

**Experimental Stress Analysis – An Overview**  
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**Lecture – 1.6**  
**Visual Appreciation of Field Information – Part – 2**

Let us continue the discussion on overview of experimental stress analysis. We have seen, photoelasticity gives the information of  $\sigma_1$  minus  $\sigma_2$  and the orientation or the principles as direction  $\theta$ . When you go to Moiré you get displacements, when you go to holography you get the displacement vector and so on. So, essentially what we saw was, each of the experimental techniques give you a particular kind of information based on the physics that we exploit in getting the information. The other aspect is in first level course in strength of materials, you try to understand what is stress? And you have to understand stress is a tensor. When you say stress is a tensor, we would like to know state of stress at a point of interest. At a point of interest, you are focusing on what happens on all the infinite planes passing through the point of interest. This is what you get in a study of Moiré circle.

So, you understand stress is the tensor although, many times you get only the stress component there is a danger that you could construe stress as a scalar. The moment you come to any of the optical techniques, you have the advantage of getting the information on the entire field. So, you get the whole field information and you get this whole field information in the form of construes. So, when we go further what we are going to do is how to do the experiment what physics is employed and how to interpret we will study. To start with it would be desirable to see these construes for a class of problems from known situations to unknown situations that are what we are going to see.

And we have discussed in the last class, the problems considered are Beam under four point bending, Cantiliver Beam, Disc under diametral compression, Clamped circular disc with a central load and finally, Spanner tightening a nut.

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**EXPERIMENTAL STRESS ANALYSIS** Courses of Experimental Stress Analysis

### Typical Results for Various Problems

The problems considered are

- ★ **Beam under four point bending**
  - Closed form solution by Strength of Materials is possible
- ★ **Cantiliver Beam**
  - Engineering analysis possible by Strength of Materials.
- ★ **Disc under diametral compression**
  - Only Theory of Elasticity can provide closed form solution.
- ★ **Clamped circular disc with a central load**
  - $w = \frac{\Delta u}{\Delta x} \frac{\partial^2 u}{\partial x^2} + \frac{\Delta u}{\Delta y} \frac{\partial^2 u}{\partial y^2}$  obtainable from theory of elasticity
- ★ **Spanner tightening a nut**
  - Due to complex nature of the geometry only a numerical solution is possible

In all these cases relevant experimental results (recorded or simulated) are shown to appreciate the nature of fringe contours.

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So, this is based on increase in degree of complexity. Beam under four point bend easy to solve from strength of materials approach, you have a closed form expression. The moment you come to Cantiliver Beam you have shear, but you do a engineering analysis by strength of materials and when you come to disc you cannot do by strength of materials, but theory of elasticity can provide closed form solution. Clamped circular disc you could find out the displacement, slope and curvature from a study of theory of elasticity theory of plates. Finally, spanner tightening a nut we have seen earlier that it is a down to the earth problem, surprisingly you cannot solve it by analytical methods, you have to solve it either by numerical or by experimental methods alone.

And now, we take up the problem of Beam under four point bending and what you have here is, I have a beam and this is shown as x axis, this is y axis and you have taken a central portion of the beam.

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EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

Typical results for various problems

Beam under pure bending – Analytical solution

$$\frac{M_b}{I_z} = -\frac{\sigma_x}{y} = \frac{E}{\rho}$$
$$\sigma_x = -\frac{M_b y}{I_z}$$

Stress tensor

$$[\sigma] = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Entarged view of stresses acting on the cross-section of the beam

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Which is subjected to bending on  $M_b$  and in this you have the stress variation over the depth as like this, it shows the variation is linear and the central core does not contribute to load sharing. You have a famous flexure formula here, and which is in slightly different form than what you might have seen in some other books. Many times people simply say  $M$  by  $I$  equal to  $F$  by  $Y$  equal to  $E$  by  $R$ . It is written in a different fashion; know the difference it is one and the same. This is much more precise mathematically we show that this is a bending moment, indicated by  $M$  subscript  $B$  and the moment of inertia is indicated as  $I_z$  depending on the axis that is chosen for the problems.

And when, you come to a second term, you also have a sign attaches to  $\sigma_x$ . This is minus  $\sigma_x$  divided by  $y$ . Why this is so the sign conventional opted is on a positive surface, anti clock wise moment is taken as positive. For a loading like this, the top fiber of the beam is subjected to compression to indicate that you have a negative sign. Once you have this it is easy to find out what is the value of stress component. You can directly write it its child play please write down that expression and this turns out be minus  $M_b y$  by  $I_z$  and what you get here? You get as the function of  $y$   $\sigma_x$  changes linearly and you also find the  $\sigma_x$  is not a function of  $x$ .

