Fundamentals of Gas Dynamics
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Week – 02
Lecture – 04
Conservation Equations

This lecture we are going to derive the mass conservation, momentum conservation and energy equations from Reynold's transport equation what we had learned in the last lecture.

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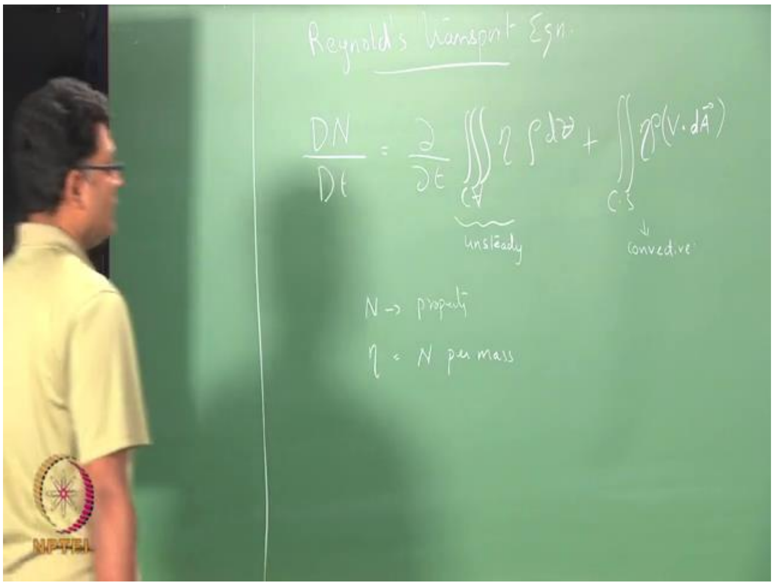

So, I start with Reynold's transport equation, which relates the material derivative of any property N in the following fashion, which contains two terms, the unsteady component over the control volume eta rho dV plus the convective part integrated over the control surface eta v dot da into density. So, this is the unsteady component, this is the convective component and N is the property, that we are trying to find, property of interest and eta as N per mass, the property per unit mass. So, if we are trying to find the transport of mass, N is the mass itself. So, eta is mass per mass or this is 1.

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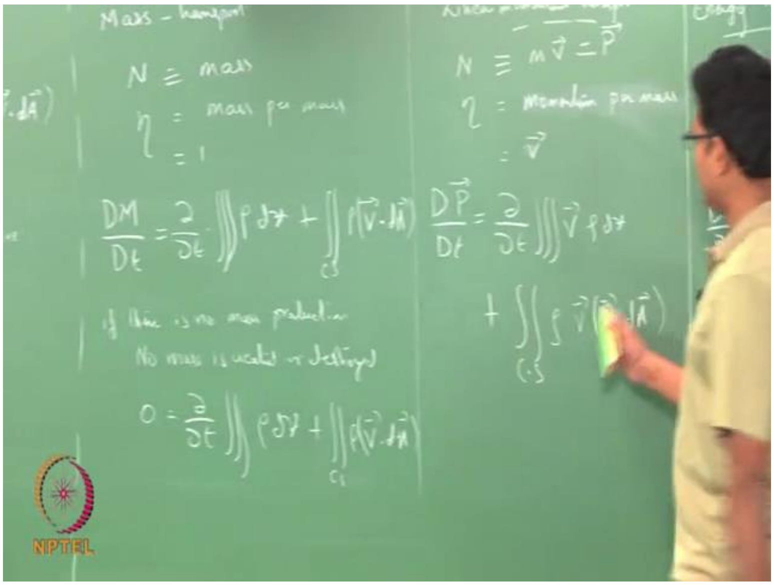

Now, if we are trying to find linear momentum, N is the linear momentum, which is m into v, v is the velocity and eta would be momentum per unit mass, which is v. So, this is per mass.

