Fundamentals of Gas Dynamics
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Week – 03
Lecture – 06
Concept of stagnation

This class we are going to discuss a new concept, which is something called stagnation quantities or stagnation concept.

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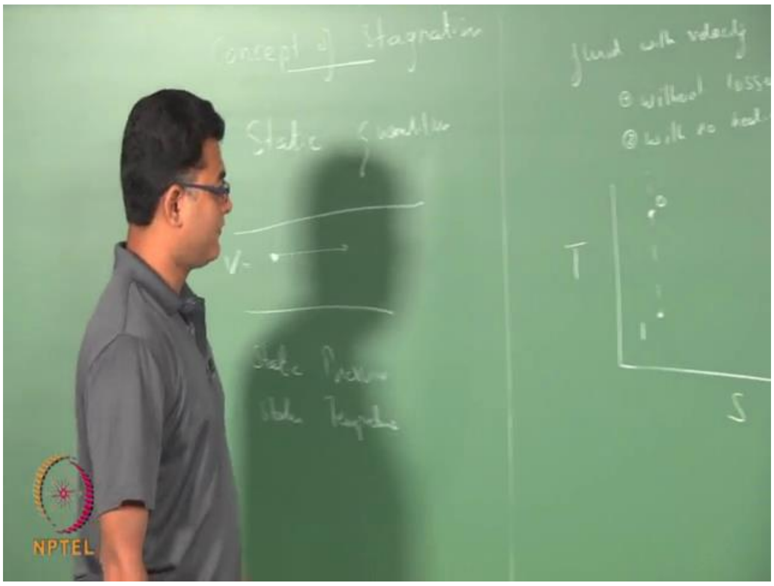

So, if I have a flow and if I measure quantities while traveling along with the flow, what I would be getting is something called static quantities. So, if I have a flow with velocity, some velocity of the fluid and if I travel along with the fluid with the same velocity and measure my temperature or my pressure, I call those quantities as static pressure or static temperature and hence, other quantities measured like that, if it, if they are measurable. So, we need a reference quantity, which is now we call as stagnation.

So, what do I mean by stagnation quantity? So, if I have a fluid with velocity v, with some velocity and if I bring that fluid to rest at, without losses and with no heat or work; so, if I bring the velocity of the fluid to 0, with no losses or no heat, so I am bringing the fluid isentropically to 0 condition. So, graphically if I have a point here. So, let us consider this as T-S plain, I have a sate one here, the velocity at that particular point or at that particular state or the fluid is some V, now I am bringing this fluid to 0 velocity without any losses, which means, it is isentropic process. So, it would be a straight line somewhere here and with no heat or no work done. So, this is my, say, 0 condition the stagnation condition.

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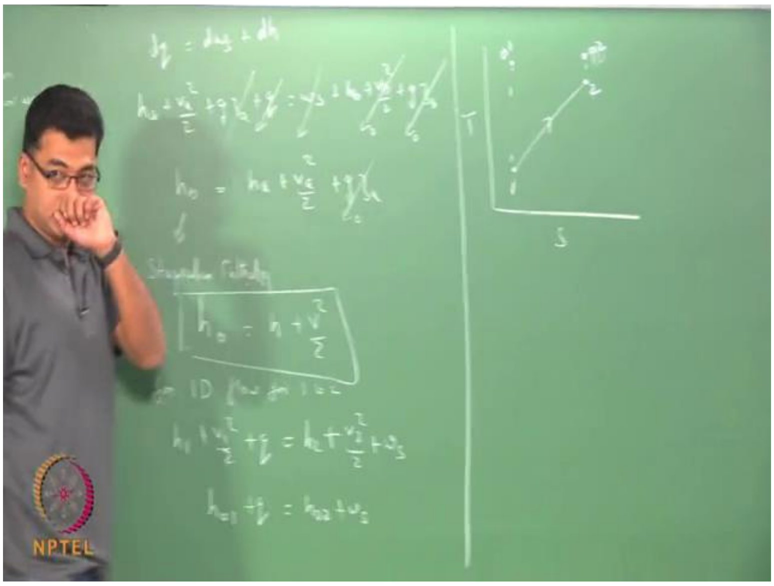

