

**Experimental Stress Analysis - An Overview**  
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**Lecture – 2.3**  
**Completeness of a Numerical Solution**

Let us continue our discussion on Overview of Experimental Stress Analysis. What we have primarily focused in the previous class was for typical problems for which you know the solution. We have looked at the kind of patterns you could get from some of the experimental techniques. The problems considered were Beam under four-point bending, Cantiliver Beam, Disc under diametral compression and also Clamped circular disc with a central load.

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The slide is titled "Typical Results for Various Problems" and lists five mechanical problems with their corresponding solution methods:

- ★ Beam under four point bending
  - Closed form solution by Strength of Materials is possible
- ★ Cantiliver Beam
  - Engineering analysis possible by Strength of Materials.
- ★ Disc under diametral compression
  - Only Theory of Elasticity can provide closed form solution.
- ★ Clamped circular disc with a central load
  - $w, \frac{\partial w}{\partial r}, \frac{\partial^2 w}{\partial r^2}, \frac{\partial w}{\partial \theta}, \frac{\partial^2 w}{\partial \theta^2}$  obtainable from theory of elasticity
- ★ Spanner tightening a nut
  - Due to complex nature of the geometry only a numerical solution is possible

At the bottom left, a note states: "In these cases relevant experimental results (recorded or simulated) are shown to appreciate the nature of fringe contours." At the bottom right, there is a copyright notice: "Copyright © 2007 Prof. K. Ramesh, IIT Madras, Chennai, India".

In all these cases, you have analytical solution possible for the stress field. So you have got a close form for the stress field, you have got the strain field and also the displacement field. What we did was, we did a sample of experimental methods. Some of them were directly from experimental results, some of them were simulated result to give a feel of how the whole field information looks like. Now, your eyes get tune to how to interpret whole field information to an extent possible.

And finally, what we will do is we will go to the problem of spanner tightening a nut. And as I had told you earlier due to complex nature of the geometry only a numerical solution is possible for this problem. That is what we are going to see and what I am going to look at?

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The slide is titled "Spanner tightening a nut" and is part of an "Overview of Experimental Stress Analysis" presentation. It contains a list of five points discussing the complexity of the problem and the need for a numerical solution. An image of a spanner is shown on the right. The slide includes NPTEL logos and a copyright notice for Prof. K. Ramesh, IIT Madras, Chennai, India.

EXPERIMENTAL STRESS ANALYSIS

Overview of Experimental Stress Analysis

### Spanner tightening a nut

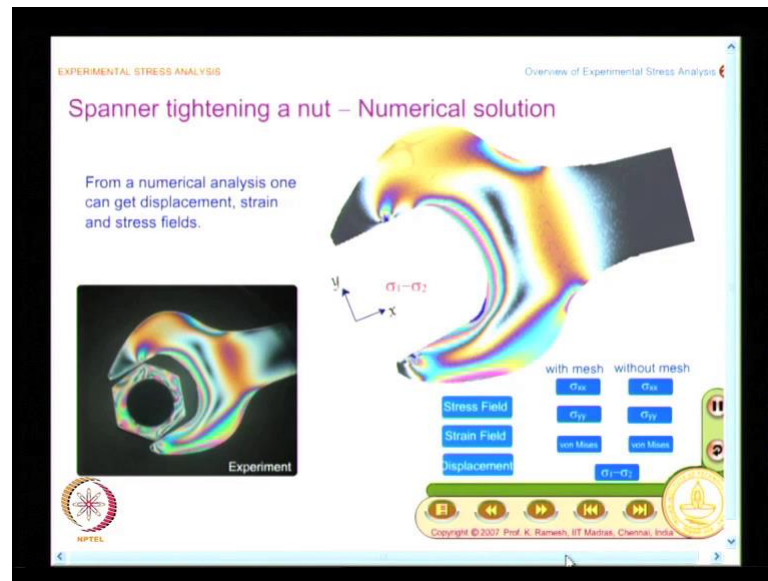
- Consider one of the day to day application of the problem of a spanner tightening a nut.
- Let us consider the evaluation of the stress field in the spanner.
- You will be surprised to know that the problem can't be solved by Strength of Materials or even by Theory of Elasticity.
- This is because the shape of the spanner is quite complicated.
- Only a numerical solution is possible.