So, it is constant over the entire length of the beam which is taken slightly away from the

points of loading and this is what you get in a first level course. Expression for  $\sigma_x$  when you look at its only a component, its only a component that you are looking at and what you have to understand is, you have to understand, that this is just not a number, you have to understand this as a tensor. So, when you say tensor, I have to plug it in a 3 by 3 matrix appropriately this component and fill the other components. In this particular example what you have is, you have the stress tensor only the component  $\sigma_x$  is exist all other components are 0.

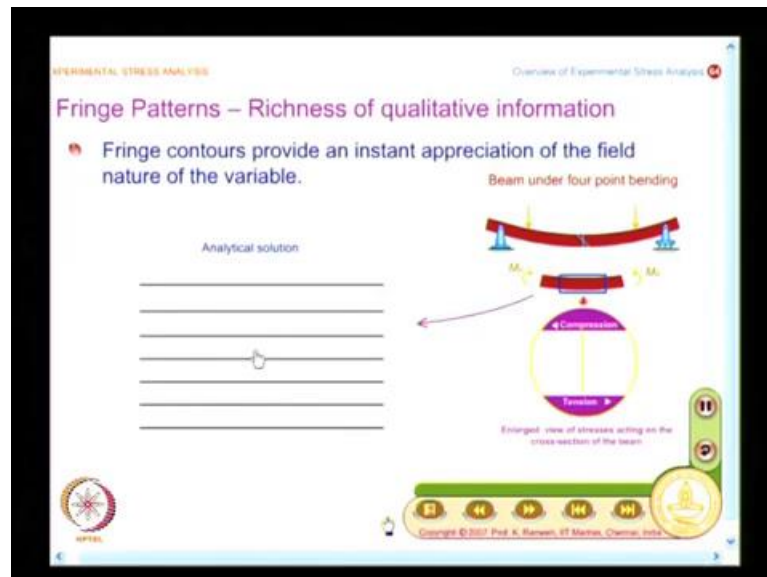
And what you have to important is the 0s are very important, and you know if you look at the life history of Ramanujam, when he was a young student he was sitting in a class of mathematics and he was observing zeros. When you put the 0 before the number the value of the number decreases, when you put the 0 after the number the value of the number increases. This is a very, very important and pertinent observation, zeros play a very important role, and if we look at the history of science, Indians are credited with giving the concept of 0 to the entire world. So, though in this case you have zeros present, you have to respect those zeros. They serve a very important purpose and when I have a failure analysis then, those zeros also play a very important role the failure planes could be different depending on the material on hand.

And once you have this I have the analytical solution and it is possible for you to plot the value of the type of contours that you could get. We have seen earlier that we would like to we get only  $\sigma_1$  minus  $\sigma_2$  in the case of photoelasticity. Since you have the stress tensor you know how to find out the principle stresses. Then you can find out what is  $\sigma_1$  minus  $\sigma_2$  and you can comfortably plot it. When you say contour what is that you mean along the contour the value or the variable is same.

So, you plot discrete contours and I would like you to make some effort because the problem is very simple, and find out how you can plot this contour, what way it will look like. Let me give you few minutes of your time, take few minutes of your time and then try to see how you can plot this contour. This is fairly straight forward because, if you know how to plot contours, then you will know how to interpret the experimental results. Because you need to have a visual appreciation of the variable over the domain and if you have done that, what I will do is; I will show you, some of you must have done it. I

see some of you must have done it already. And if you look at I have in this zone the contours are essentially horizontal lines. In fact, if you go and look at many of the engineering problems you get very interesting contours.

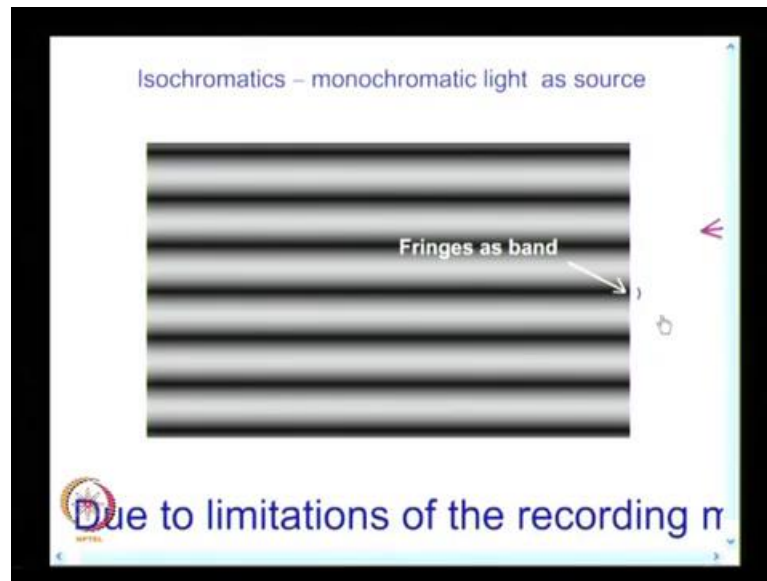
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In some cases straight lines some cases circles, concentric circles, circles touching edge. So, they give beautiful patterns. In fact, Durley has written an article on art and science and where he has shown, how the fringe patterns could be related to piece of art. So, fringe pattern gives the lot of information. So, if you are an artist, you look at the artistic value of it. If you are a stress analyst, you look at the value pertinent to your requirement.

