

Experimental Stress Analysis – An Overview
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Lecture – 1.7
Visual Appreciation of Field Information – Part – 3

Let us continue the discussion on Overview of Experimental Stress Analysis. We have discussed in the class, the problems considered are Beam under four-point bending.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Typical Results for Various Problems

The problems considered are

- ★ Beam under four point bending
 - Closed form solution by Strength of Materials is possible
- ★ Cantiliver Beam
 - Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
 - Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
 - $w = \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$ obtainable from theory of elasticity
- ★ Spanner tightening a nut
 - Due to complex nature of the geometry only a numerical solution is possible

For all these cases relevant experimental contours (recorded or simulated) are shown to appreciate the nature of fringe contours.

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EXPERIMENTAL STRESS ANALYSIS

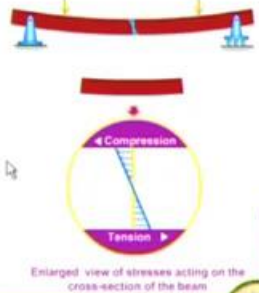
Overview of Experimental Stress Analysis 20

Typical results for various problems

Beam under pure bending – Analytical solution

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$
$$\sigma_x = -\frac{M_b y}{I_z}$$

Stress tensor

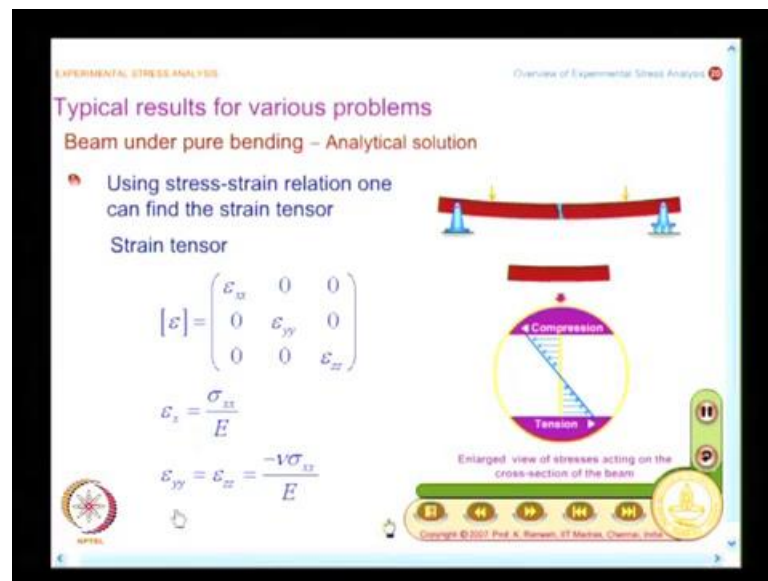
$$[\sigma] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$


Enlarged view of stresses acting on the cross-section of the beam

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Now, we come back to the stress field analytical approach. Now, you have got the stress field, you have also seen the stress tensor. With your knowledge of mechanics of solids it is possible for you to find out the strain tensor, it is not difficult at all. Can I request you to look at, brush your old memories and find out how do you find out the strain tensor, because you have the stress tensor is available from stress tensor you can easily find out the strain tensor. You have to be very careful while applying this step. What do you find here is, stress tensor you have only one component it is uniaxial state of stress.

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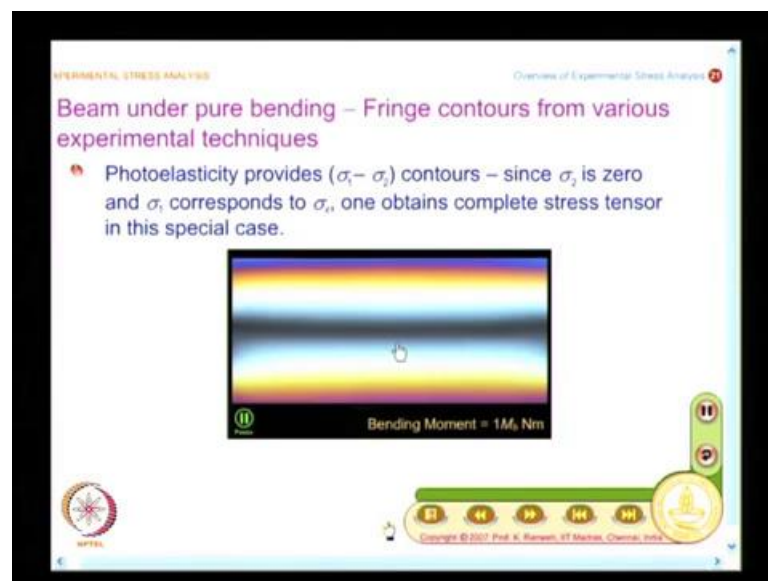