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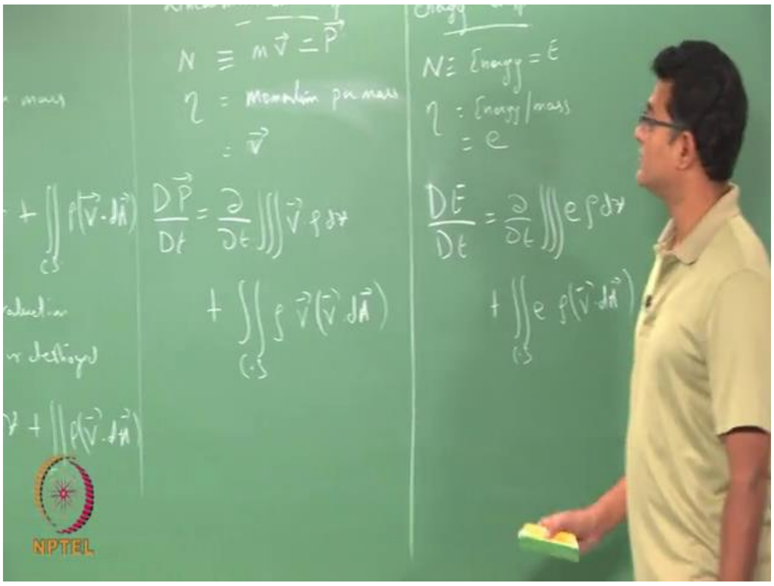

Now, if you are trying to find the energy transport, N is the energy, total energy for the entire mass and eta is per mass. So, this is indicated by capital E and per mass is indicated by small e. We will discuss what exactly that in few minutes from now.

Now, let us substitute these values in the Reynold's transport equation and see what happens. So, the mass transport dM by dt is dou by dou t, eta is 1. So, it is triple integral rho d volume plus the double integral rho into V is the velocity dot d a. So, if I substitute eta equals v here, the rate of change of momentum, so let us substitute this as, let us call this as P, P is a vector. So, the momentum vector is transported in the following form, dou by dou t triple integral, instead of eta now we substitute v, so it is v vector rho d v plus surface integral rho v v vector dot d a.

When it comes to this, the d material derivative of energy or energy transport is give as dou by dou t triple integral small e rho d v plus the surface integral e rho v vector dot dA. We will discuss in detail these two later, now we will stick to the mass conservation. If there is no production of mass or conversion of mass into energy and other things, we will conveniently assume no mass is created or destroyed within the control volume, then this is 0 equals dou t integral rho d v plus surface integral rho v dot dA.

Now, we will move forward. I am rubbing this for the time being, we will come back to this in the short while.

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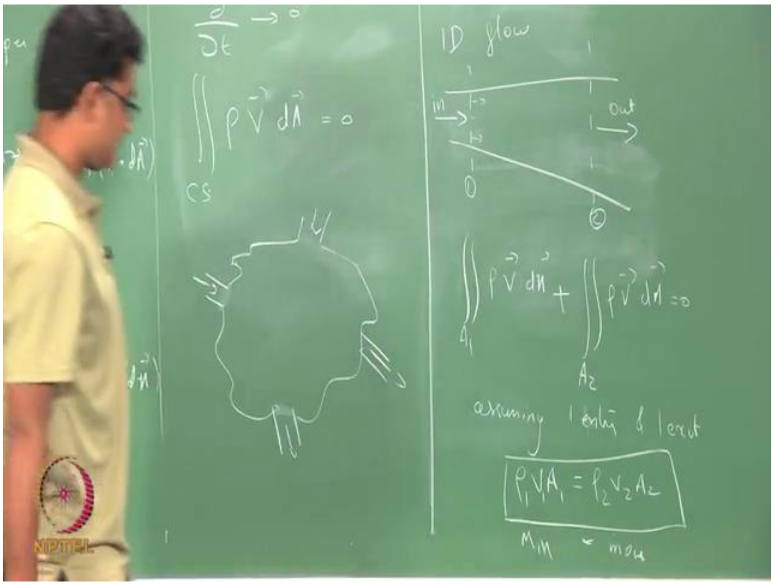

Let us, if flow is unsteady or the process is unsteady, process is steady, sorry, the process is steady, then your time derivative is 0, which means, that the surface integral rho v dot is 0.