So, let us go back to the equation and see how it behaves. So, we have dq or delta q equals d w s plus your dh. So, if this condition is my state a and this is my state b, I would have h a plus v a square by 2 plus g is at a plus q equals w s plus h b plus v b square by 2 plus g is at b. Now, what did I assume? I assumed no losses isentropically and no heat, no work. So, I have no heat, no work and I have brought the velocity to 0. So, my, be 0 and I do not have a potential difference between these, the potential energy is also brought to 0. So, my h b is h a plus v a square by 2 plus g is at a.

Let us assume flows without the potential energy, which means, the quantity is also assumed 0 for most of the cases that we are going to discuss in this course. So, let us just stick with the kinetic energy. So, the enthalpy of point b where I have brought my fluid to rest isentropically is given by this. So, I call this as my stagnation enthalpy, which is typically denoted h 0 into h plus v square by 2 if I consider any point. So, any point here is brought to rest at some point here as the stagnation point. So, any point associated with any point, there is a stagnation point. If I do this process, I bring back the velocity to bring the velocity to 0 isentropically. So, this is my stagnation enthalpy.

If that is the case, I can rewrite my original equation, say for example, for 1D flow, in the previous lectures we have seen between two, between two points or for the process 1 to 2 I have written h 1 plus v 1 square by 2 plus q equals h 2 plus v 2 square by 2 plus w s, which means, I have a process 1 to 2. In the T-S diagram, I have a process 1, I have process state 2, I have a process that is happening between 1 to 2 and the energy is related in this particular fashion for 1D flow. Associated with 1 there is a stagnation point, associated with 2 there is a stagnation point.

So, I we can rewrite this as h 0 1 plus q with h 0 2 plus w s. So, h 0 1 is the stagnation point associated with point 1 here, this is 0 1 and h 0 2 is the stagnation point associated with point 2. So, if this is 2, this is 0 2.

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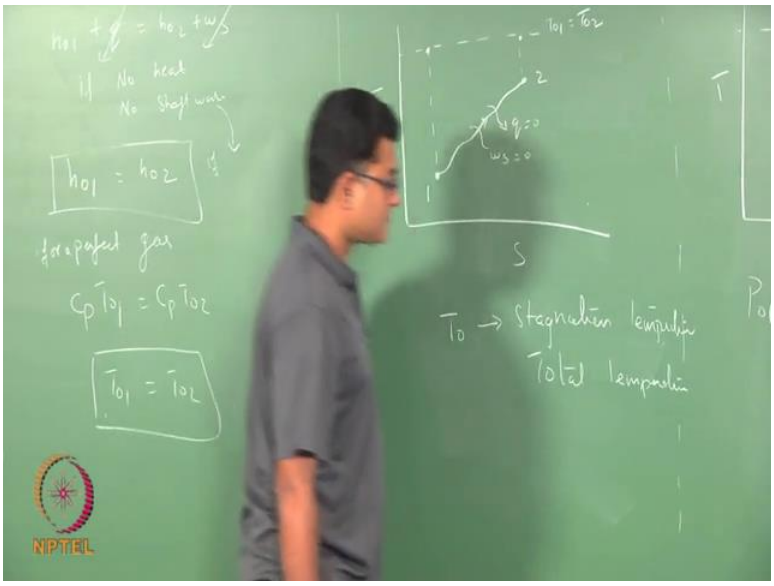

Now, between these two points h 0 1 plus q equals h 0 2 plus w s, if no heat is supplied or no shaft work, you get this term and this term 0. So, if there is no heat supplied in the process 1 to 2 and no shaft work, so the q is 0, w s is 0 in this process. If that is the case, then I can strike off this, which essentially mean, h 0 1 equals h 0 2.

