Fundamentals of Gas Dynamics

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Week – 03
Lecture – 05
Conservation Equations (Cont'd.)

Today we will discuss the differential form of energy equation and we will derive the pressure energy relation.

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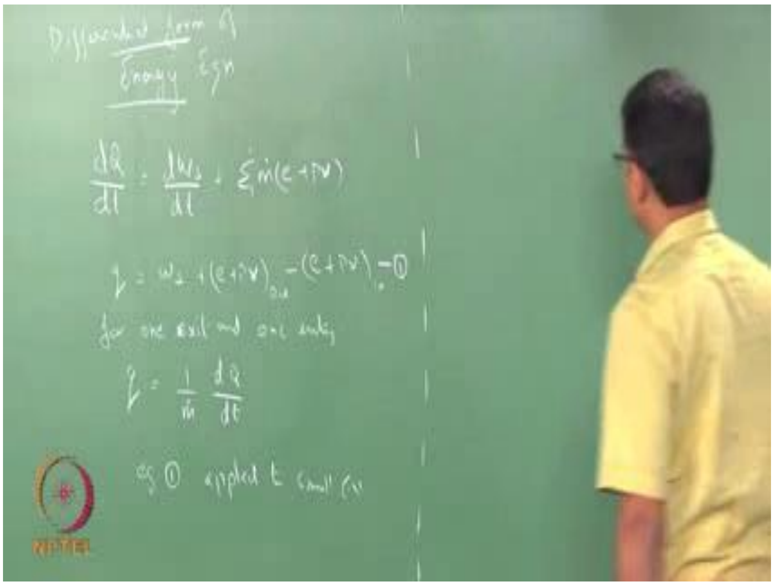

So, we will, I will start from where we had left in the last class. So, we have seen this equation, where Q is the total heat supplied, W s is the shaft work, e is the internal energy and V is the volume which we have reduced to this form, out minus in, for one exit and one entry. So, that is a reduced form, 1D, 1D equation of that, we will apply this to infinitesimally small control volume and derive the differential form.

Q here, Q is 1 by m dot dQ by dt and W s is 1 by m dot dW s by dt. So, equation 1 apply to infinitesimally small control volume and that is how we derive the differential form of any governing equation, any kind of equation.

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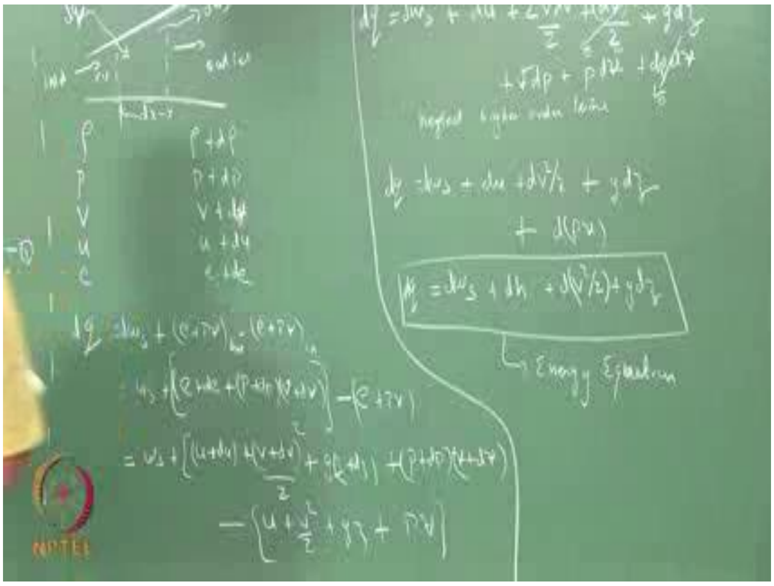

So, I have a control volume which is infinitesimally small, the dimensions is given, the characteristic dimension is dx. I have an inlet, outlet. This derivation is also true for 3rd dimensions as long as we take the 3rd dimension equation. So, let us stick to 1D equation. So, at the inlet the density is given by rho, pressure P, velocity, velocity v and what are the quantity, internal energy u and the total energy small e. At the exit, it is rho plus d rho P plus dP v plus dP u plus du and e plus de.

So, we write this equation, Q W s plus e plus Pv in out minus e plus Pv in W s plus, e. We substitute the out quantities, out are these quantities, so instead of, e plus de, e plus de plus P plus dP multiplied by v plus dv, which is the out quantity, minus the in quantity, which is e plus Pv.

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Pardon.

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W s, I expand this, e and e cancels out, I, we will expanded as u plus du plus v plus dv the whole square by 2 plus gz plus dz plus this quantity, which is pressure and volume P plus dP multiplied by v plus dv minus u plus v square by 2 plus gz plus Pv. So, Q equals W s. So, this u and this u cancel out, what is left is du. So, this v square and v square cancels out. So, you have 2v dv by 2 plus dv the whole square by 4 plus gdz. So, Pv Pv cancels out, you have v dP plus P dv and (Refer Time: 08:37).

So, I have expanded this and this and cancelled out all these terms.

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2, there is, this is, this is correct. dv square by 2, yes, this is dv square by 2, neglect your higher order terms. So, this goes, this goes, so you are, have left with Q equals W s plus du plus dv square by 2 plus gdz plus this is d of Pv. So, that is nothing but W s plus dh plus dv square by 2 plus gdz.

So, this is something that is applied to this infinitesimally small volume and even if it is 3rd dimension you just consider this differential as the third dimension. Here, the Q and ds is also the small differential because what is going in as Q is dW for a small differential volume and work that is obtained is W s. So, ideally this should be dQ and ds. So, this would be dQ and ds that would complete the differential form.

