Fundamentals of Gas Dynamics

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Week – 02
Lecture - 06
Tutorial 2

From the last class we discussed the Reynolds transport equation. We derived the mass conservation, momentum conservation, and energy conservation equation. So, those equations I will write it down here.

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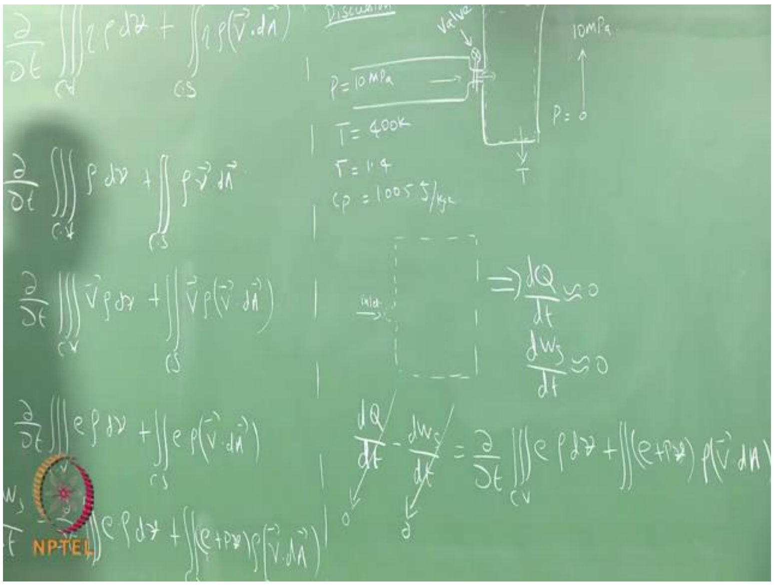

The material derivative of any property, as the time variation of that property within the control volume, plus the convective part, the property that is coming in and going out of the control surface, and the property is mass, we have seen the value of eta is 1, control surface rho into v dot d A. If the property is the force, it is essentially is sigma force, as the eta is now take the velocity as plus the control surface.

Control surface v rho v cross d A and if it is an energy, we have seen d e by dt as rho w by dt integral cv. The total energy per mass rho into d v plus control surface total energy per mass rho v dot d A, and we have also substituted this in the second law of thermodynamics, and we have obtained this as d Q by dt minus d w shaft by dt equals dou w T by del e rho d v plus integral control surface e plus p v, v is the specific volume v and rho density into v that d A.

Now we will try to understand these equations further. So, and we will try to do some examples, and try to get a feel of these equations. So, I have a question here. So, I have a long pipe which is going into a cylinder. So, this pipe has pressure of say 10 mega Pascal, and the temperature of around 400 Kelvin, let us assume l. So, your gamma is 1.4 your c p is 1 0 0 5 joule per kg Kelvin. Now the gas goes into the cylinder, there is a valve here.

The valve opens and the gas flows in, so the pressure moves from 0 to 10 mega Pascal. So, when it reaches 10 mega Pascal the valve automatically closes. So, what would be the temperature inside? How do we apply this equation to this problem? For this thing first we have to identify the control volume. So, our control volume is the cylinder. I am re drawing that here. So, this is in, and there is no outlet. So, there is just 1 inlet. So, I apply the energy equation or the conservation of energy equation from the.

\So, application of Reynolds transport into the second law of thermodynamics we will give you this result which we have derived in the other, class last class and we are going to apply this there. So, d Q by dt minus d w s by d t. So, d w by dt contains 2 components 1 is the shaft work d1, and the force d1 by the surroundings to push their into the control volume. So, that is the pv which we have seen in the last lecture.

So, dou by dou T integral control volume e rho d v plus control surface e plus pv rho v dot d a. Now, let us assume there is no heat transfer, there is no shaft work. So, your q addition is 0, and shaft work is 0. So, these 2 terms vanishes. So, if these two terms vanish what is left here is, and your e is u plus v square by 2 plus g z into rho d v plus control surface u plus v square by 2 plus g z plus pv into rho into v bar dot d A equal 0.