The moment I go to strain tensor, I find something different. I find strain tensor is no longer uniaxial but it is triaxial. Can you say what is the value of epsilon x x, epsilon y y, and epsilon z z. Epsilon x x there is no problem, you would be able to directly write from your understanding of even simple tension test and what many people may ignore is the effect of Poisson's ratio. They may ignore the presence of epsilon y y and epsilon z z. You have to be very careful about that there is a possibility that you may ignore the presence of these two strain components. So what do you have is, you have a stress tensor which gives you uniaxial state of stress. On the other hand, when I go to strain tensor I have a triaxial state of strain.

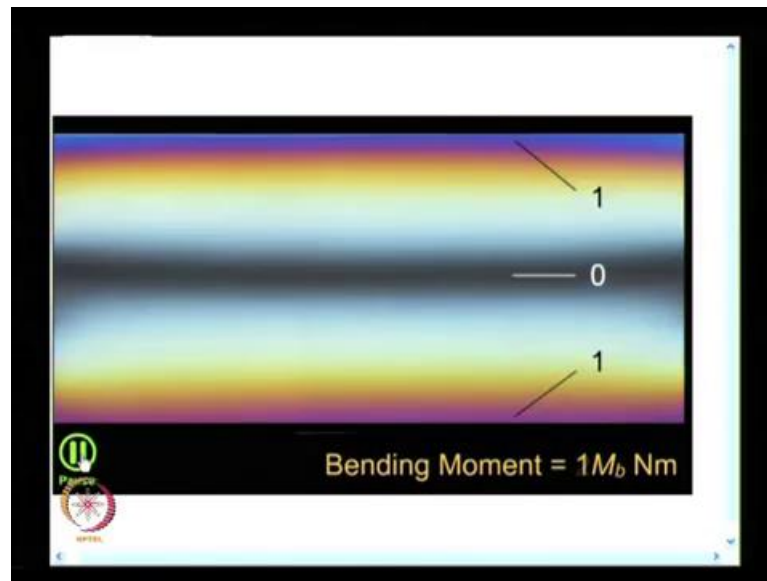
You may ask, suppose I make a mistake and I go and interpret experiments will the experiments understand this or not. I have always been saying experiment is truth, you may make approximations in your formulation to simplify your mathematics and in some cases if the values are reasonably small we may also ignore it, but the moment you come to experiments they always reveal the truth. What is the role of this strain components epsilon y y and epsilon z z, you have a Poisson ratio here and when you have the beam at the bending I would like to show you the model, if I take the model and bend it and what I have here is the beam bends, the beam bends here and this is one curvature you have and this is what you do it in your simple strength of materials approach.

I have said that in addition to this, you also have strain because of Poisson effect, and what you find is if you look at sideways, what I find is I have a bulge at the bottom and it is not bulging out in the top. Why this has happened is, in the way I have bend this fiber is subjected to tension and the inner fiber is subjected to compression and what happens because of the Poisson effect the bottom portion bulges out and top portion narrows down. And you also see one more interesting thing, you have additional curvature here because of this and this curvature is known as Anticlastic Curvature. None of this you might have heard in your first level, in strength of materials if people have not gone deeper into the analysis of beams. Only when you go deeper into the analysis of beams you will find that these concepts you have to look at it. What happens, the experiment looks at it, experiments reveals this.

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And that we will see very closely when you see the fringe pattern. We will see the fringe pattern we will take a second for it to come. And what you will see is when I magnify it you will see in a very settle fashion what you have on the top fiber.