If this is true, if the gas is the perfect gas, for a perfect gas I can rewrite this as C P T 0 1 equals C P T 0 2 or T 0 1 equals T 0 2, which means, in the diagram T-S, point 1, point 2, my stagnation temperatures are same. So, the stagnation temperature associated with state 1 is same as stagnation temperature associated with state 2; so, these are same. So, remember, there is no q and no w s, shaft work is 0, the heat applied or heat transfer is 0 for this to be valid.

So, if I have a nozzle flow, the gas has, I have a nozzle flow is from this side, there is no q, no w s. In such a scenario I can assume this. But if I have a propeller here, I cannot assume this that is the meaning of, now, constant stagnation temperature. There is no loss in stagnation temperature. So, this T 0 1 or the T 0, that is called your stagnation temperature or, or your total temperature.

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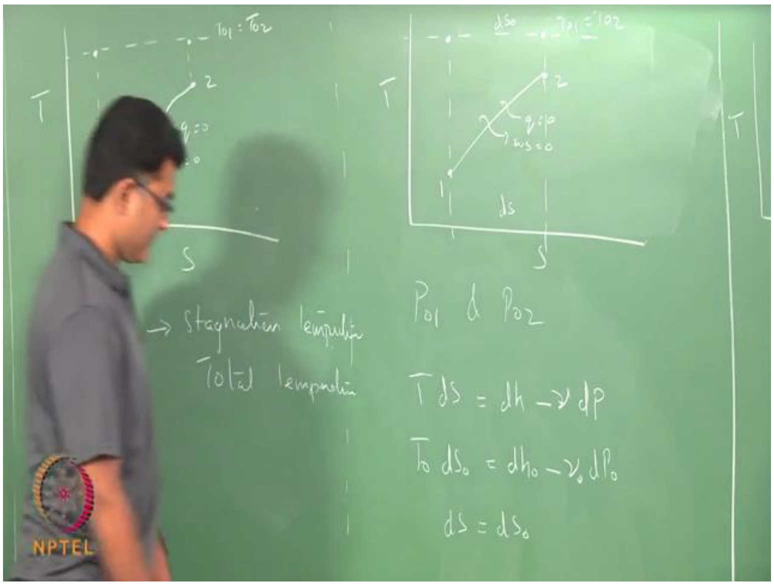

Now, consider process from 1 to 2 where there is no q, no w s, so my T 0 1 is same as T 0 2. This is 1, this is 2, my q is 0, w s is 0. Now, let us see if, let us see what, what is P 0 1 and P 0 2. So, associated with this static state we, I have a static pressure and now I have defined a reference quantity, which is stagnation quantity. So, along with T 0 1 I have a P 0 1 and same as for the second state, I have a stagnation pressure. Let us see what happens to P 0 1 and P 0 2.

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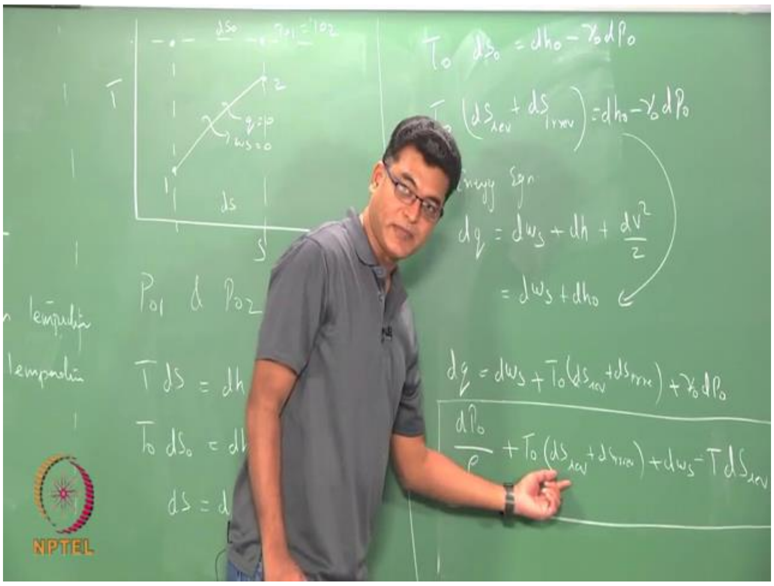

So, I start from my Gibbs equation. TS equal dh minus v dP. I am talking about stagnation quantity. So, this is my T 0 dS 0 equals dh 0 minus v 0 dP 0. Now, the dS associated with this is same as the dS associated with these two points, from the picture, from the graph. So, my dS is same as dS 0. T dS 0 dh 0 minus v 0 dP 0. Like before, I split my dS into dS reversible plus dS irreversible equal dh 0 minus v 0 dP 0.