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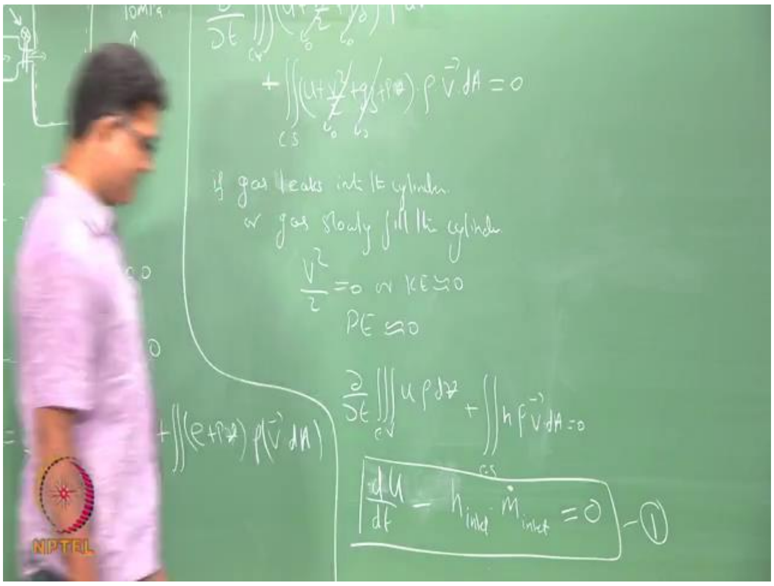

Now, if the gas is leaking if the gas, leaks into the cylinder, or if the gas slowly fills the velocity into the inlet crossing the control volume is 0. So, you could assume, or your kinetic energy is assumed 0, and you can also assume your potential energy to be 0, which essentially means this term is 0, this term is 0. So, you are left with dou by dou T triple integral internal energy rho d v plus h into rho v dot d a. now this is the internal energy per unit mass, which means that this is d by dt of your full mass. So, u is nothing, but your u into m dot here. I will rub this plus there is only 1 inlet.

So, this is the only inlet. So, you have h inlet into your m dot inlet. Now this is something that is going into the system. So, you could write this as a minus sign this goes into this system which is minus sign equals 0. So, that is the balance of energy; that is crossing this inlet and filling this control volume. Now the same set of analogy for the mass conservation would give me, mass that is going into the control volume minus my m inlet. So, that is my equation 2. So, I substitute equation 2 and equation 1.

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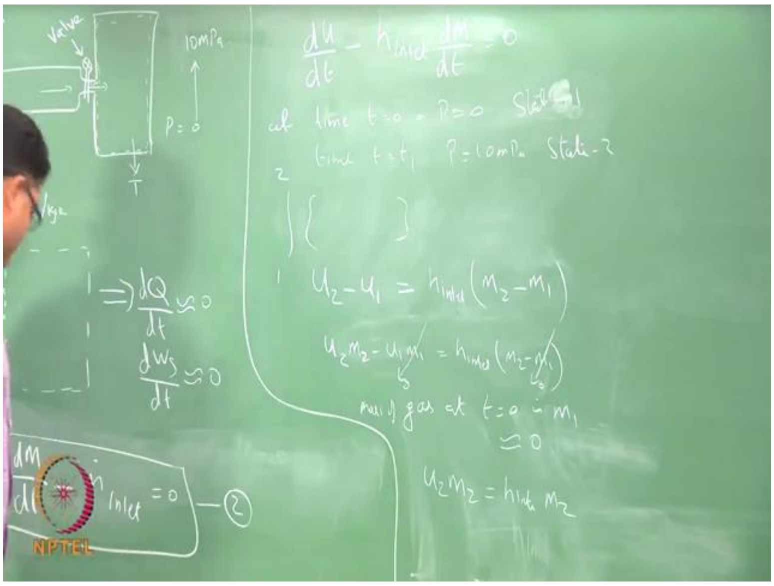

So, minus d u by dt minus h inlet into d m by dt equals 0. So, that is the balance of energy equation that is happening inside the control volume. Now, at time T equal 0, your pressure is p 0 p equals some value at time T equals some T 1, your pressure is 10 mega Pascal at which time the valve closes. So, it is from state 1 to 2 state, 1 to state 2. So, the system goes from state 1 to state 2 in time T 1. So, you are actually integrating between 1 and 2; the whole set of equation, which will give me u 2 minus u 1 equals h inlet into m 2 minus m 1.