So, what do I mean by that? If I have a control surface, okay, assume 3D control surface and there is mass that is coming in, mass that is coming in, mass that is going out. So, if I integrate this quantity along the surface, the net value is 0. So, the addition of all the masses that is coming in is equal to all the masses that is going out. So, essentially what I mean is, mass that is coming in is mass that is going out.

So, if it is a 1D system. So, let us have a 1D system where there is a mass that is coming in, fluid that is coming in, fluid that is going out, which means, that velocity at any cross-section, at, at the cross section is same. So, the velocity here is same as velocity that is here, velocity is constant across the cross-section. So, what I can write here is integral for A 1 rho v 1 d A 1 plus integral A 2 rho d equal 0.

So, if that flow, the fluid that is coming in is considered the positive and that is going out as negative, then thus you can rewrite. Assuming 1D flow with two exit, one, one entry and one exit, then rewrite this as rho 1 V 1 A 1 equals rho 2 V 2 A 2. So, this is the 1D, one exit and one entry continuity equation or the mass conservation equation. So, there is this mass in equals mass out. So, rho A v where v is the velocity, rho A v is constant, so the mass contained in the control volume is constant. So, I can further write this. So, this is for a steady flow.

If it is unsteady, you should have the other unsteady part also into the equation.

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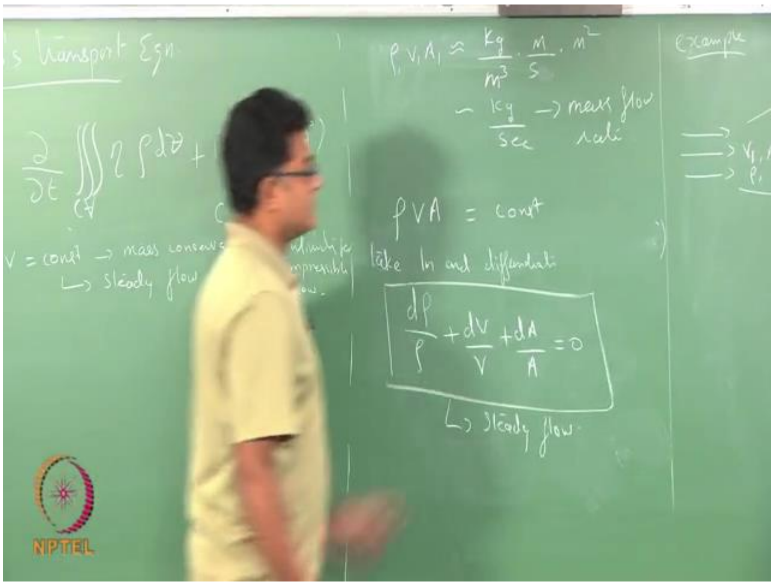

So, what is the unit of this density is Kg per meter cube, velocity is meter per second and area is meter square. So, this is Kg per second. So, this is mass flow rate, the rate at which the mass is flowing in any 1D system is what is given here as rho A, A 1. Now, again I write this equation, take the logarithm and differentiate, I could get this form though this is the differential form of the same equation which is applicable only for steady flow. So, this is also called as the continuity equation for the, for compressible flow for a steady. Again, remember it is a steady flow.