From my energy equation, I have dq equals dw s plus dh plus dv square by 2 or I can write this as w s plus dh 0. Now, I substitute this quantity here. So, my dq equals dw s plus T 0 dS reversible plus dS irreversible plus u 0 dP 0. So, I am substituting this here. So, I have taken the energy equation and from the Gibbs equation I have substituted that. (Refer Time: 16:33) So, v 0, which is a specific volume, I can rewrite this as rho 0. So, that is the stagnation density associated with some state plus T 0 dS reversible plus dS irreversible plus dw s minus dq.

Now, dq I can replace, as my T dS reversible from the 2nd law of thermodynamics equals 0. So, I had taken the Gibbs equation, I had taken the energy equation, substitute at these stagnation enthalpy change from Gibbs equation into this and I have replaced dq with T dS reversible from the 2nd law of thermodynamics, that is all I have done

Now, let us look at this equation. So, your change in stagnation pressure depends on the irreversibility, the losses, the heat supplied, which is your the reversible and your shaft work.

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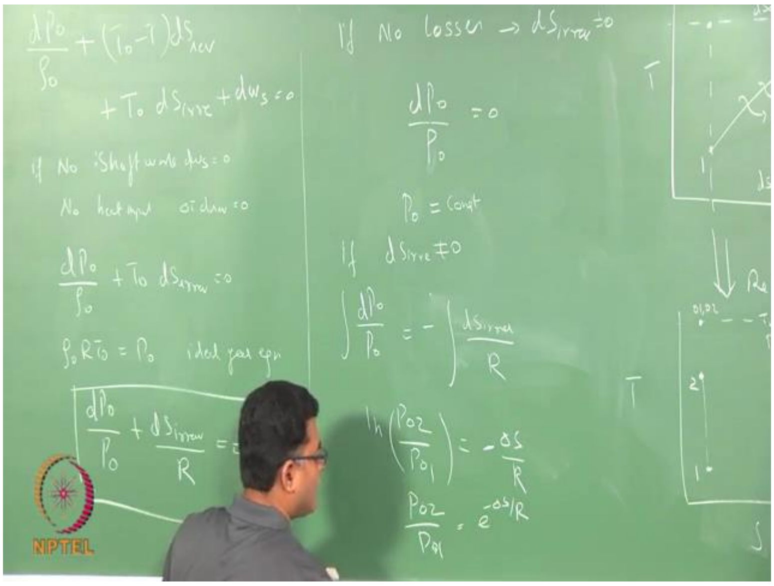

So, I write this; plus T 0 minus T reversible plus T 0 dS 0 reversible plus dw s equal 0. If no shaft work, w s is 0; no heat input, so my 2nd term is 0, d reversible 0. So, what is left is dP 0 by rho 0 plus T 0 d irreversible equal 0. Now, rho 0 or t 0 is my P 0 from my ideal gas equation. So, I substitute rho 0 here, I should get dP 0 by P 0, 0, plus dS irreversible by R equals 0. So, how do I represent this equation graphically?

So, between two processes if there are no losses your d, revers, irreversible is your losses. So, if there are no losses, dP d 0, this is going to be 0. So, between these two process, losses dS irreversible is 0, which means, in the picture here, from process 1 to 2, if there are no losses, there is no entropy change. So, dS, this quantity is going to be 0, which will lead to this equation, which essentially means, P 0 equals constant or I redraw this picture, T-S.