The enthalpy of the fluid that is crossing this control surface is always the same, because you have a constant supply of 10 mega Pascal 400 Kelvin, yes. So, that is always constant. So, u 2 is u 2 into m 2 into minus u 1 into m 1 h inlet introduction m 2 minus m 1, what is m 1. m 1 is the gas at T equal 0. So, the gas at mass of gas at T equals 0 is here m 1 which is 0, because initially the cylinder was empty which means this is 0 this is 0. So, your u 2 m 2 equals h inlet into m 2. So, now, you have a simplified equation for this case which we are discussing, which is u 2 equals h inlet.

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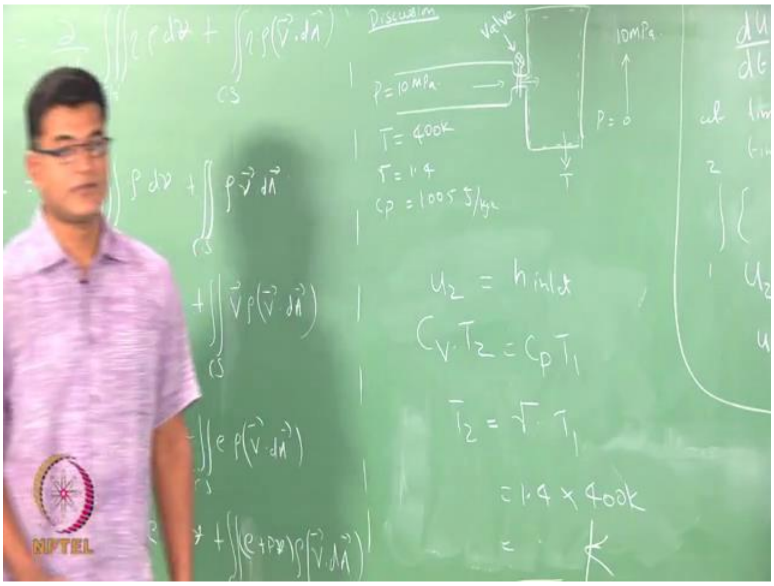

What is u 2 Cv T 2, h inlet is this value, which is your c p say T 1 or c p, c p temperature at the time, when the gas pressure is 10 mega Pascal inside the cylinder, and this is the temperature at which the fluid is crossing the control surface. So, your T 2 is nothing, but gamma into T 1, which is 1.4 into 400. So, that is yours. So, that is how you apply these equations to this flow. We will have few more problems. I will rub this.

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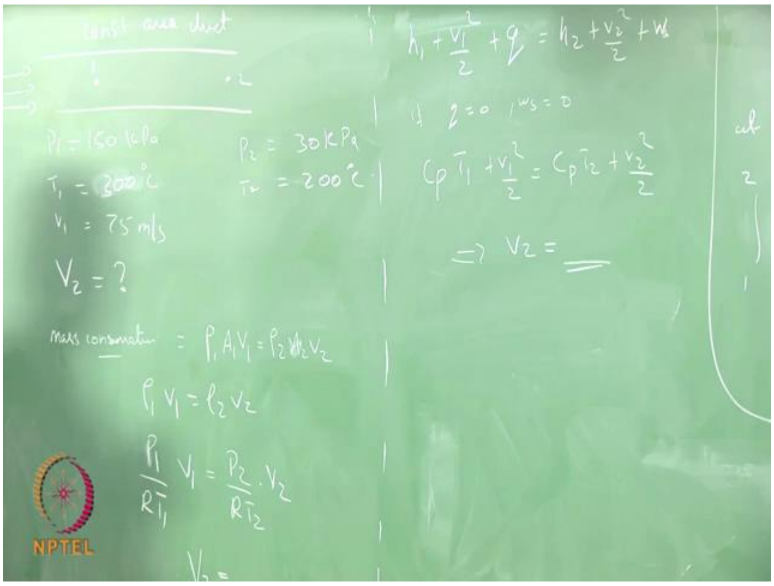