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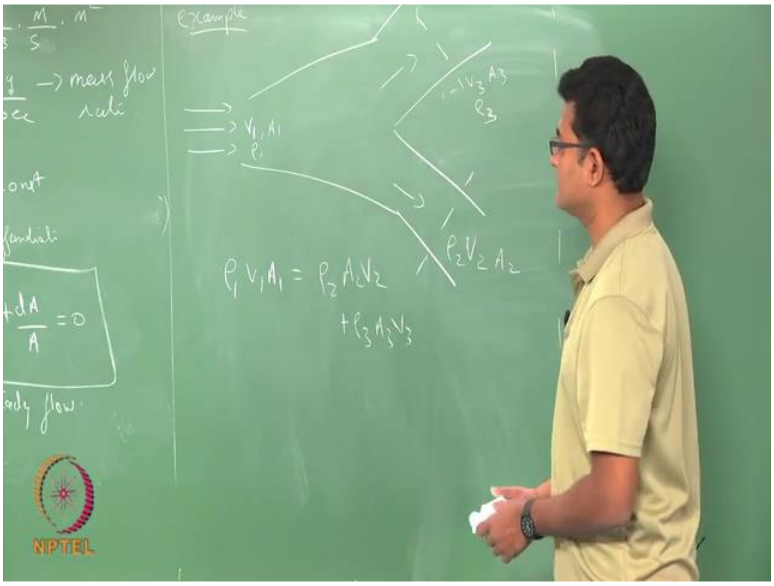

So, where do you apply this equation? So, if I have a system in which I have a flow, then this deviates into two. So, it is an example. So, there is a flow that is coming in and there are two ways and which are, the flow is taking a deviation, you could apply this equation. So, your rho 1 V 1 A 1 is equal to rho 2 A 2 V 2 plus rho 3 A 3 V 3.

So, we will see further on this after discussing the momentum equation and energy equation. So, I will write that equation here before rubbing it.

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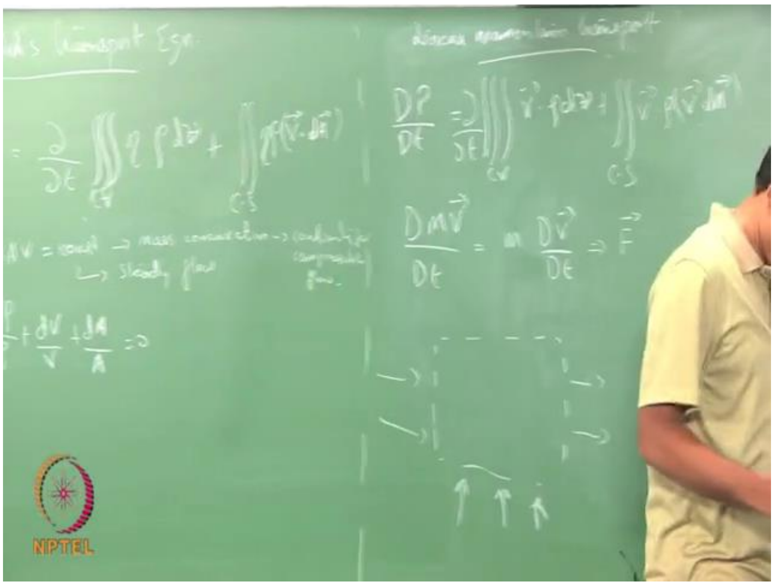

We will write the equation that we had written a few minutes back, equals dou by dou t of a control volume integral, surface integral, eta is v bar rho v bar d A. So, what is the rate of change of momentum? It is actually the force from Newton's 2nd law. So, your D m V by Dt as m DV by Dt, which is your force from Newton's law.

So, what you are trying to find here is the force balance essentially. So, if I have a control volume, an arbitrarily shaped control volume, there are forces that is acting on these control surface and forces that if work done by the control surface and away from the control surface or by the surroundings. So, you are trying to find how, that some of those forces are balanced by this particular quantity, which we had derived.