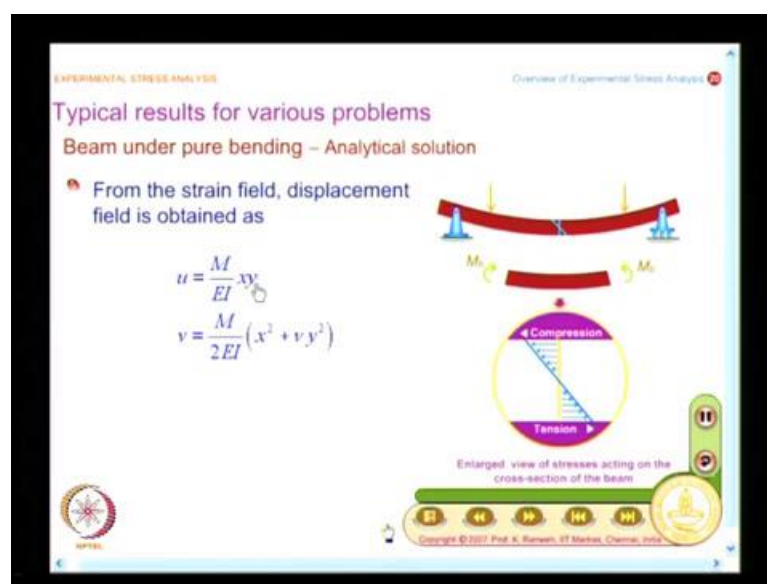
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In this particular example the top fiber is subjected to compression and bottom fiber is subjected to tension. You will have the fringe order slightly greater than what is there at the bottom. So, it is not perfect symmetry, so the experiment captures this you may call it as anticlastic curvature but experiment reveals this information. So, you will not have perfect symmetry there will be small deviation and you should not come back and say I have done the experiment, experiment is wrong it is not so. In your analytical evaluation of stresses, if you also bring in the effect of Poisson ratio effect then the stress patterns will be like this. So, experiment always gives you truth and that you should never forget it.

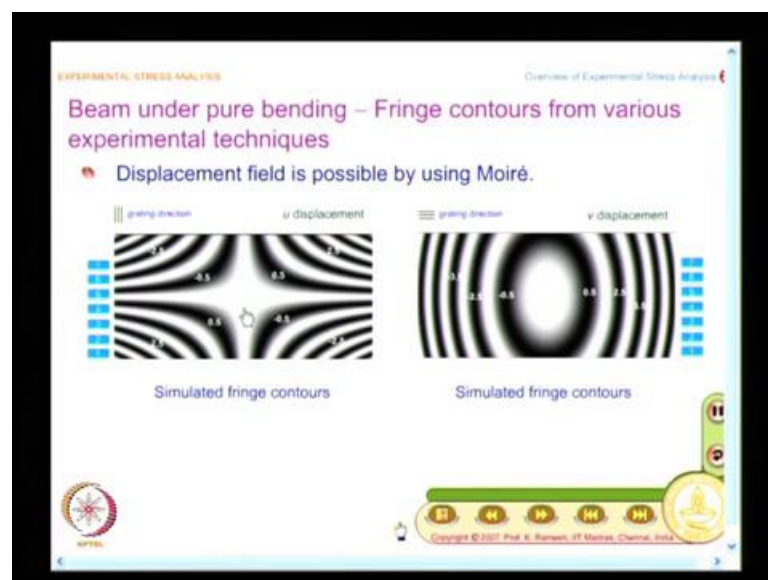
Now what we are going to do is, we will again go back to the slide and what we will do is we will we have seen the stress tensor, we have seen the display of strain tensor, now with that information using mechanics of solids that is using the equations of theory of elasticity, you could integrate the strain quantities and get you the displacement values. I do not think you would have done it in the first level course, because you are happy with the stresses, you are happy that in bending central code does not contribute a load shedding that is all you focus on that and you do some experiment some exercise on finding out beam deflection beyond that you do not go and study deeper.

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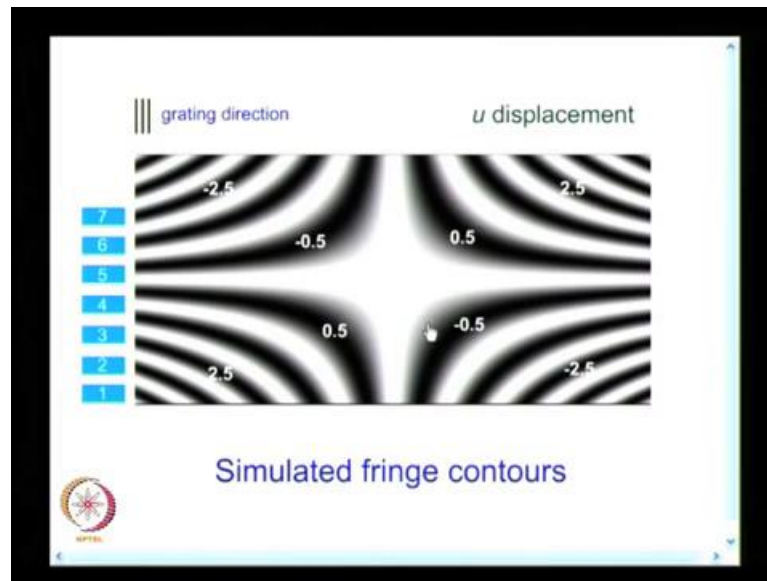
Now, when I look at this I can get the displacement field and displacement field is like this it is fairly simple, I have this u equal to M by EI into x y and this is nothing but an equation of a hyperbola. When you plot the contours, you will have the contours looking like hyperbola. You also have an expression for the v displacement which is given as M by $2EI$ in the x square plus u times y square. In your deflection calculation you may not have calculated u displacement at all, you would have only found out the v displacement, you would not have found out the u displacement you would have determine only the v displacement. Now what we will do is, we have seen the fringe pattern from photoelasticity, which gives you essentially contours of σ_1 minus σ_2 . Now, our interest is to see other contours.