I will discuss question 2, I have a long pipe. So, the fluid is going from station 1 to 2, the pressures p 1 T 1 and v 1 p 2 T 2. So, let us put 150 kilo Pascal, some T 2 is. So, I have a long pipe between station 1 and 2 I have to evaluate my velocity change. So, I have pressure p 1 T 1 and v 1 at location 1, and pressure and temperature at location 2 what is your v 2. So, mass conservation equation is of the form rho 1 a 1 v 1 equals rho 2 a 2 v 2, your area is same. So, it is rho 1 v 1 equals rho 2 v 2.

Now how will you find rho 1? You could use your ideal gas equation, which is p 1 by r T 1, into v 1 equals v 2 by r T 2 into v 2. So, you know all these quantities; p 1 T 1 p 2 T 2 and v 1, from which you could find your v 2. or you could use the energy equation, which gives me h 1 plus v 1 square by 2 plus q equals h 2 plus v 2 square by 2 plus w s. If q equals 0 and w is 0 I could write this as C p T 1 plus v 1 square by 2 equals c pt 2 plus v 2 square by 2 from which you could find your v 2. So, if it is a different gas other than air you use the appropriate value of the c p and you find your v 2. So, we will go the next.

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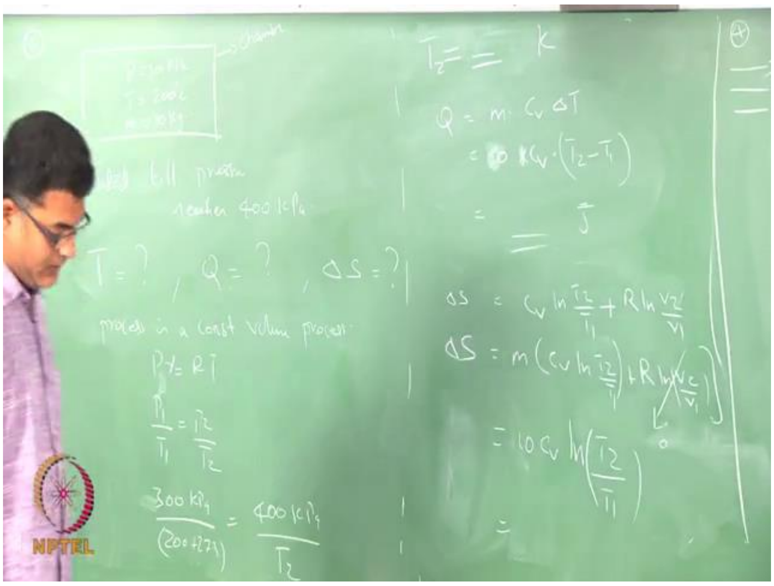

So, I have a chamber, I have a pressure and a temperature here; say some 2 hundred degree Celsius and say 10 kilo Pascal. For example, I have s 300 kilo Pascal, the mass that it contains is. So, let say 8 KGs. Now the chamber is heated till pressure reaches 400 kilo Pascal. Now, find your t, find the q required to heat this chamber two the pressure 400 Kelvin, find your entropy change. So, how do we do this? First we identify the process is constant volume process. So, the ideal gas equation; p 1 p equals pv equals r t. So, at between state 1 and 2 this would become p 1 by T 1 equals p 2 by T 2. So, now, you have p 1. Now you have p 2 you have T 1 find T 2. So, p 1 is 300 kilo Pascal by 200 plus 273 Kelvin equals p 2 is 400 kilo Pascal divided by T 2. So, you can find your T 2.

So, what is q? It is a constant volume process. So, your mass and the cv into delta t. So, mass is 10 multiplied by the appropriate cv into T 2 minus T 1. So, that should give me the mass the heat that is supplied in joules, this would be in Kelvin. Now what about delta s, cv ln T 2 by T 1 minus r ln v 2 by v 1, is this a minus or plus, it is a plus that is these specific entropy change. So, if you want in the total entropy change, you multiply this with m which is ln T 2 by T 1 plus R ln v 2 by v 1, since v 2 by v 1.

