Experimental Stress Analysis - An Overview
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Lecture - 1.1
Introduction to Stress Analysis - Analytical and Numerical Approaches

This lecture is on overview of experimental stress analysis and these light shows in nutshell, what experimental stress analysis is all about?

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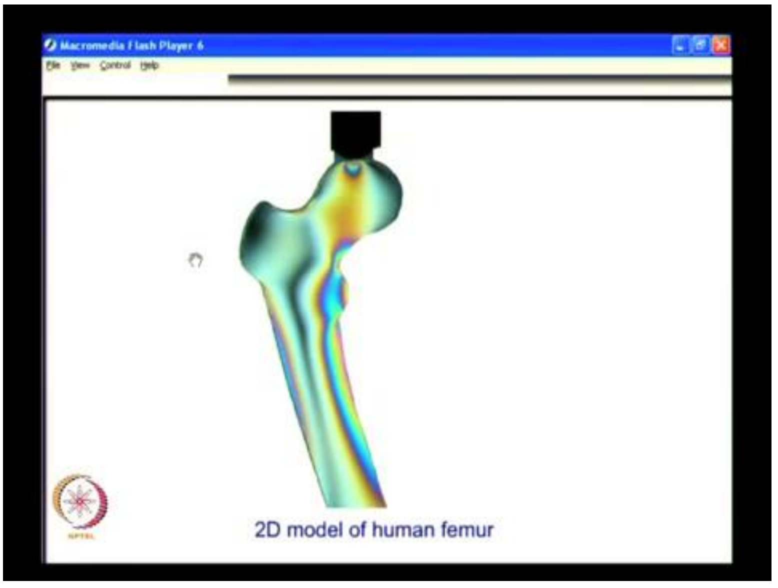

What we see here is stress patterns observed in a 2D model of a human femur. So, you get stress information in stress analysis, and you also get displacement information in stress analysis and what you find here is, for a circular plate clamped at the boundary with a central load. You get the displacements and you can also have the slope. You can also get curvature.

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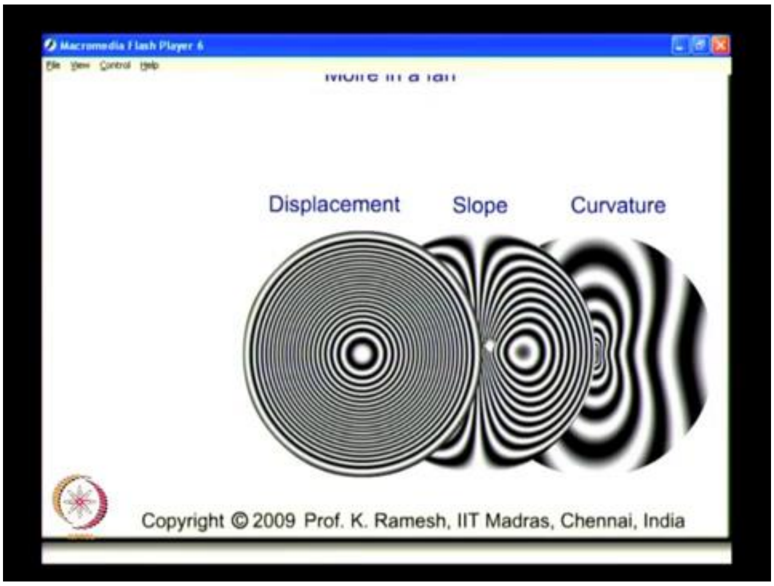

But what will have to appreciate here is for each of these information, you need to use a specific optical arrangement. You would not get them in one shot, but you have to have effort to do that and what you see on these 2 corners show that you need to use physics in applying experimental techniques.