(Refer Slide Time: 24:49). Now, I have state 1, which just means, there is no entropy change. So, I have state 1, I isentropically take it to state 2 so that there is no loss, there is no shaft work, there is no heat input. So, from 1 to 2, I take the state from 1 to 2. Associated with 1 there is a stagnation point, associated with 2 there is a stagnation point, which we have seen both are same. If there is no shaft work and no heat input, so your T 0 1 equals T 0 2. Now, this also tells me my P 0 1 equals P 0 2.

But if, if there is losses is not equal to 0 what do I do? I need to integrate this, dP 0 by P 0. So, if there is losses, if there is, if there is some loss due to friction or some heating, you need to integrate this equation. So, I integrate dP 0 by P 0 plus dS irreversible by R equals some value or some constant or I can take it to the other side. So, this would be ln P 0 2 by P 0 1, this would be delta S, which is S 1 minus S 2 by R with the minus sign. So, if this is, if I take the exponential P 0 2 by P 0 1 is exponential delta S by delta R, this also mean something.

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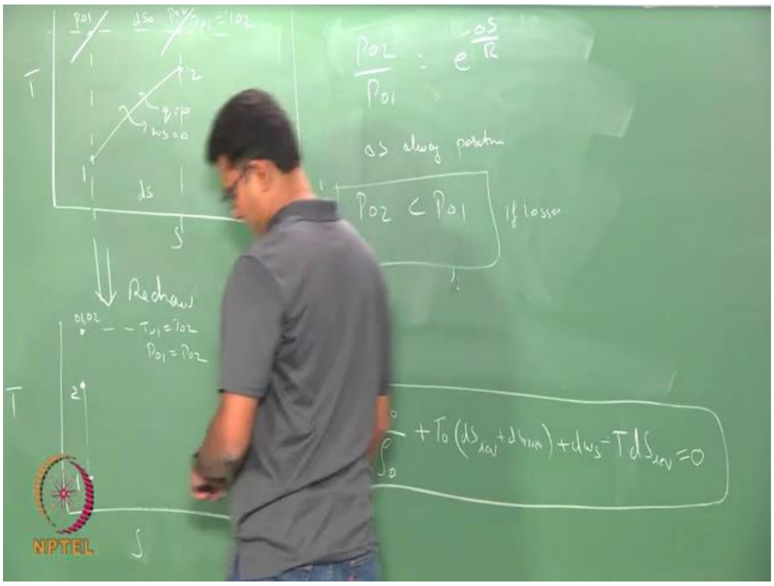

So, I rewrite that equation here, P 0 2 by P 0 1 equals exponential minus delta S by R. So, all your P 0 change is due to your irreversibility condition given that there is no shaft work, there is no heat input.

If those two conditions are satisfied, the changes in P 0 1 and P 0 2 is entirely due to your irreversibility, irreversibility or the losses that is happening in the flow. So, if delta S is always positive, which means, your P 0 2 is less than P 0 1. So, if I have a P 0 1 here, this pressure is less than this pressure, if there are losses; if losses.

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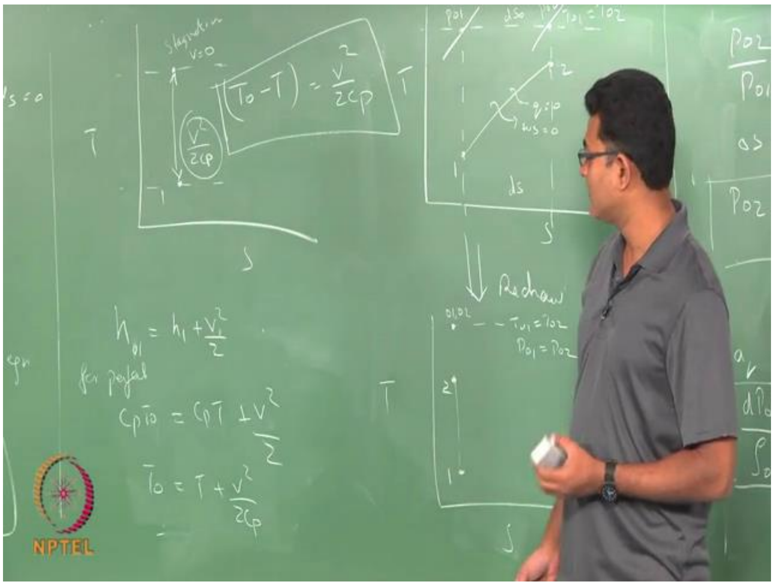