Student: Instead of v 2 (Refer Time: 28:59).

It is a constant volume process.

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So, this would get cancel out. So, what is left here is this. So, since v 1 and v 2 are same, this would cancel out and this is what is left is this. So, that would be your? So, m is 10 appropriate cv ln, whatever value you get for T 2 plus T 1.

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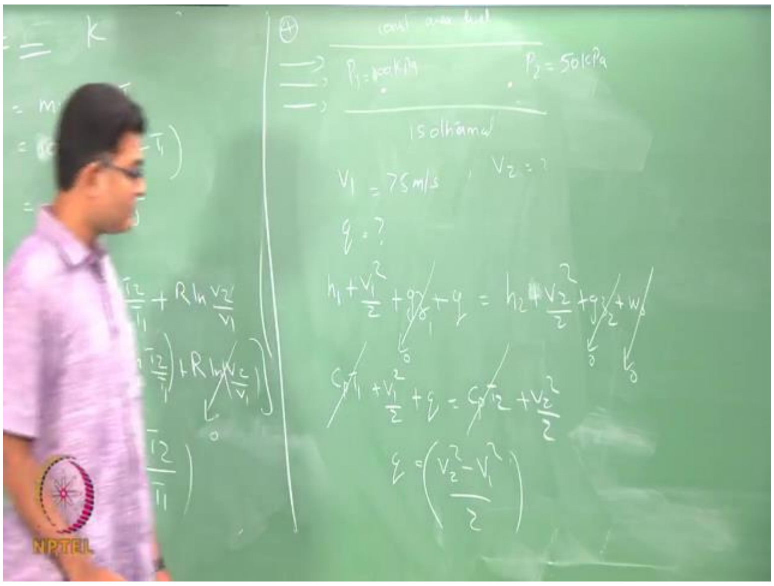

I have a constant area duct kept isothermally, a fluid that is going in. So, the pressure at 1 is 2 hundred kilo Pascal and the fluid goes to station 2, where the pressure is, say let say fifty kilo Pascal what is the velocity at station 1, velocity at station 2, what is the q change, let say. So, you apply your energy equation. So, you have 1 exit and 1 entrance.

So, your energy equation is h 1 plus v 1 square by 2 plus g z plus q equals h 2 plus v 2 square by 2 plus g z 2 plus w s. So, there is no a data change. So, g z minus g z 2 0, there is no shaft work. So, ws is a 0 it is an isothermal process. So, your c p T 1 is same as your c p T 2. So, those 2 terms would also cancel out. So, your q is nothing, but v 2 square minus v 1 square by 2. Now, what is v 2? You could use the mass conservation equation, to area cancels out.

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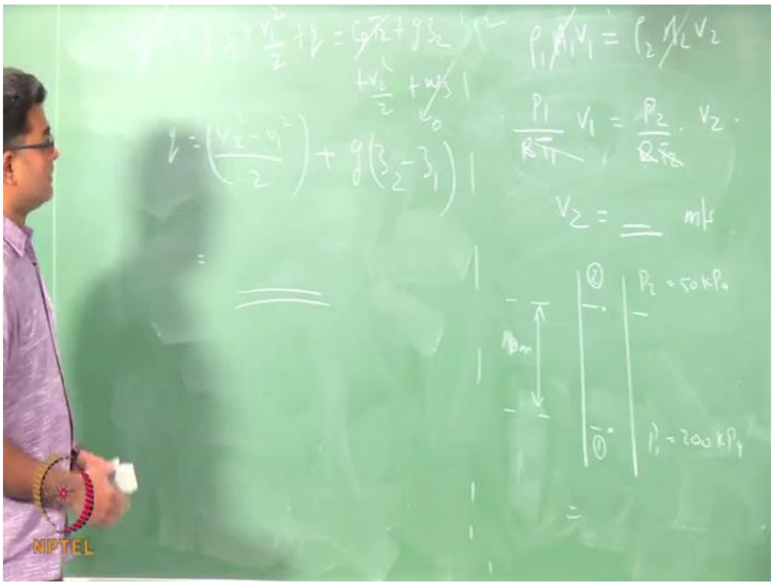

So, your v 2 is. So, your rho 1 you substitute from the ideal gas equation p 1 by r T 1 is your rho 1 v 1 equals p 2 by r T 2 into v 2 T 1 and T 2 are same. So, this cancels out. So, you could get v 2 you know p 1 you know p 2 you know v 1. So, v 2 can be evaluated in meter per second substitute that, and here you would get your heat change.

