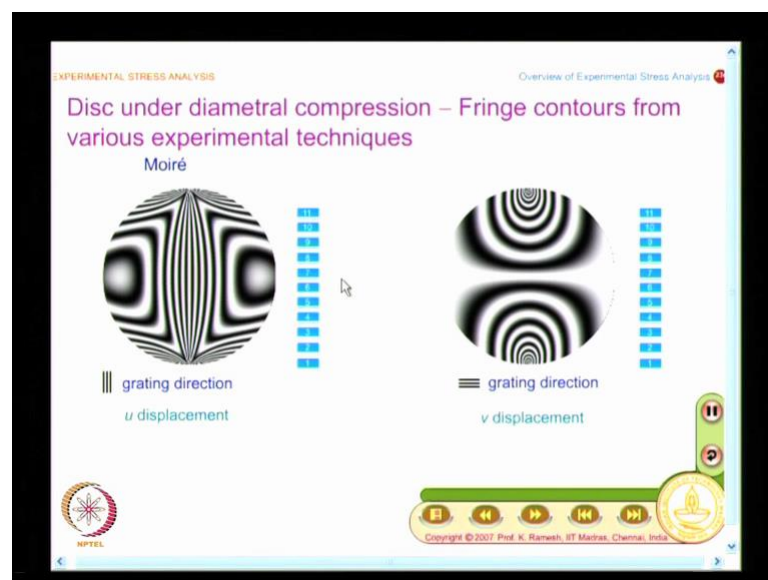
So, we will see the photo-elastic contours and we will also see another set of an experimental arrangement, where you get contours of sigma 1 plus sigma 2. In photoelasticity you get contours of sigma 1 minus sigma 2 and I said one experimental technique will not give all the information, suppose I want to find out information of individual cis components; one approach could be do a holographic experiment get simulated, I get isopachis recorded, in this case it is simulated, but get the isopachis recorded. So, when I have photoelasticity contours as well as holographic contours, I can process these two and find out individual magnitudes of sigma 1 and sigma 2 on the entire field.

And what you have not noted down is between photoelasticity and holography; I have showed colored contours for photoelasticity because you use white light and get information in color which is unique to photoelasticity. Although, it also gives monochromatic information, many techniques depend on monochromatic light source and you get black and white information which is processed.

Holography, you do it only on a single wavelength and the contours are very similar and I would like you to have a reasonable sketch of this, any one of it which gives you an indication; how the fringes look like and here you have this as a shape of 8, this you have to note it down, this we will use it in our experimental interpretation later and you can also see as a function of load applied, how the fringes develop. So, what happens fringes develop here and more outwards that you could see from this simulation; this also gives you an indication of doing an experiment right at the laboratory. Same thing you can do for photoelasticity, fringes develop and move outward and this also gives you an indication how you can go about in labeling fringes, which is a very complicated exercise; we will see later.

So, in this example we have also shown result from holography and if you want to go for displacement information; What is the technique that you will use? We have been seeing moiré.

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So, I would see the moiré contours, what I get for u displacement, as well as v displacement. How do I decide that, I have the grating direction and which it is recorded and you get these contours beautifully and we saw this is very difficult expressions and obviously, when I do it; it will be complicated like this and I can also change the load 1,

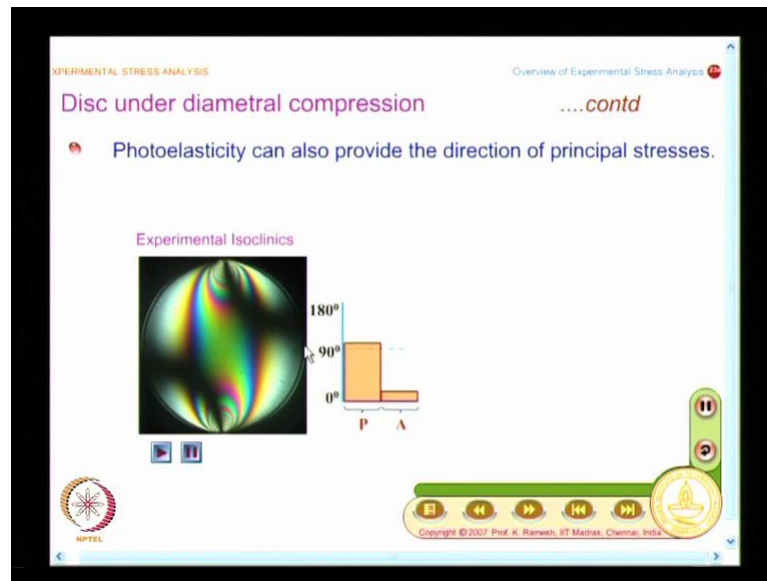
2 and gradually increase load. I see more and more fringes appear and probably this sketch you can make because you do not have many fringes, but it gives you the geometric shape of the fringes reasonably well, goes at a later point when you have occasion to see the experimental result, you could easily say I have seen this patterns earlier and they would be for a disc it is only u displacement.