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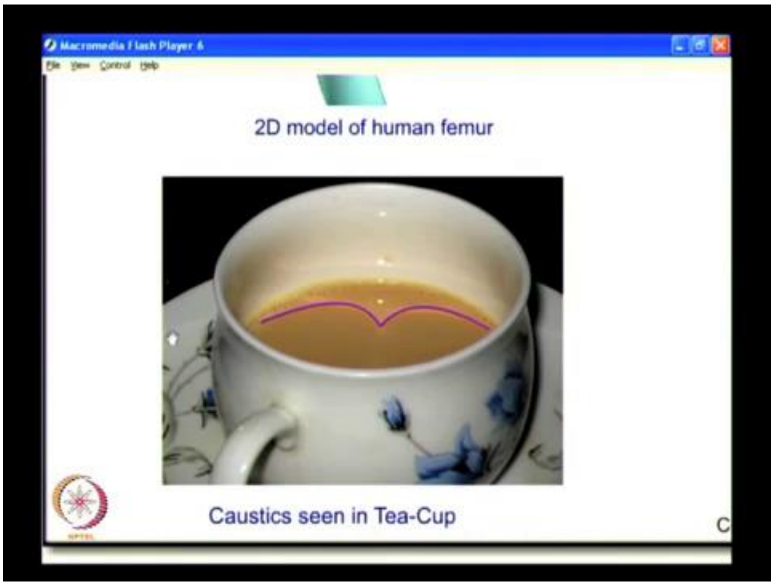

So, what is done is any physical information which could be exploited for measurement is identified and that is translated into an experimental technique. What you see here is a cusp in a Tea-Cup and this you see as a silver line, what happens is when light gets reflected on the curved surface, it gets reinforced and you have a cusp. This can be seen on any shallow filled containers with appropriate lighting and this phenomenon is called Caustics.

This physical information is exploited in a technique called Method of Caustics, which is used for measurement of high stress concentration. How it is done, we will see as part of the course. What you see here is another down to earth example, where you see in the case of a fan. When you have 2 grid super imposed, you see a nice moving patterns and this is called Moiré and this is used for measurement of in plane displacements, out of plane displacement and so on.

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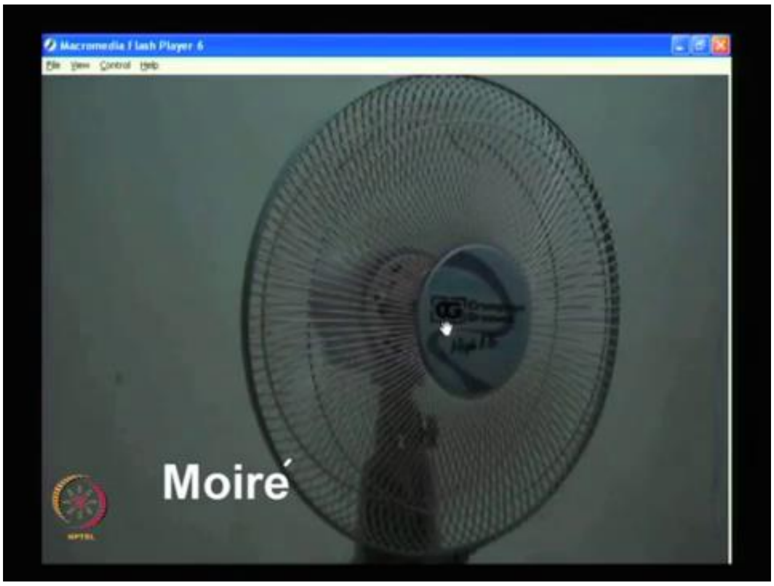

So, what is emphasized here is, when you have experimental stress analysis I can measure stress information, I can measure displacement information, I can also measure strain information though it is not shown here and what is emphasized here is for each of the experimental technique, there is physics behind it. So, you need to appreciate the physics, understand how this physical information is exploited and that is what we are going to see in the lectures to follow. But before we proceed further, we need to know what stress analysis is all about?