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We have already looked at day to day application of the problem and surprisingly you do not have solution from Strength of Materials or even by Theory of Elasticity. This is primarily because, the shape of the spanner is complicated and you cannot define the outer boundary in a convenient fashion for you to do a Theory of Elasticity solution. In this case, only a numerical solution is possible. I do not have a close form expression, so I have to solve this problem numerically. Let us see how the numerical solution is?

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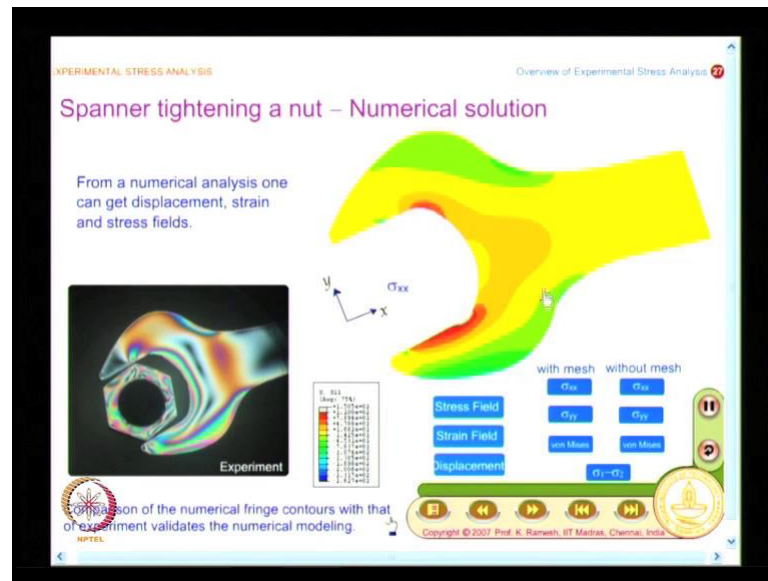
You have very rich set of results that you get from a numerical solution. In this case, the numerical method adopted is finite element method, and what you have done is you have meshed the spanner. What you find here is, this is the experimental fringe pattern, and this is the stimulated final (Refer Time: 03:13) result of sigma 1 minus sigma 2 contours. Here, I want to point out a few things; when I do a numerical analysis I can get the stress field, I can get the strain field; I can also get the displacements. If you go to commercial packages you would be able to plot specific contours. I have a sigma x contour and the software chooses its own colors, and then gives you an indication the red means a maximum and shade of blue what you have here it goes to less value of the stress information. You could see this without mesh; you could also see this with mesh.

For this you know you do not have to take the trouble of sketching it, because it is too complicated for you to sketch. The idea is to visualize what way you have the information available from a numerical solution. I have the mesh here and you have a nicely done mesh. These are all quadrilateral elements and I have the sigma x stress contour. Similarly, I can also get the sigma y stress contour. And I can also get Von Mises Stress Contour. If you look at von mises, in certain aspects it captures certain geometry feature of the photoelastic fringe, but the colors are totally different because the color is dictated by the standard finite element software package.

What was done was we have developed a in house software, what it will do is it will evaluate  $\sigma_1$  minus  $\sigma_2$  and also mimic the colors that you get in an experiment and this is plotted. An approach like this helps you to quickly come to understanding that you have very good comparison between what you observe in the experiment and what you observe in the numerical method. If you look at very closely I have this as a stress concentration region and what you have got and what you see in the screen, you know my students have taken little time to apply the boundary condition appropriately until the experimental fringe pattern matches closely with the numerically simulated results.