So, this is the energy equation in the differential form.

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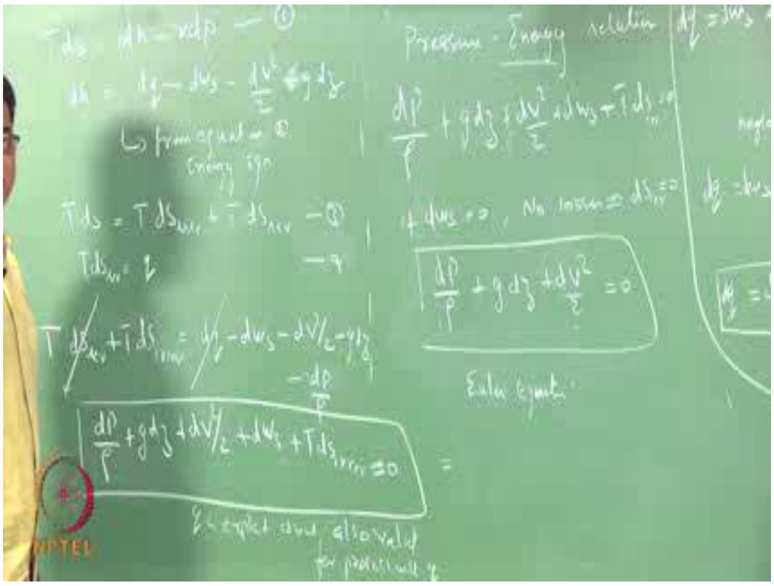

Now, we will proceed further. vdP, so this is, one of these are called Gibbs equation. So, dh we substitute from this equation. So, dh from equation 2, which is dq minus Ws minus dv square by 2 minus gz from equation 2, which is your energy equation.

Now, Tds contains two components, two components are Tds irreversible plus Tds reversible and from the 2nd law we have Tds reversible equals q, the heat input, okay. So, substitute these equations, say 3 and 4 in the 1st equation, such say, this us your 1st equation. So, this is Tds reversible plus Tds irreversible equals dq minus dWs minus dv square by 2 minus gdz minus vdP, which I replace it as dP by rho. So, now, dq is this T irreversible. So, I cancel out this term. So, my, sorry, so I cancel out dq with the irreversible entropy change. So, what is left here is dP by rho plus gdz plus dv square by 2 plus dWs plus Tds irreversible equals 0.

Even though q is explicitly absent, this is also valid for process with q because we did not assume q to be absent here, we just cancelled out q with the reversible entropy change, okay. So, if we have a system with q input, this equation is still valid and this equation is, I rewrite this here, 0.

So, if shaft work is 0, dWs is 0 and if no losses implying d irreversible is 0, so entropy change due to irreversible process is 0. I lose these two terms, in the end what is left is dP by rho plus gdz plus dv square by 2 equals 0, which is your Euler equation, 1D Euler equation. So, the differential form of energy equation from the differential form of energy equation, we get these equations.

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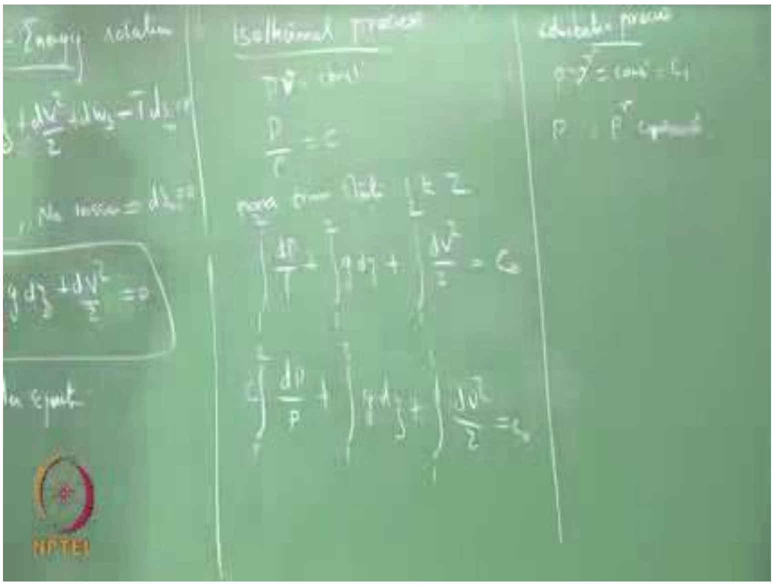

So, if the process is isothermal. So, if the process is, I have Pv equals constant, P volume is constant or P by rho equals constant. So, for process 1 to 2, process from state 1 to 2, you integrate this equation to get, so from process 1 to 2 dP by rho plus integral gdz 1 to 2 plus integral dv square by 2, 1 to 2 equals some constant where you substitute, instead of rho I substitute dP by P into some constant c plus this.

So, instead if the process is adiabatic, Pv power gamma equals constant. You substitute the appropriate value of P; substitute the value of rho in terms of P with this equation and get a different form of equation, okay. So, these are the energy equations for isothermal and adiabatic process.

For an incompressible flow the density is constant. So, it is convenient to take it out and you can integrate and get the Bernoulli's equation. So, this, this constant is different constant from this constant. So, this is c 0 this again is a different constant. So, this would be c 1 and you integrate this equation with the appropriate value of P.

So, what we are done today is the differential form of the energy equation and from which we have derived the pressure energy relation and we had seen some special cases. We will be using this equation further in our gas dynamic course. So, that is for the day.