So, I repeat what we have learnt today. So, what I want to do here is, so I have a state 1, which I bring it to a quantity with 0 velocity, where v is 0. I have defined my h 1 to be h 0 1 to be h 1 plus v square by 2. This is what we have seen in the earlier part of the lecture.

Now, for perfect gas, this is C P T 01 is C P T 1 plus v 1 square by 2. For any process, I can remove the suffix 1, for any state. So, my T 0 is T 1 plus v square by 2 C P or my T 0 minus T is v square by 2 C P. So, this difference here is just v 0 by 2 C P for a perfect gas. So, I have a flow in which I have, I take the condition, the static condition to a stagnation condition, meaning, I take the flow to a 0 velocity isentropically with no heat and no shaft, then the difference in temperature is given by this quantity, v 0 by 2 C P. We are going to have few other reference quantities and we will see where they are placed in the T-S, T-S diagram.

What we are learnt today is the stagnation, concept of stagnation condition, the possibility of a new quantity, stagnation temperature and associated with it you have a stagnation pressure and hence, other quantities as applied to your energy equation as well as your ideal gas equations. So, those are related, though relations are derived from your, again from your thermodynamic flows and energy equations and your Gibbs relations.

So, for case where there is no heat and no shaft work, if the process do not involve losses, you have T 0 1 equals T 0 2. The stagnation temperature at, for state associated with 1 is same as your stagnation temperature associated with state 2 if they undergo a process without heat, without shaft and no losses. And we have also seen how the stagnation pressure changes and why irreversibility is affecting the stagnation pressure. So, the, the difference in temperature, stagnation temperature and static temperature depends only on your velocity and the kind of fluid that is undergoing. So, that is the final message from the lesson where the difference in you stagnation temperature, difference between the stagnation temperature and static temperature is just your kinetic energy and the associated fluid property.

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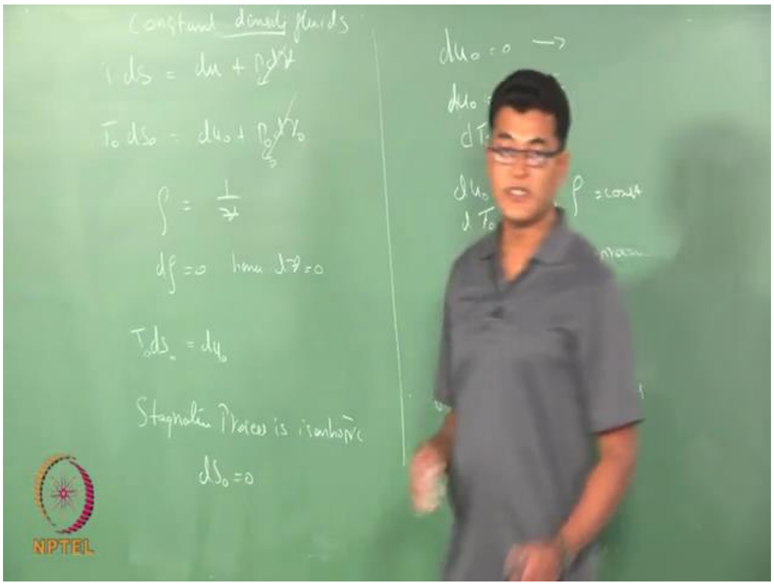

We will try to see what the constant density would bring in these pro, in the relation of stagnation quantities. So, what is stagnation pressure for a constant density fluid? What is a stagnation temperature for a constant density fluid? So, we start with Gibbs relation T dS equals du plus P dv. Constant density fluids, so that is the topic of this short lecture.