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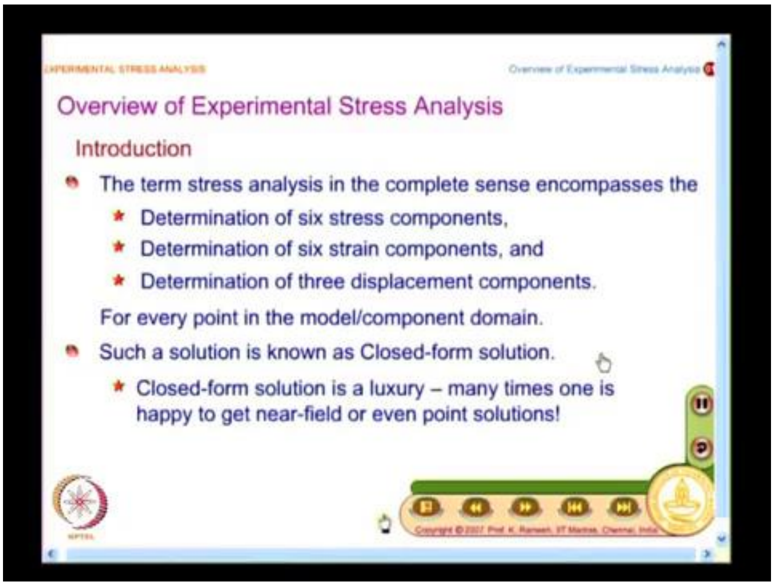

So, the term stress analysis in the complete sense encompasses the determination of six stress components. Why they are six? Why do I say that you have six stress components? Suppose, I take a member like this and then apply a tension, you all know the stresses are developed and what is the value of stress? Someone can answer. Suppose, I apply the load as P and area of cross section is A. What is the value of stress?

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Whereas six components? You are only saying only one and what you have in this simple example problem is stress is a tensor of rank 2, so you have nine components. Of the nine components, because of symmetry you have only six independent components. So, in a simple tension test like what you do on the slender member, you evaluate stress and it appears to be a scalar. P by A appears to be a scalar, and that is what your learnt in a simple course in strength of materials, but what you should change your thinking is go back and fill in the components in the stress tensor. So, what you find is, I find one stress component all other stress components are zero. So, actually you have because stress is a tensor of rank 2, you have six independent components.

So, when I say stress analysis I should know stress components. Then I also need to know strain components, and strain components are again six because, strain is also a tensor of rank 2. So, I need to get stress components, I need to get six strain components, it is also desirable that I get, displacement components. And displacement components are three and what way I want this information?

Suppose I take this rod and I pull it I want this information at every point in the model. So, what I want to do is for every point in the model are component I want all of these information and in this simple case the stress component is only one, if I keep it is sigma y, if I keep it horizontal, you will label at a sigma x. If you have x and y as horizontal and vertical you will have this as sigma x stress components or you will put it as sigma y stress components. So, want you need to understand is stress is a tensor of rank 2. So, whatever the kind of loads that you apply you should be able to identify the components, and what I want is I want this information for every point in the body.

Right now, let us not worry about the values of these near the place where I hold them, where I grip them and do it. Away from the point of loading we will worry about it. When you say this P by A, I know it at everywhere it is constant and such a solution if I give the x y location, I am able to find out the values of stress. I call that as closed form solution because at every point which I want to find out I have the answer. Can you get closed form solution for all problems? It is not so, it is only a luxury for certain problems you are able to get all the six stress components, six strain components and displacements and in certain class of problems you are satisfied with near field solution, particularly in the case of problems with crack, where fracture mechanics is focusing. This you get analytically solution close to the vicinity of the crack temp.

From a design point of view you are happy with even point solutions. You like to know where is the maximum stress, when you want to find out a stress concentration, when you want to go for optimization. You also want to know, where the minimum stress is because from there I can scoop out the material. So, what you have to understand is, the term stress analysis in it is complete sense encompasses determination of six stress components, six strain components and three displacement components and it is a luxury.

So, we have to go for that kind of solution, which you require for a problem on hand. Always you do not require all these 15 components and how this stress analysis can be carried out.