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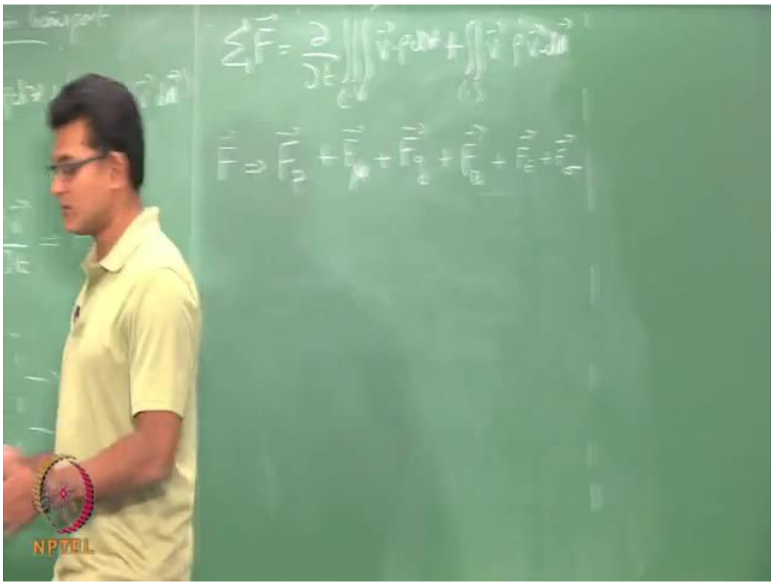

So, essentially you are trying to find this quantity, which is acting on the control surface. So, the sum of all the forces is equal to your unsteady part from the Reynold's transport equation and the convective part, which is the integral over the area rho V dot d. So, your F, we did not assume any form for the force, so this can be forces due to pressure, this can be forces due to viscosity, this can be forces due to gravity, this can be forces due to say, magnetic field, this can be forces due to say, electric field or any such thing that is you can also have surface tension force, all sort of forces, when you add should balance this equation. So, you have an unsteady part and you have a convective part.

So, we will leave it at that the momentum equation and we will go on to derive the energy equation and see how we will do few examples and see, how these are useful in evaluating the forces. So, we will write that equation here, sigma F V rho vector plus surface integral V vector dot rho V dot d A.

Now, we will try to find the energy equation. This is slightly more, we need slightly more discussion on energy equation than the mass conservation and force balance equation, which you might have already done in your fluid mechanics course.

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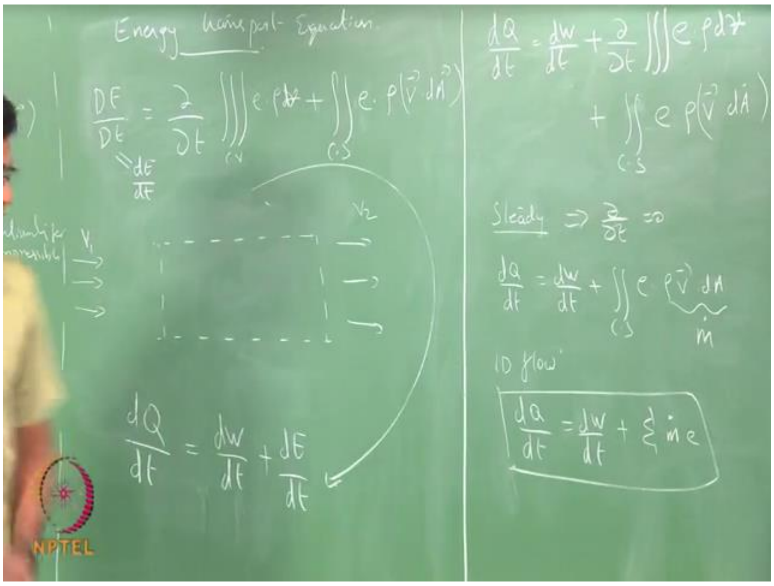

Energy transport equation, so we, so the Reynold's transport equation is, I have written with identifying eta as small e, which is energy per unit volume unit mass, eta is u e rho. So, now let us take a control volume; so, let us take a control volume. So, some fluid is coming in with velocity v. So, let us say v 1, some fluid is going out with velocity v 2. So, we have written the energy, the rate of change of energy within this control volume to be this from.