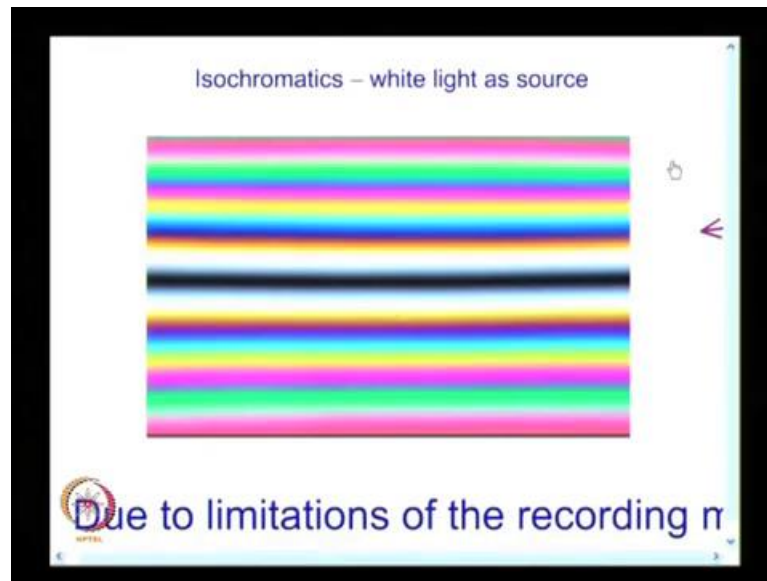
And now what I do is; let me look at an experiment what does it give. I will also magnify this picture and you have this has analytical solution. Now, what I do is I take a model put it in optics and find out what the contours look like.

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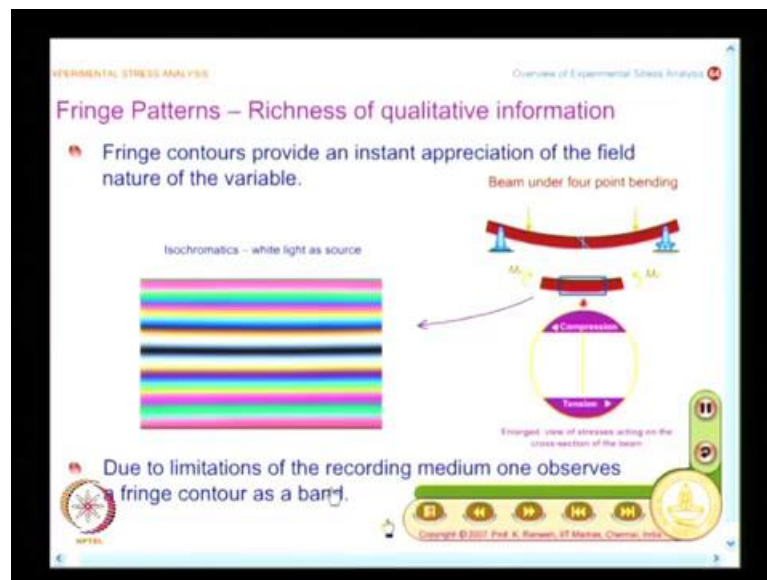
I do not get straight lines as thin as possible. I get only a band, I get essentially straight bands. So, this result matches with what you have seen in an analytical solution with the difference. The difference is, I see the fringes as a band rather than a single line and I see this as black and white contours mainly because I have used monochromatic light source for elimination. In most of the optical techniques it is desirable to use monochromatic light source because, you have single wave length and data interpretation is far simpler. On the other hand, if I use wide light which is the uniqueness of photoelasticity I get the same fringes appearing as beautiful play of colours. And the advantage with colour is, using the colour it is possible to label and order the fringes and why do you see the contours as bands.

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You see contours as band mainly because of the deficiency in the recording medium.