So, if it has, if you are going to deal with the stagnation quantities, we write this. So, if it is a constant density, density is inverse of your specific volume. So, your d rho is 0 and hence, your dv is also 0 for a constant density fluid. So, if I put that here, I would lose my second term on the right hand side. So, my T dS is du or the T 0 is (Refer Time: 31:53).

If this is the case my stagnation process is isentropic, which means, my dS is 0. So, if my dS is 0, my du is 0. So, for a constant density fluid my u 0 is C v du 0 is C v dT 0. So, if du is 0, my dT is also 0; if du is 0, my dT is also 0. So, my du is 0, my dT 0 is 0 for density constant fluids for a stagnation process where my dS is 0 for the stagnation process, which is also true if I write du is 0 and dT is 0. Essentially, it means, for constant density fluids my u equals u 0 and T equals T 0.

There is no distinction between stagnation and static quantities for a constant density fluid, that is what it means, because the stagnation process is essentially isentropic and if you substitute that in Gibbs relation, you directly get the inference as static quantities is same as your stagnation quantities for u and T.

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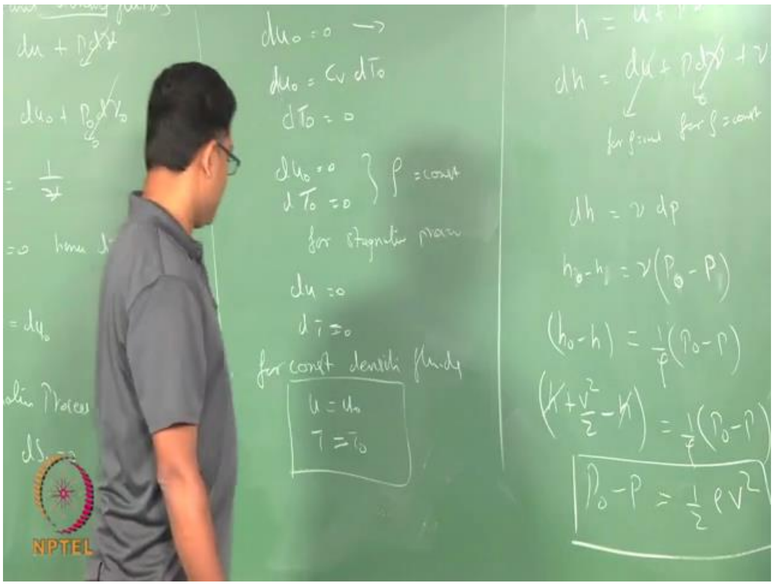

What about P? My definition of enthalpy is u plus P v and dh du plus P dv plus v dP. Now, dv is 0 for a constant, constant. We have also seen du is to be 0 for rho equals constant fluids, if I am, if the fluid is undergoing a stagnation process meaning, I am taking the fluid to a state where the velocity is 0 and the process is isentropic with no heat and no shaft work.

So, I am left with dh equals v dP. dh is, say, h 2 minus h 1 equals v into P 2 minus P 1. So, the stagnation process my two is 0 and one is no suffix, so P 0 P. I can replace v as 1 by rho P 0 minus P h 0 minus h. Now, h 0 is h plus v square by 2 minus h equals 1 by rho P 0 minus P. So, my h and h cancels out. So, my P 0 minus P is 1 by 2 rho v square.

A constant density fluid undergoing the stagnation process, constant density fluid, which has a velocity v is taken to a situation where the velocity is 0 isentropically with no sheet and no shaft work, then this relation holds where your P 0, which is the stagnation pressure minus your P static pressure is 1 by 2 rho v square, which is typically called as dynamic pressure for a constant density fluid. So, your T is same as your T 0, u is same as your u 0, but P is not same as your P 0. So, the stagnation pressure minus your static pressure for a constant density fluid is your dynamic pressure.

So, that is a short lecture on constant density fluid. How you can reduce the relations, what we have learnt in the previous lecture to your incompressible flow relations.