But second law of thermal dynamics tells us, delta Q by dt equals, let us write dQ by dt for the time being, let us not differentiate between delta Q and dQ for the sake of convenience. So, dQ by dt is dw by dt plus dE by dt. This dE by dt, we will get it from the Reynold's transport equation. This is again a material derivative. So, we substitute this here. Thus, is same as DE by Dt, both are material derivatives.

So, your energy equation from clubbing the Reynold's transport equation and the 2nd law of thermodynamics, the momentum equation is clubbing the Reynold's transport equation and the laws of motion, Newton's laws of motion. Here, it is clubbing Reynold's transport equation with the 2nd law of thermodynamics. So, you have dQ by dt equals dw by dt plus dou by dou t triple integral e rho d v plus surface integral c s rho v dot dA. So, this is general form. This is valid for three dimensions and we have not assumed anything here. So, this is valid for any kind of flow, adiabatic or isentropic or any kind of flow.

Now, we will take some assumptions and reduce this equation further. So, the 1st assumption is a steady because this course, we may, we will concentrate on steady phenomena that is happening in compressible flow. So, unsteady part actually brings in other interesting dynamics, which we are not going to discuss in this course most likely. Steady part implies dou by dou t equal 0. So, your dQ by dt is now dw by dt plus triple surface integral e rho v dot dA. And if I assume 1D flow, I can rewrite this as dQ by dt equals dw by dt plus sigma of this quantity. This quantity of rho v A is your mass conservation m dot, sorry, it is m dot e. Now, we will worry about dw by dt. So, that is the work done by the control volume. So, let us look at it more closely.

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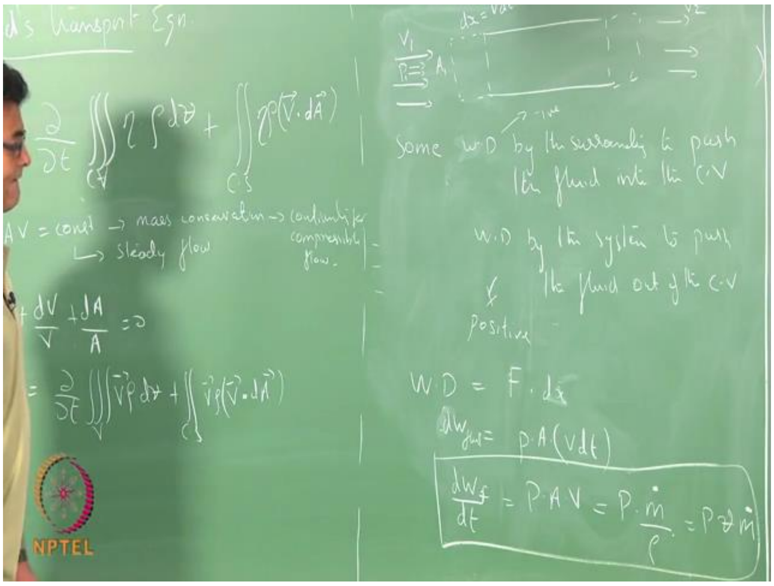

I have this control volume and fluid is coming with v 1, it is going out with v 2. So, there is some amount of energy spent by the surroundings to push the fluid into the control volume. Some force or some work done by the surroundings push the fluid into the control volume. Likewise, there is some amount of work done by the system or by the control volume to push the fluid out of the control volume. We take this as positive with (Refer Time: 30:09) work done with a positive sign and this is with a negative sign.

Now, what is this work done? So, if the fluid moves a distance dx in time dt, then the dx is v dt; likewise, here also. So, we will just concentrate on this particular aspect. So, what is the work done? Work done is force into dx, let us take 1D. So, instead of and now what is the force? If the area is A and the pressure is P, so P 1 A 1, the force is nothing, but pressure into area, and dx is vdt; v is the velocity v. So, the force is P pressure into area and dx is v into d t. So, that is the work done. So, that is your dW, dW by the fluid or I want to differentiate between two kinds of work done, which I will discuss it later.