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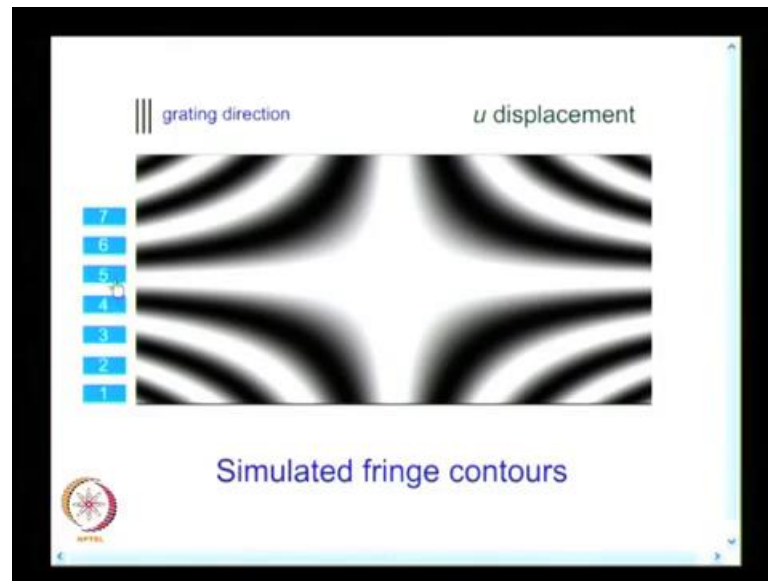
What I can do is, I can do a contour on I can look at geometric moiré and find out how the displacement field is. This is what the u displacement, and this is what the v displacement. We would see this little closely and what we will see is what you see here.

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I have the fringes labeled and you have a 0.5, 1.5, 2.5 like this and you have minus 0.5, minus 2.5. Once I go to displacement I have both positive and negative numbering of the fringes, this is first observation. The second observation is, the fringe pattern comes with an indication on what does the grating direction that has been used. When you go to moiré for me to reveal the displacement information, I have to use a grating and grating direction dictates what component of displacement I do get. I would like you to have a reasonable sketch of this. When you do this you will have an idea how the whole field pattern looks like. The many indirect learning you learnt one is, suppose I change load and you see this is how the fringes are developed.

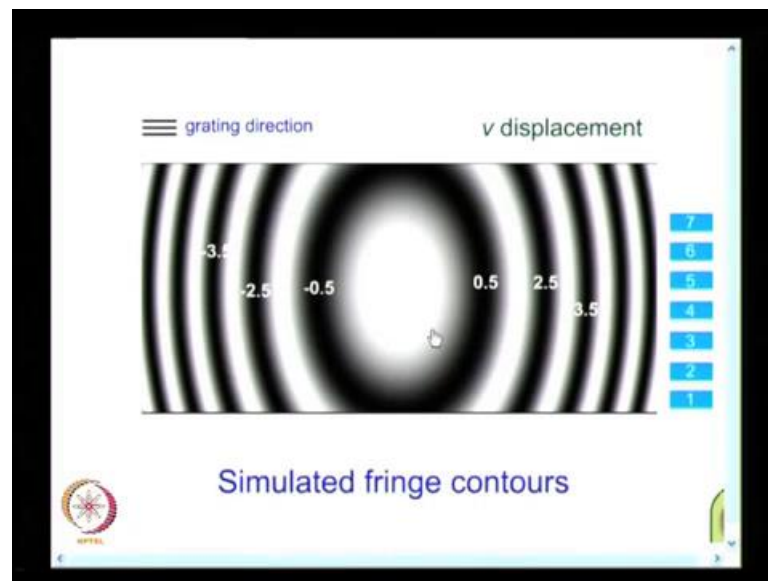
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As the load is increase more and more fringes are developed and this is very key information in most of the experimental technique to label the fringe orders as an auxiliary method to do that. Other one what you find is the fringe thickness is different. This also carry some information, this we will see later part of the course.