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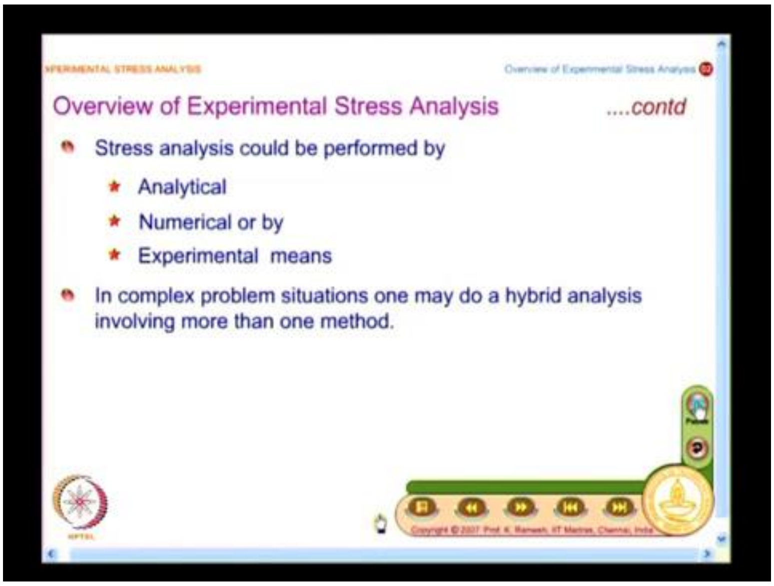

You could do this by analytical methods, you could do by numerical methods or you can also do by experimental methods. Can you take a view that I will always use analytical method or I will always go to numerical method or being an experimentalist I will use only experimental methods. We cannot take that kind of a prejudice review, each approach has certain characteristics that could be effectively exploited.

Few of this we will see as we go by, in a very complex problem situation one technique may not be sufficient, you may have to use a combination of analytical, numerical or numerical and experimental and data analysis is known as Hybrid Analysis. What we are going to do now is we will see one by one, what analytical methods, for which class of problems? Why do you do analytical methods? What is the advantage of numerical technique? And when you go to experiment in which class of problems experimental methods are ideally suitable? That is what we will see now and once you come to analytical methods what is the advantage here?

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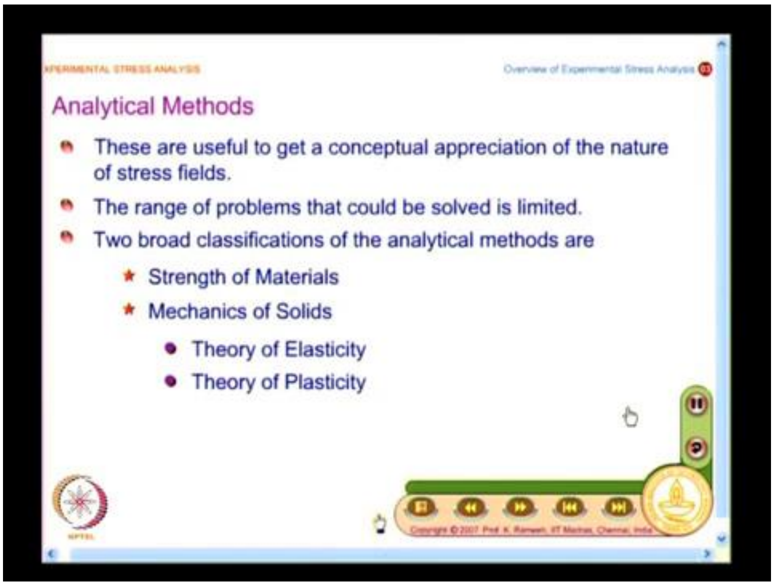

These are useful to get a conceptual appreciation of the nature of stress fields. If I want to know I take a member and I pull it, I know that it subjected to tension and I do the bending and I want to know how the stresses are developed in bending. But when you look at whether it is strength of materials or theory of elasticity. The range of problems that could be solved is limited and the broad classification of the analytical methods we call them as strength of materials and mechanics of solids. In mechanics of solids you could classify that as theory of elasticity and theory of plasticity. What we see here, they are very useful in getting a conceptual appreciation of the nature of stress fields, but the range of problems that could be solid is limited. You have all done a preliminary course in strength of materials, do you know in strength of materials, what is the basic assumption that has been used in solving the problem one hand. Can anyone of you answer?

Student: Plane sections remain plane before and after loading.

You remember that, that has been thought in your earlier course.

Student: Yes.

No, the basic, what you want to know is the basic information that we used for attacking the problems. What you find here is. So, one of the basic assumptions is “plane sections remain plane before and after loading” and with this what we have solve?