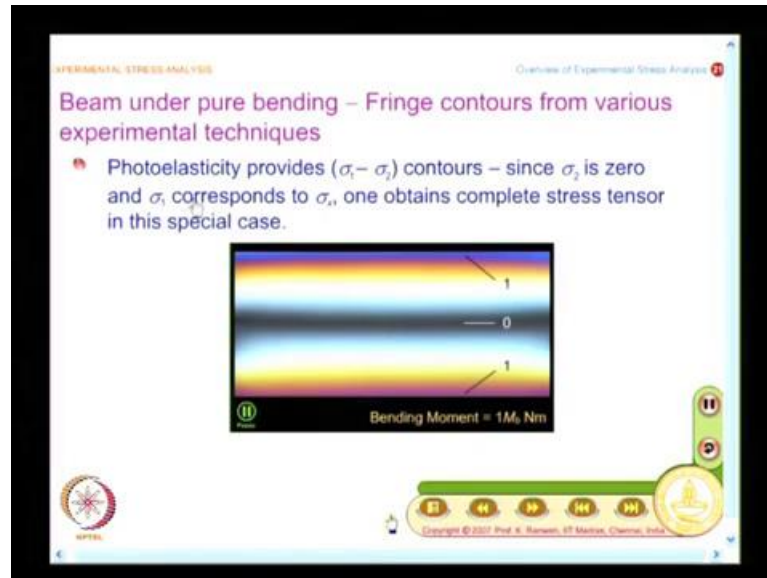
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You observe this as the band because of the deficiency in the recording medium. If I use the very fine digital camera, you would be able to find out the variation in the gray scale and pick out minimum intensity points as fringe scales interpret.

And now what we will do is it is also desirable that we go and look at how to number these fringes and what we do is.

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We now go back and see what photoelasticity gives; it gives you contours of sigma 1 minus sigma 2. In the bottom half of the beam you have sigma 1 corresponds to sigma x and sigma 2 is 0 and sigma 1 and sigma 2 contours appear horizontal and you see some deviation from straight line in this zone.



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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

### Beam under pure bending – Fringe contours from various experimental techniques

- Photoelasticity provides  $(\sigma_1 - \sigma_2)$  contours – since  $\sigma_2$  is zero and  $\sigma_1$  corresponds to  $\sigma_x$ , one obtains complete stress tensor in this special case.

Bending Moment =  $2M_0$  Nm

The slide displays a central image of a beam under pure bending with fringe contours. The contours are labeled with numbers 1 and 2 on both the top and bottom surfaces. The beam is shown in a perspective view, with the top surface being concave and the bottom surface being convex. The fringe pattern consists of alternating dark and light bands, with a central dark band (fringe order 0) and two light bands (fringe order 1) on each side. The labels 1 and 2 are placed on the light bands. The text 'Bending Moment = 2M₀ Nm' is displayed below the image. The slide also includes a navigation bar at the bottom with various icons and a copyright notice: 'Copyright © 2017 Prof. K. Ramani, IIT Madras, Chennai, India'.

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EXPERIMENTAL STRESS ANALYSIS Overview of Experimental Stress Analysis

### Beam under pure bending – Fringe contours from various experimental techniques

- Photoelasticity provides  $(\sigma_1 - \sigma_2)$  contours – since  $\sigma_2$  is zero and  $\sigma_1$  corresponds to  $\sigma_x$ , one obtains complete stress tensor in this special case.

Bending Moment =  $3M_0$  Nm

The slide displays a central image of a beam under pure bending with fringe contours. The contours are labeled with numbers 1, 2, and 3 on both the top and bottom surfaces. The beam is shown in a perspective view, with the top surface being concave and the bottom surface being convex. The fringe pattern consists of alternating dark and light bands, with a central dark band (fringe order 0) and two light bands (fringe order 1) on each side. The labels 1, 2, and 3 are placed on the light bands. The text 'Bending Moment = 3M₀ Nm' is displayed below the image. The slide also includes a navigation bar at the bottom with various icons and a copyright notice: 'Copyright © 2017 Prof. K. Ramani, IIT Madras, Chennai, India'.

Because, this is closer to the load application point in the experiment and what you see here is, I have increase the load in steps of 1 and you find that this is fringe order 0, fringe order 1, fringe order 1 at the bottom and when I double the load, I see the fringes doubled. So, it is a function of load and I can also label them and when I triple the load the fringes are increasing from 0 to 3 on either side.

What you can do is, by looking at the colour it is possible for you to identify the number associated the fringe in photoelasticity reasonably well with  $I$ . And with developments in digital image processing, you have techniques like 3 fringe photoelasticity as been developed where, you could pick out this RGB values, very comfortably and then label these numbers and one of the challenging task in any optical technique is. How do I label these number, this is not something trivial and this is where many of the automated techniques people want to have, where they would like to minimize the human intervention.

So, far we have looked at this stress fields in the Beams under four point bending. We have also seen the corresponding experimental fringe contours from photoelasticity. We have also looked at the basics of how the fringes are ordered.