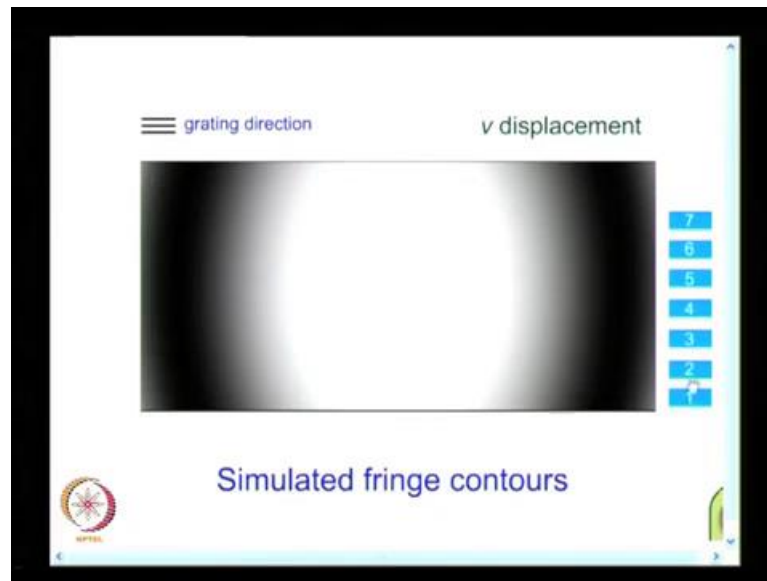
So, you have a familiarity when I want to look at a displacement field in the case of a beam under four point bending it would look like this. We will also have a look at the v displacement, have you been able to make a reasonable sketch of this.

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So what I will do is, I will see the v displacement and v displacement is like this. Here you find I have the grating direction horizontal, so by comparing these two figures you can easily understand what I say as displacement is a component perpendicular to the grating direction. And what I want you to note down is, when I have a grating direction in one way, I need to have one optical arrangement to record it. When I have the grating direction in another way, I need another optical arrangement to do it. This is what I have said earlier experiments will reveal information of a particular kind. They may have the capacity to reveal more than one information but they may require two different measurements scenario. When I have two different measurement scenarios what I need to do is, I need to have there are also methods where they combine and try to get more than one information in one go. When we look at moiré, we will find out how you can get both u and v displacement simultaneously and what you should keep in mind is, if I get two information together it is always a nuisance.

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And here again we can see as a function of load increase how the contours appear. This is very important and this will give you a sort of understanding, you also give a feeling of doing the experiment right in the class. When I increase the load I see fringes are formed and here again the labeling I have positive and negative quantities, if you have noticed very carefully in photoelasticity, we have not label negative fringe orders we have labeled all of them as positive. This is an important point that you have to note with. Suppose, I want to find out the strain what do I do? Now I have the displacement, I differentiate the displacement it is possible for me to find out the strain.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

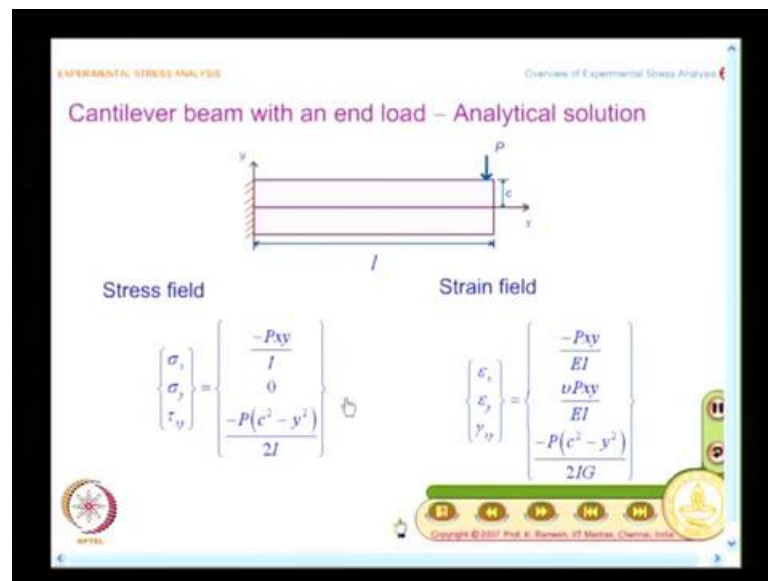
Beam under pure bending – Fringe contours from various experimental techniques

- Strain field is obtainable from the displacement fields – however differentiation in experimentation is prone to error.
- Use of Moiré interferometry is suggested to get the strain field.
- Strain at discrete points could be evaluated by strain gauges and along lines of interest by strip gauges.