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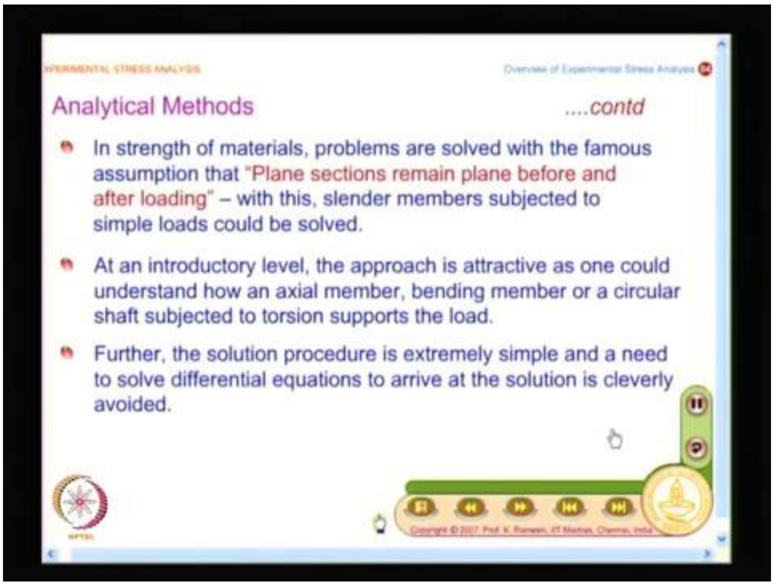

We have solved a large number of problems, which are slender in nature very, very important. Suppose I take a plate and then stretch it like this. I have no problem. Suppose I put a hole there, I cannot do it with strength of materials. The moment I put a hole, the assumption plane sections remain plane before and after loading no longer exist. So, I have to be careful, that is why you always solve problems that are solvable. So, in strength of materials course what you get is, you understand how an axial member, a bending member or a circular shaft subjected to torsion supports the load.

In the case of axial member what do you understand? Suppose I take the cross section A and suppose I apply the load P. How the force is distributed? It is uniform the entire cross section participates in the load sharing. On the other hand, if you go to beam under bending what happens? What is the difference between axial member and a member that bends?

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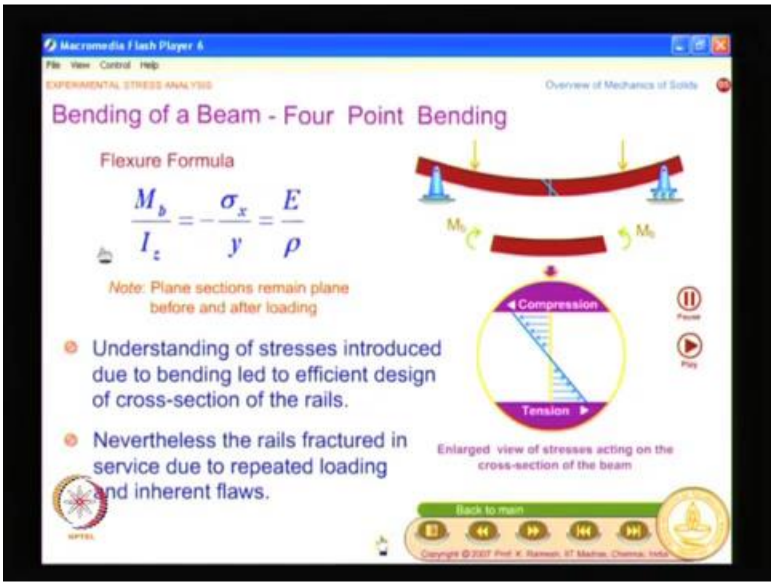

So, this is where you have the answer. So, what you have here is. I have the member which bends and I find that stresses are varying linearly over the depth and what we find? This is a very famous flexure formula and what way you did it? You never went in solve differential equations. You cleverly avoided solving differential equations and you hide behind the assumption plane sections remain plane before and after loading. If you watched this problem, this is also very clearly shown that you have the end loads here, but you have only looking at a region which is interior to that where you have only pure bending is applied and the discussion is confined to pure bending. If I go to cantilever, I have a shear and plane sections do not remain plane before and after loading. They are not couple, so you are able to still leave with flexure formula and if you go for deep beams you have to bring in shear effects. So, in a first level course you learnt how an axial member supports load. That is why you have a lot of stresses that is being used for in stadiums and very large halls.