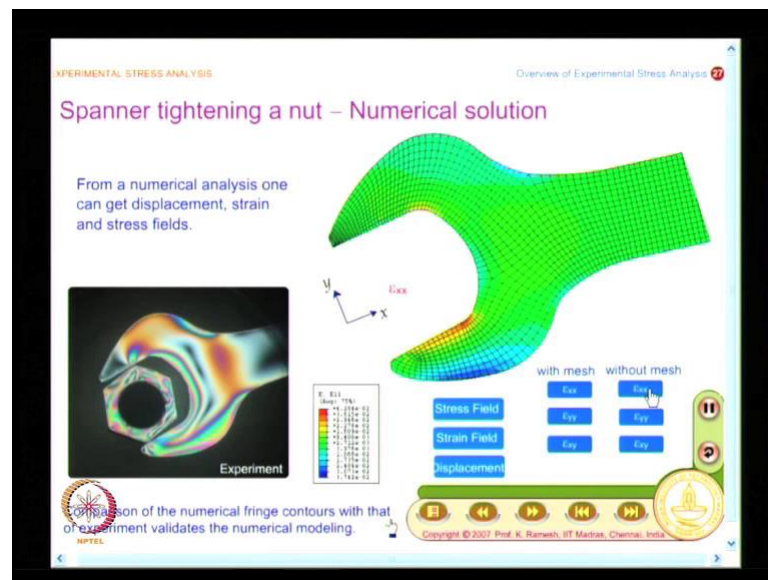
So, both the choice of elements, discretization and also the boundary conditions are improved until you get a close match between experiment and numerical solution. So you have this, what is the difference here? Here, when I have to find out I can get  $\sigma_x$  but I can do this only by interpolation and based on finite element formulation. I cannot go to the location of the coordinates plug simply  $x$  comma  $y$  and get these values directly. So that advantage you had in the case of analytical solution. Analytical solution the main advantage is, if you have a possibility to solve it analytically there is nothing equal to it because the amount of computational effort required is very small I simply plug in  $x, y$ , I get the value which I want. The movement I come to numerical techniques the greatest advantage is the shape of the geometry of the problem on hand does not post any restriction, but you have to do a lot of computational efforts. But you get whole field information.

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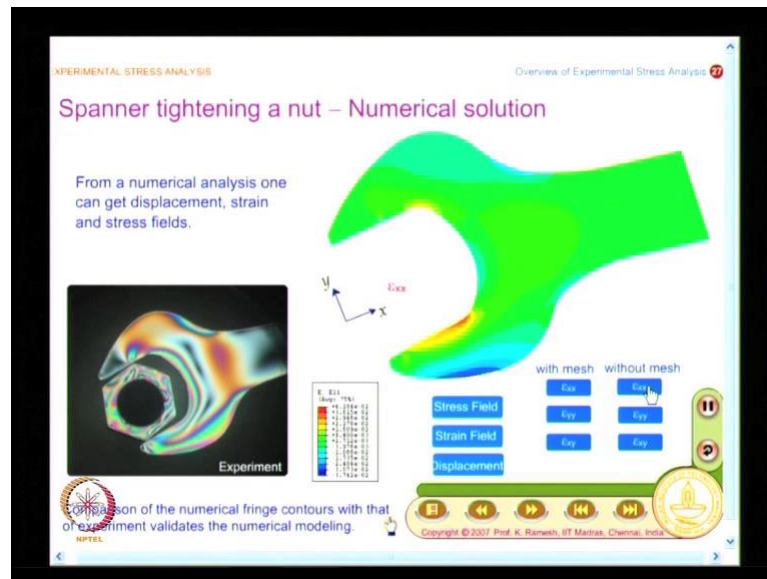
So I can also get the strain field and till now you are not seen strain field as a plot.