Suppose, somebody gives you a photograph without the grating direction on an unknown situation, if they want you to interpret you will not be able to give an answer immediately, unless you know how the experiment is conducted, how the values are recorded, you will not be in a position to interpret the fringe patterns. Interpretation requires additional information, labeling of fringes is not at a simple task and for that we will have to know what are that various techniques available to label the fringes and some of the later examples I have not shown the fringe ordering deliberately so that you can do this as an exercise and label the fringes after we have learnt the course. So, when I increase the load, the more and more fringes appear and that is what I see in the case of a disc under diametral compression and this gives you u displacement.

Now similarly, we will also see the v displacement and this is what you have here, the grating direction is like this and as I increase the load, I go from this gradually go from this. You find this is the load application points and all the fringes come from this and what you also find you have a very broad fringe and as you increase the load, you have the fringes move and occupy different positions. So, it is possible for you to label them appropriately, if you have this knowledge what you find is this fringe as moved at very a high load, so if you have a number attached, the number also will move along with it and here again you can make a reasonable sketch for a intermediate load, to know the nature of the displacement field.

So, that gives you certain level of familiarity and you feel closer to an experimental technique and also get knowledge how to appreciate visual information and for this example, I also have another set of contours what you can get from photoelasticity.

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Photoelasticity can also give you principal stress direction and that is what you see here and we will see this closely. So, what I find here is I have experimental isoclinics and what you find here is, the whole image looks blur. Here black contour moves over it, that is what you seek, a black set of contours move over colored bands. Can you identify the color band; because, you had seen this earlier. Can you identify the color band? And, you can identify the color band. What the color band show?

Student: Isochromatics.

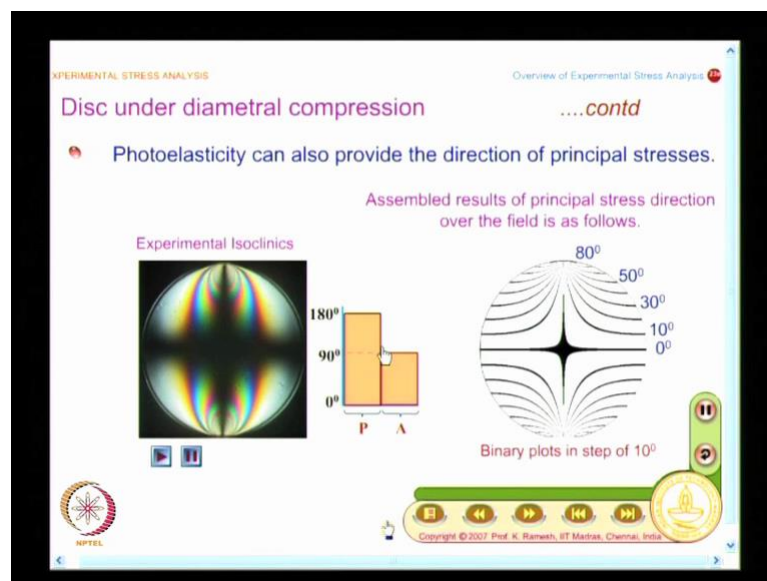
Yes, contours of?

Student: $\sigma_1 - \sigma_2$.

$\sigma_1 - \sigma_2$. So, what you find here is, in this optical arrangement I get two information; one information is $\sigma_1 - \sigma_2$; another information is the orientation of the principal as plane and what do you find if I have two information super imposed, the clarity is lost. So, you would always like to have independent information then processing, data collection, everything becomes much simpler.

And what do you see in this example, I have one set of contours that move and this you may not be able to understand at this stage; what it indicates is a direction of polarizer, analyzer, these are two optical elements. They are kept at mutually perpendicular degrees, for these angles these contours move. So, what you will have to do is from a data collection point of view, I have to set it at fixed angles and try to make a reasonable sketch of what this contours are and that is what to shown here.

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So, what you have here is, I have the set of contours which is a binary plot in steps of 10 degrees. This you make a sketch of it so I have to process this information and extract this in this form, I do not have an optical arrangement which would give me this set of contours directly. So, I have a 0 degree isoclinic, 10 degree isoclinic, 20 degree isoclinic, 30 degree isoclinic and so on and we call this as isoclinics; iso means constant, clinic means inclination; isoclinic means contours of constant inclination; constant inclination of what, constant inclination of principal stress direction.

So, from a photoelasticity experiment, it is possible for me to get $\sigma_1 - \sigma_2$ contours and you can also get by an appropriate optical arrangement, contours of principal stress directions and what I find is, here I have $\sigma_1 - \sigma_2$ contours super imposed over isoclinic contour and I do not see all isoclinic in one shot, I have to

scan the image and pick out this information, get this as a separate image of isoclinics. So, without getting into the experimental details, I have tried to project what is in store for you.

In some experimental technique, you get the information separately, in the of case moiré; I can get u displacement, I can get v displacement separately and getting them separately is better though I have to do one more experiment, getting them separately in an optical arrangement is far better from processing point of view than getting the image of σ_1 minus σ_2 super imposed on the isoclinic pattern.

Thank you.