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But I may not want to do that if I have my record from geometric moiré, because geometric moiré gives with less accuracy and I have to go to moiré interferometry and then to the differentiation and get the strain field. Strain at discrete points could also be evaluated by strain gauges. Suppose, I want for a series of points then I have what is known as a strip gauge, in one banking, several strain gauges are available at fix lengths given by the manufacture, this is done by at the time of manufacturer itself one grade containing several elements of strain gauge. So I can get the strain from strain gauges from point to point or along a line of interest. We have seen reasonably various contours that is possible from experiment for the problem of beam under four point bending.

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Now, let us move on to cantilever beam. What is the difference? Because of the load here, you have bending moment varying across the length of the beam that is what is seen in the first term here, you do not have a bending moment but the bending moment is a function of x that is what you find here. And in addition you also have a shear stress, and for convenience, the stress tensor is simply given as stress field, where I give only the component σ_x , σ_y and τ_{xy} . A fairly straight forward this you have solution from strength of material. What you may not have done is you may not have calculated the strains and the strain fields looks like this I have minus Pxy divide by EI . Epsilon y is ν times Pxy divided by EI . Gamma xy is minus P times c^2 minus y^2 square divided by $2IG$.

And obviously, here the stress field is little more complex, strain field is little more complex, and you will also anticipate the displacement is going to be complex than what you have seen in the cases of a beam. We will have a look at it. The point to note here is, when I do an analytical method I am able to get the stress field, I am able to get the strain field, I am also able to get the displacement field and that is what you see here.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

Cantilever beam with an end load – Analytical solution

Displacement field

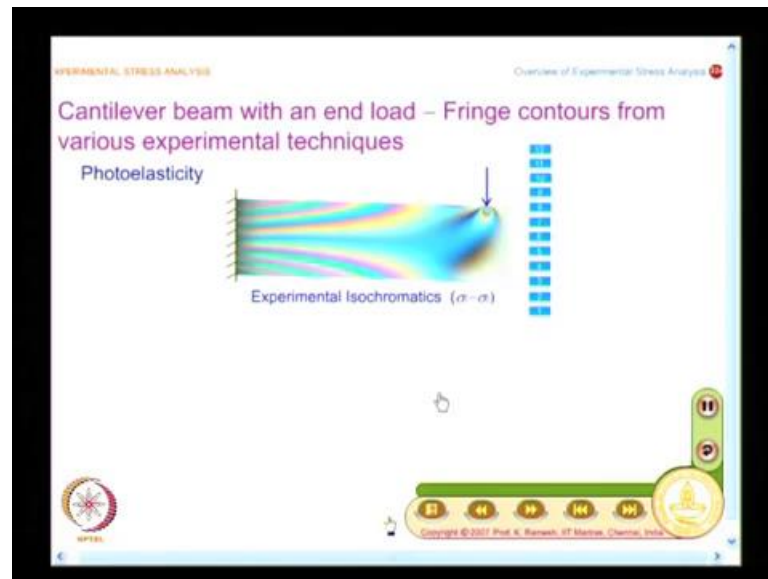
$$u = -\frac{Px^2y}{2EI} - \frac{\nu Py^3}{6EI} + \frac{Py^3}{6IG} + \left(\frac{Pl^2}{2EI} - \frac{Pc^2}{2IG} \right) y$$
$$v = \frac{\nu Pxy^3}{2EI} + \frac{Px^3}{6EI} - \frac{Pl^3x}{2EI} + \frac{Pl^3}{3EI}$$

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Displacement field is slightly more involved, and you would have worried in your basic course in strength of materials only on the v displacement. And when x is 0 you get Pl^3 cube by $3EI$ and you do not find out what happens for the v displacement as a function of y . What you need to keep in mind is you have simply taken a two dimensional beam as simply a line and then analyzed it, you have not consider this as a beam and then analyzed it. When you considered it has a beam you will also have to have a variation over the depth y , and that is why you have the u expression if you look at, it is minus Px^2y divided by $2EI$ minus ν times Py^3 divide by $6EI$ plus Py^3 divided by $6IG$ plus within brackets Pl^2 square by $2EI$ minus Pc^2 square by $2IG$ multiplied by y .