So, you effectively utilize, role of material completely because all the material contributes to load sharing. The moment you come to bending, you understand that, stress is varying linearly. So, the inner core is not contributing to load sharing. What is seen when you go to, how this is used in design, so if you go and see a rail cross section. You do not have a square blocked because you know that this is subjected to load on the top phase and you know it is essentially transmitting a bending load, since the inner core is not participating you are removed the material.

So, what you have done is your analytical approaches to problem you are not solve the problem of a rail to start with, but your bending understanding that what you have stresses vary linearly and inner core is not contributing is effectively utilize in arriving at a shape of the rails. Now, what we will see is, the introductory level the approach is attractive, as one could understand how an axial member, bending member or circular shaft subjected torsion supports the load and once you come to circular shaft you all know people used hollow shaft.

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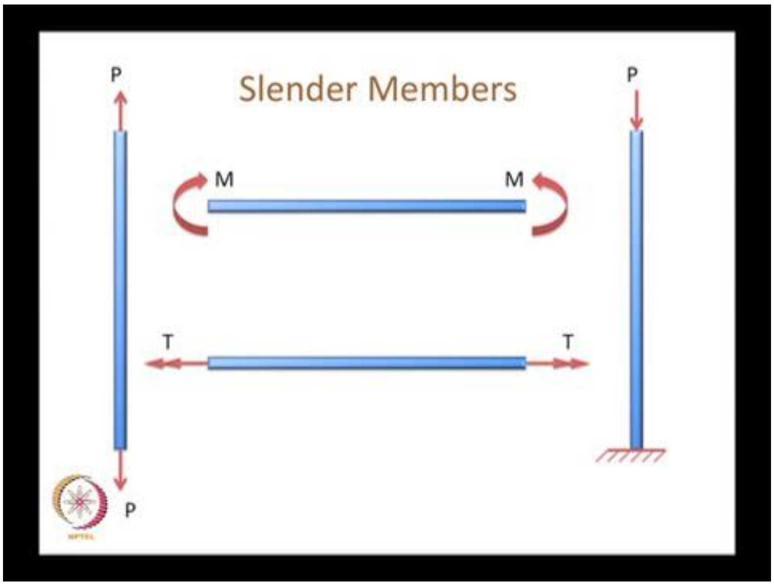

Very similar to bending there the shear wave linearly. So, in pure torsion you only talk of a circular shafts subjected to torsion. Suppose you have a rectangular shaft subject to torsion that is shifted to the next course. It is not taught in the first level course because, we always want to have a very simplified approach to problem solving and we have cleverly avoided solving differential equations, but nevertheless we have understood how an axial member supports load, what happens in the case of a bending? What happens in the case of torsion?

So, analytical methods are very crucial in giving a conceptual appreciation, on very simple problems, but you will restrict to class of slender members. You will not go to any other class of problems and this is very effectively used in designs. If we go to the second level course you always use a design based on very simple idealization based on strength of materials.

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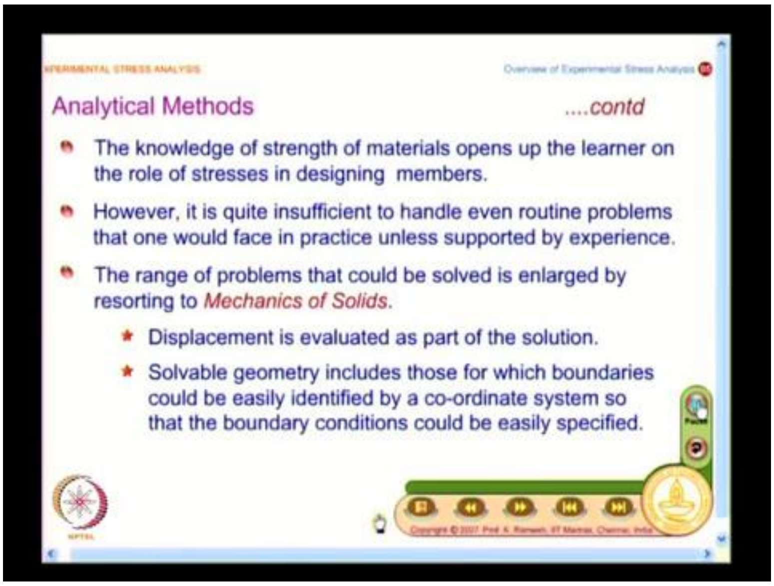