Now, this is the work done by the surrounding fluid into, the surrounding fluid to push the fluid into the control volume. So, dW by dt, say, let us take fluid equals P into A into velocity, which I can rewrite it as P into m dot by rho, which I rewrite again as P into specific volume v, small v, into m dot. So, that is one component of the work done. So, I am rubbing these equations here, which I had written initially.

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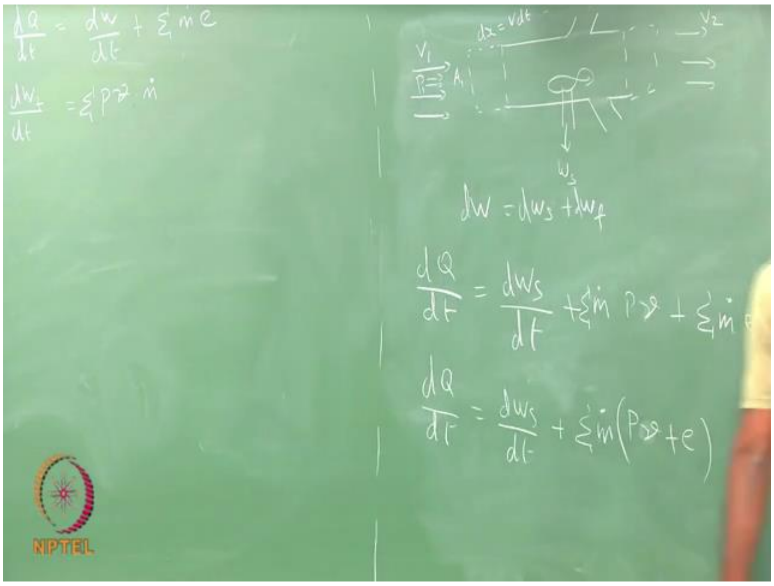

So, I have dQ by dt to be dw by dt plus sigma m dot e and my dW by dt contains two components, one is the work done by the fluid, which is rho, which is P v into m dot. Now, we will worry about the other component of work done.

If I have a propeller here, because of the motion of the fluid that will rotate and I extract some energy out of it that is typically denoted as work done by the shaft or the shaft work. Along with this my total work done is work done by the shaft plus work done by the fluid. So, my dW is d ds by d fluid. I substitute that here. So, my dQ may be, I will leave this here and substitute in here, dQ by dt is dW shaft by dt plus this quantity m dot PV. Again, there is a sigma here because there is an in and there is an out and it can be, you can have several ins and outs. So, I can have another in here or another in here. So, sigma of all those should give me this. So, there is a sigma here and then the other term, which is sigma m dot e. I rewrite this; PV is a volume specific volume, V plus e.

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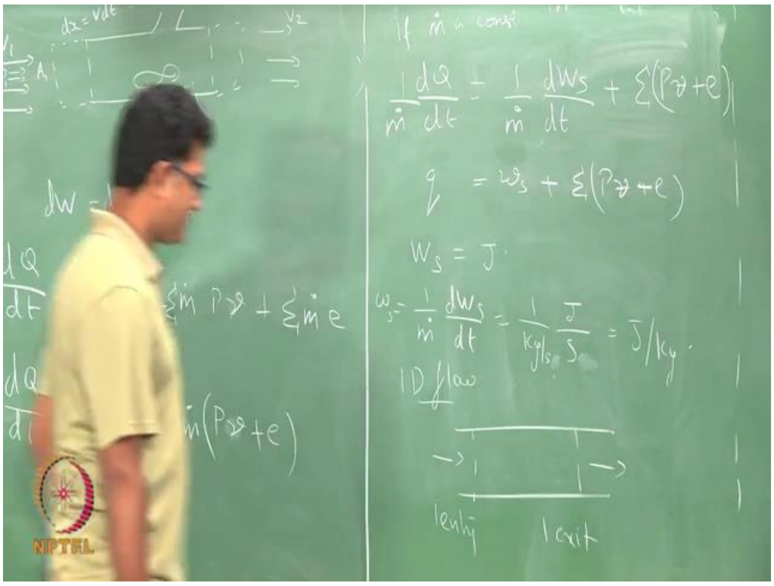