What is the interesting thing about this expression, when y is 0, u is 0. You do not find out u displacement at all in strength of materials mainly because, you are considering a two dimensional problem as a one dimensional problem. So when you come to theory of elasticity you have made an improvement. And obviously, now we will go and look at what the experimental methods give as contours.

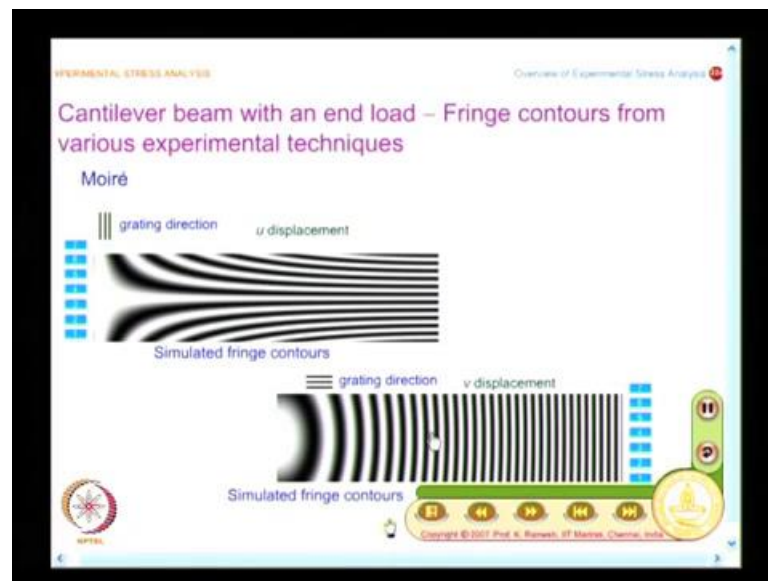
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So, first thing what we have always been doing is photoelasticity and you all know that it gives you contours of $\sigma_1 - \sigma_2$ and these contours you would not be able to calculate right at the class you need, it is better that you have a computer software to evaluate it and also have a post processing where we can go and plot it. As before, I would like you to have a reasonable sketch of these fringe patterns and what you find here is these fringe patterns are definitely different from what you have seen in beam under four point bending. So what you find is the stress field is different which your contours directly bring out, you have the affect of shear which is embedded in this stress field.

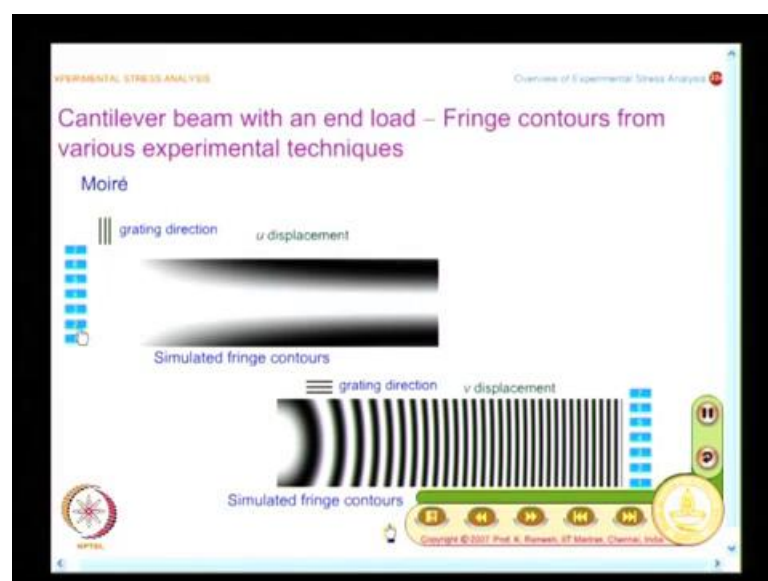
Another important point to note is, on the center of the beam you have a light shade of blue, it is not black like what you have in your four point bending. This is the load application point where you have fringes concentrated, and this is a (Refer Time: 17:42) and this is how you get Experimental Isochromatics. So, experimental isochromatics are different from what you have seen in the case of beam under four point bending, and naturally the displacement contours is going to be still more complicated.