So, what you find here is the knowledge of strength of materials opens up the learner on the role of stresses in designing members. Like I have shown you that you have the rail where you have removed the material from the central portion because it takes the load only on the top and bottom fiber, maximum. Whether you have understood or nature also is understood solid mechanics.

Even if you go and see you have bones, bones are hollow. The inner core is where you have bone marrow, where you have hemoglobin is developed and that has to be protected. So, you have a hard shell and your bones, if you will take a thigh bone. It subjected to bending and in appropriate loading it will also have some more level of torsion. So, nature is already understood that when it has to resist bending or torsional loads it can optimize it is structures beautifully, because bone marrow is a very soft material and if you really go and look at bone people now say that it is a functionally graded material. It is not even just hard and soft, it is a functionally graded and they say tooth is also functionally graded. So, a nature has understood all this mechanics, solid mechanics and utilized it in many of its structures and as humans we go try to unravel it and embedded in the form of mathematics and develop as engineering tools.

Now what happens suppose I the basic restriction is I assume some displacements in the case of strength of materials and you cannot live with that if you go to a fluid mechanics course they start with differential equations. They do not have any problem, where they avoid differential equations. Only in solid mechanics what we find is you can have one full course without touching differential equations, only in the second level course you relax this you do not make assumption on the displacement, but you evaluate displacement as part of the solution. This is what you have it in mechanics of solids.

By relaxing this if you find, whether you have been able to solve all problems on hand which is not so. The solvable geometry includes those for which boundaries could be easily identified by a co-ordinate system, so that the boundary condition could be easily specified. Now I have shown that you have a plate with a hole, I cannot solve plate with the hole by theory of elasticity unless the hole is sufficiently small and what you have here is, here is a photo elastic model and here you have a plate with a hole and plate with a very small hole. From a mathematical point of view, it is possible to assume or idealize that this is a small hole in an infinite plate. Physically it is finite, but mathematically it could be modeled as infinite and you would be able to find out analytical solution for this using theory of elasticity. On the other hand, if I take a model with the slightly larger hole then the solution is no longer valid.

I have the same width; the width is same, in one case it is very small hole, in another case the hole is about 10 to 12 millimeters and this is comparable to the width. So, this becomes the finite body problem. So, when I go to theory of elasticity, where I am able to idealize I look at a situation, where I remove the restriction on displacements and I evaluate displacement as part of the solution, even there I need to have infinite boundaries for a problem like this, only then I will be able to solve. Mathematically, the distances are far away, but physically it could be a very small hole this closely resembles and this solution is possible and plate with the hole is a very, very important problem. You are all learned see in many one of your design courses when I have reverted joints I have a series of holes.

So, the idea here is as an analytical method, you have been able to find out the presence of a hole, how does it influences, the stress field near the hole. You are able to get an analytical expression and this will be a function of x and y. If you plug in x and y you will be able to find out the solution, at every point in the domain, which is not the case when I take a finite geometry. This finite geometry it is not possible to solve by theory of elasticity, on the other hand other geometries like, suppose I have a circular disc and the diameter compression. This you get closed form solution from theory of elasticity and compare I have a axial member I pull it I can do it by strength of materials, but the same axial member suppose I have a hole, I cannot solid it by strength of materials. I have to depend on theory of elasticity.

Even in theory of elasticity the hole should be as small as possible. On the other hand even if you change the geometry, you cannot do it by I do not have a hole here, but the geometry is changed because the way I apply the load plane sections do not remain plane before and after loading. So, if you look at when I move from strength of materials to theory of elasticity. I have enlarged my domain of solving problems; strength of materials has given me a very good approach, where in without solving differential equations I could understand axial member, bending and torsion. When I go to theory of elasticity I could find out what is the meaning of stress concentration. I can also solve a class of small problems where, I am able to find out the boundary conditions and then apply it carefully.