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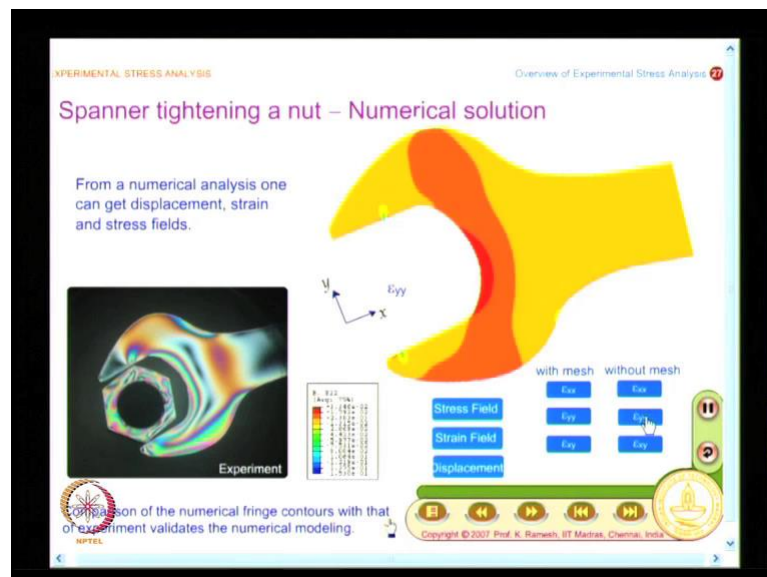
And this is with mesh and you can also see without mesh.

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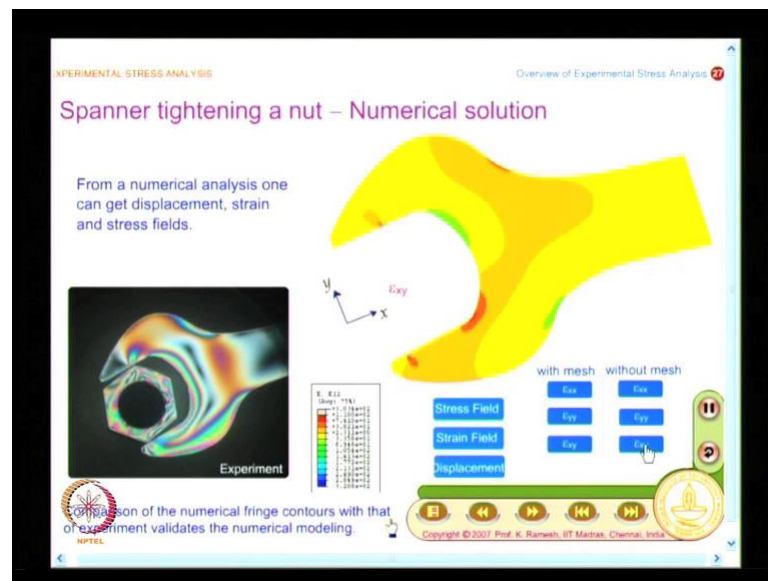
This is epsilon x x.

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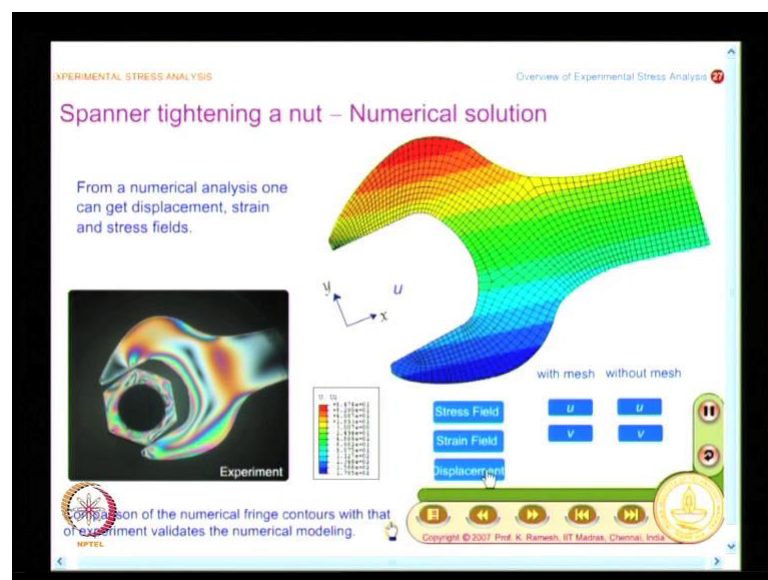
Similarly, I can get epsilon y y. I can also get shear strain epsilon x y here.