Now, if the same system is vertically up going from 1 to 2 p 1 is same p 1 is 2 hundred kilo Pascal and p 2 is fifty Pascal; the same problem except that there is a data change. So, what do you do, you have to include the data change the potential energy change. So, your h c p T 1 plus g is z 1 plus v 1 square by 2 plus q equals c p T 2 plus g is z 2 plus v 2 square by 2 plus w s. So, w s is assumed 0 c p T 2 and c p T 1 are same, because the temperature is same because it is an isothermal condition. So, your left with q is v 2 square minus v 1 square by 2 plus g is z 2 minus z 1. So, you need the distance between these 2.

So, let say that is 10 meters, v 2 you can get it from the same process. We have induced anything here. It is just the mass conservation and the ideal gas equation those these 2 are going to be the same, the v 2 is going to be same. So, this quantity is going to be same plus some addition. So, that would be your? So, potential if you have a potential the heat exchange is going to be this much. Now let us look at something similar.

So, what we are discussing here is, just the mass conservation and the energy conservation, and the momentum conservation. So, these are some of the examples, some of the areas where you could apply these equations which we have derived in the last class. So, another location or another case is, something called rocket engine. Say suppose you have an oxidizer here, you have a fuel here.

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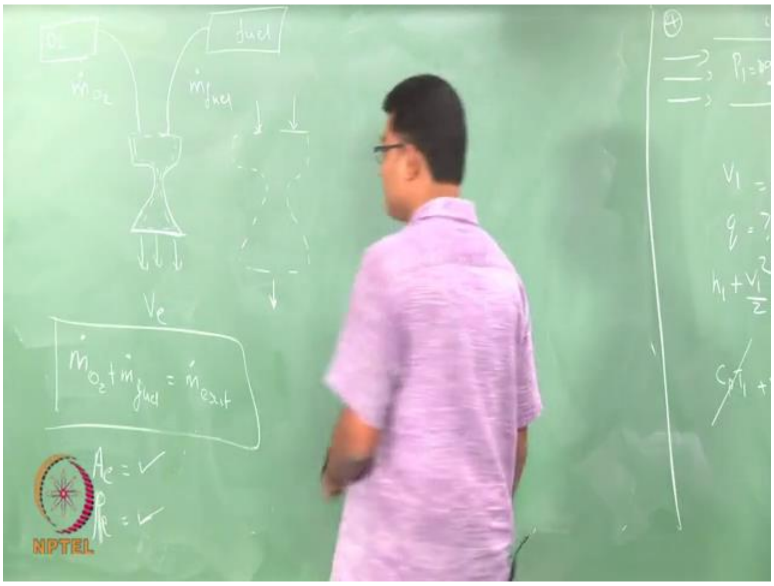

This goes into a rocket engine. So, the oxidizer is pumped into the rocket engine at some mass flow rate, and the fuel is again pumped into the engine at some mass flow rate. Here to find what is the exit velocity. So, this would be your control volume. So, the control volume is your rocket engine. You have 1 2 inlets and 1 outlet. So, the mass that is going in should go out.

So, the mass of o 2 plus mass of fuel should be your mass that is going out of the exit. So, you apply your. So, if you know your velocity with which it moves in, or density, the area of the nozzle exits. So, if you know area of the exit, and the density at which the fluid is coming out of the nozzle, you could find the velocity of the exit. So, you could put numerical values to these quantities and do the problem yourself.

I think we will stop this tutorial lecture with this. So, what we have discussed this week is, the thermodynamics, applying the second law of thermodynamics to the Reynolds transport equation, then applying the Newton's law to the Reynolds transport equation, we derived some equations energy equations and the mass conservation equation, we applied that to some of the some of the cases that we know off. So, you could expect similar numerical problems in your assignments.