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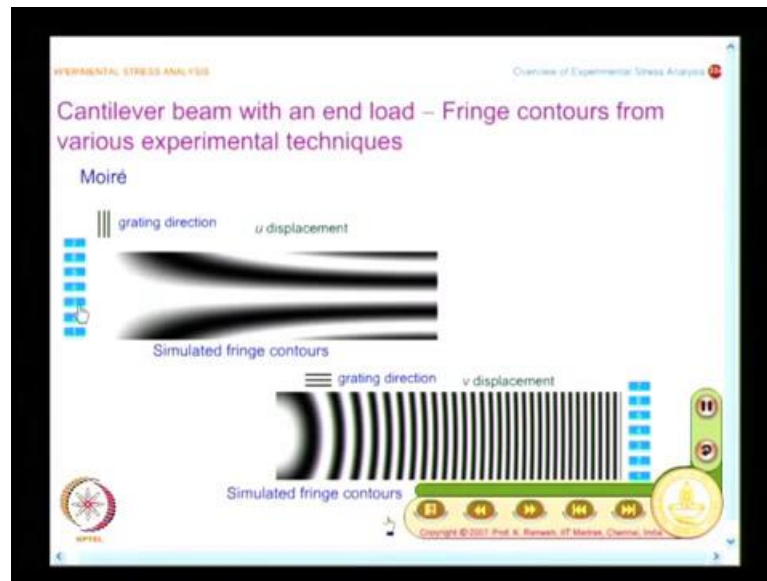


So you have u displacement, you have v displacement. What you have here is, I again have a grating direction, and knowing this grating direction it is possible for you to say it is u displacement, knowing this grating direction it is possible for you to say this is v displacement.

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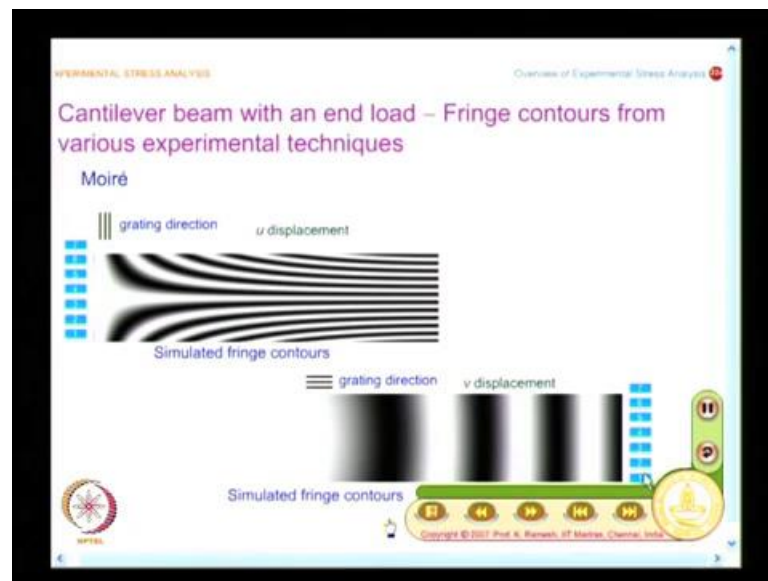


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And you also have a feel of doing the experiment by seeing what happens as a function of load. I gradually increase the load and more and more fringes get formed. You could make a simple sketch of it, tried to get only the skeleton you do not have to draw the band, you get the geometric shape and a few fringes, it will give you an idea how do I get the fringes in the case of a cantilever beam subjected to a end load and I can increase a load more and more fringes get formed. Then I can also look at the v displacement. This also I can look at as a function of applying load.

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And you will see as the load is increased, you notice this zone more and more fringes develop and they become denser in appearance. I increase a load 2 I increase a load there, increase to 4, 5, 6 and 7. So what you find here is, it become almost like parallel lines more fringes are seen in this zone, and when you draw the sketch it is not necessary that you draw all these lines, you just draw a few lines to indicative that you have fringes like this and slightly changes curvature as it goes close to the clamp end. These are simulated fringe contours.

As I said, some of you who have an exposure to computer graphics can make an attempt you have those equations, you could go and make a contour plot and bring it to me and you will have a doubt how to get the fringe width. Let me keep that as a secret for the moment and we will see how to get this band also when you do the plotting.

In this class, we have looked at the strain fields of four-point bending, and the effects of Poisson's ratio which is settle but quite important for data interpretation. Then we have looked at how the shear stress influences the color of the central axis in the cantilever beam. We have also looked at the displacement fields for a cantilever beam and the corresponding fringe patterns from moiré.