So, you have a large class of 2 dimensional problems have been solved, I have a disc, you have a solution. But if you look at a 3 dimensional problem and if you want to approach and find out whether you have analytical solution. Those problems you can count them in fingers a very, very few. So, the problem is very complex, and I want to find out, I want to take a spanner and I want to tighten a nut. because this as arbitrary geometry I cannot approach it from theory of elasticity and get the solution and this is what we do day in and day out. So, what you find here is even though you have methods which provide you conceptual understanding.

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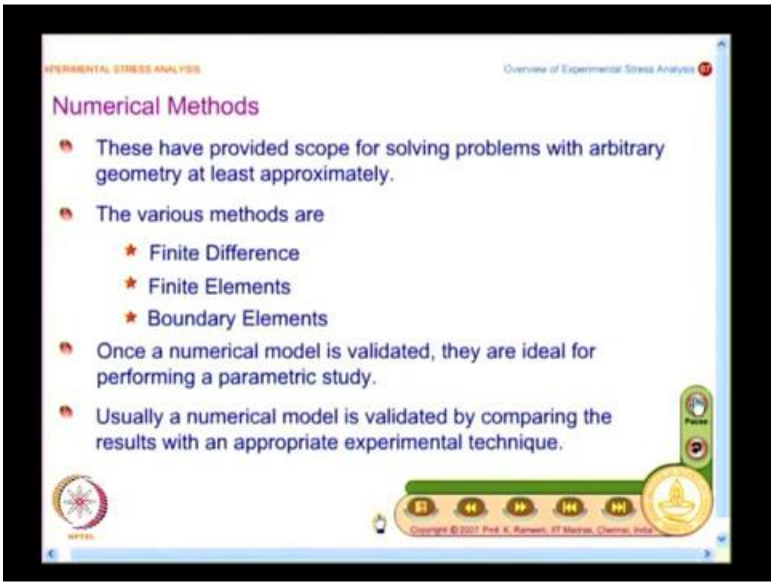

If I go for arbitrary shape, numerical methods come very handy and you are able to get the solution at least approximately for arbitrary geometry. So, that is where you find that the numerical methods are very attractive and you have many numerical methods. Many of you might have done finite difference then you have most versatile finite elements, you have many packages available for you to attack a problems and you also have boundary elements and very recently you have mesh (Refer Time: 24:29) methods that have come and what you find is when I am having a numerical approached problem. The numerical model has to be validated. Once unless numerical model is validated, if the boundary conditions are not verified, you are solving a different problem.

So, for any numerical approach you have to validate whether the procedure is all right, but the greatest advantage of numerical method is once the procedure is validated, it is an ideal way to do a parametric analysis. In a design, if you have several geometry factors you would like to vary them or you would like to change the load case and find out how the system is going to behave, conducting experiments for each one of the configuration would be difficult. So, you do an experiment, verify the discretization, verify the boundary condition then do a parametric analysis by a numerical approach.

Analytical approach provides you conceptual appreciation of what is axial load supporting members, how do they behave, in bending how the stresses vary over the depth, in shear how the stresses vary, that understanding is very crucial, in translating many of the design what you see across. They could be simplified to any one of these slender members as a first of approximation and you can get some insight what is happening. But if you want to get a detailed solution, you have to go to theory of elasticity where, you do not have the restriction on the displacements in you have a simply connected and multiply connected problems in multiply connected problems you have to bring in uniqueness of displacement as a input otherwise, you cannot solve the problem, but living that a path you do not put any restriction on displacement.

You evaluate by solving differential equations and when I have complex geometry, I can always go and attack the problem for numerical approach, but numerical approach has to be handled very carefully because you may go wrong in understanding and implementing the boundary conditions. So, any numerical approach has to be validated by experimental inputs, once you validate the numerical approach, they are the ideal choice for a parametric analysis.

We have discussed in this class the analytical and numerical approaches for stress analysis.