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And I can also go and see the displacement field and this is what I get here.

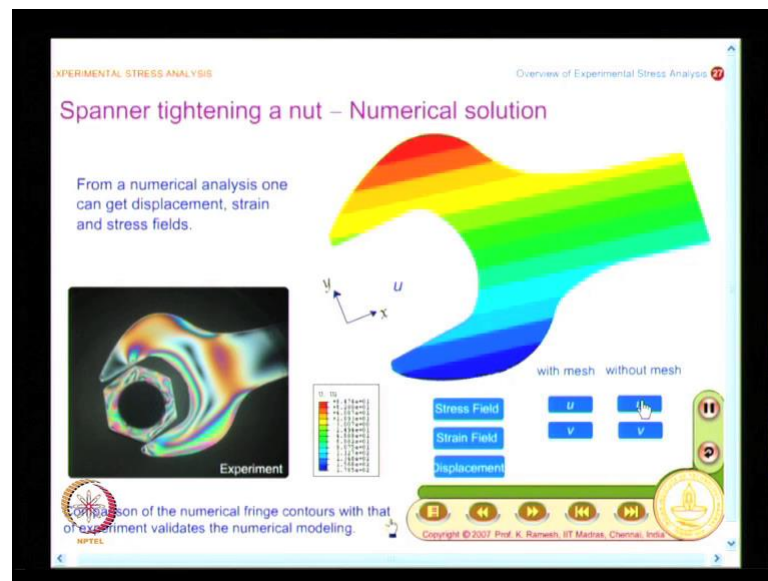
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I have the u displacement.

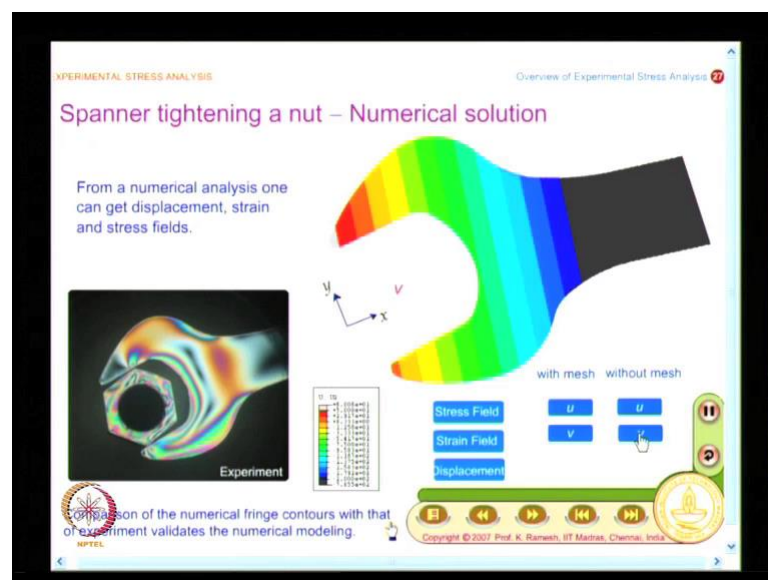


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I also have the v displacement.

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So what I find here is, when I have a numerical approach I could get all the 15 quantities comfortably. But a very important aspect is I must match what I have in the experiment very closely by choosing the boundary conditions correctly. Once I have done this then I



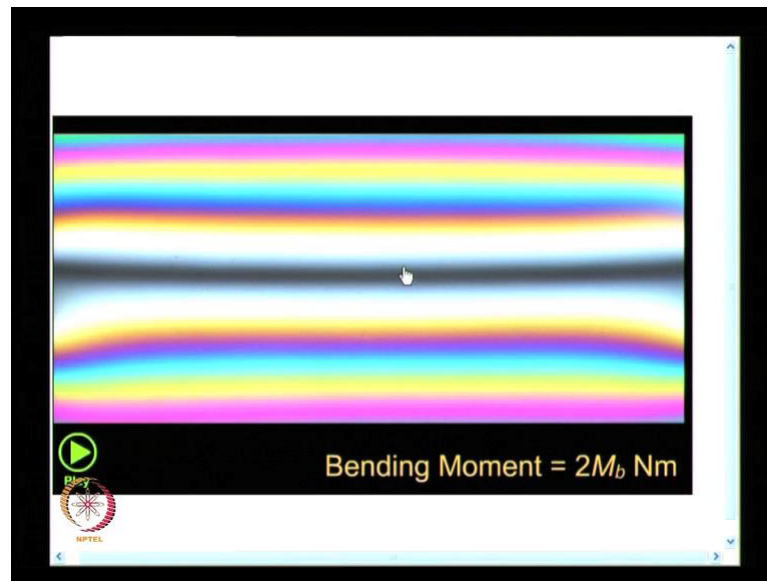
have solved the problem satisfactorily and a parametric analysis is very convenient when I go for a numerical methodology.

What is seen here is, here I have taken the effort of plotting fringe contours what you get in an experimental technique and that requires special software to be developed, it is not readily available in standard packages and we have this and this the best way to compare results of photoelasticity with the actual experimentation. When you do a numerical analysis you can compare with photoelasticity comfortably.

And what I also want to emphasize at this stage is you know though we have taken simple problems, we have gone and also studied in the process what are the approximations you do in your analytical modeling. You know we have taken a beam under four point bending and it bends like this, and it is 3 dimensional objects, it has a cross section. But what you have managed to do in strength of material is you just take it as a line that is all you do the analysis, and when I go to theory of elasticity you do not consult as a line but you consult that as 2 dimensional object.

But in reality, because you have fluxing what you find here is, you also have the Poisson ratio effect becomes very prominent, and what you find here is, this is the compression side and this is the tension size and this compression side bulges out because of Poisson ratio effect. This may be very difficult to model from analytical point of view, but experiment looks at all this. So when I get the fringe pattern, the fringe pattern is defective of all the three dimensional effect that happens to the model. Some you may have ignored it for the point of view of simplicity, so that is why I said that experiment is giving you truth.

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And we also noticed that fringe pattern observe in the beam, there was slight variation on the tension side and compression side if you look at it in a settled fashion. For a quick look it will appear asymmetric symmetric, but for a very closer look it will have small deviation which could be neglected as a second order effects and carry on with it. So the point emphasize here is the moment you come to experiment do not discord the raw data, raw data is very, very important.

You may have an explanation to understand what the re raw data means, if you do not have the explanation try to go and find out whether you have made any approximation, whether you can refine any of this, because in engineering what we do is, we never want to solve the problem in 3 dimensional with all the complexities. I have also mentioned earlier the success of engineering is approximation, and if there is possible I would like to work with a one dimensional solution. If one dimensional solution is not feasible I go for 2 dimensional solutions. Only we are pushed to the wall that without 3 dimensional solutions, you will not get satisfactory result we go and attempt 3 dimensional solution. The movement you go to analysis of plates and shells, it is actually 3 dimensional problems you bring in plates and shell theory approximation and try to leave in 2 dimensions you do not want to go in 3 dimensions. So that is the knowledge that you will have to get.

So, what we have done is still now we essentially looked at; what is an analytical method? What is a numerical method? And what is an experimental method? What is the information I get directly from an experimental technique. The idea is you may be able to combine more than one experimental technique and try to get all the 15 quantities some quantities of your interest. But what you get directly is from the physics of the problem, physics of the experimental technique on which this based. We try to look at what whole field information is, because you have to graduate from stress as a tensor at a point of interest, you have to go and find out how the stresses vary over the domain of the model that was the primary focus.

In the next class what we will do is we will go and find out what is the physical principle each of the techniques is based on.

Thank you.