And between these exits and entries if m dot is assumed constant, constant with respect to time dQ by dt and 1 by m dot equals 1 by m dot equals dW s by dt plus sigma PV plus e. This I rewrite as small q plus small w s plus sigma P plus e.

So, if you look at the units of Q. So, what is a unit here? W s is Joule, so your 1 by m dot dW s by dt is 1 by Kg per second into Joule by second. So, this is Joule per Kg. So, that is unit of your W s. So, the unit of each of these terms is Joule per Kg and there is no seconds here. In some of the text books you would see this along with this equation, but the unit is still this. So, let us, to avoid confusion we will use without the dot here. So, it is the energy, specific heat supply, specific work done and the other term we will discuss with the other term, fine.

Now, we will see what is. So, if I have one entry and again one exit, exit and it is a 1D flow. So, the velocity at each of these points in one cross-section is same. So, there is no dependence on this direction. The velocity changes only in one direction.

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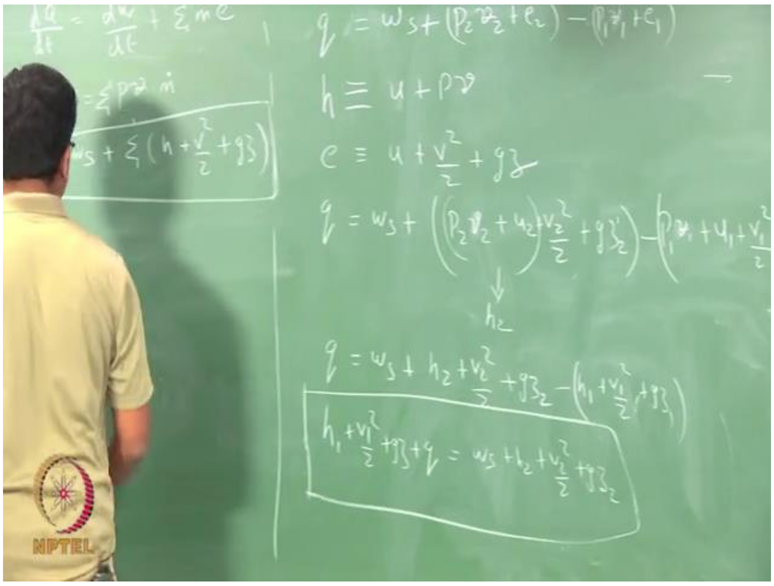

Now, if I write rewrite that, so q plus, q equals W s. Remember this is the heat input, this is the work done by the system plus P 1 V 1 plus e 1. So, this is one does into the system, the work done by the surrounding, so this will come with a minus sign. So, I will write the positive quantity first, which is P 2 V 2 plus e 2 minus P 1 V 1 plus.

Now, we have also defined enthalpy as u plus P v and e as, a total energy e consists of internal energy plus your kinetic energy plus your potential energy. Now, I substitute these two there, P 2 v 2; v 2 is u 2 plus v 2 square by 2. Let us leave the potential energy or maybe we can add plus g z 2 minus P 1 v 1. So, this is small v 2 plus internal energy u 1 plus kinetic energy v 1 by 2 plus g z 1. And this quantity is my h 2 from equation, from the definition of enthalpy. Q equals W s plus h 2 plus v 2 square by 2 plus g z 2 minus h 1 plus v 1 square by 2 plus g z. Now, I take this quantity here. So, this would be h 1 plus v 1 square by 2 plus g z plus q equals W s plus h 2 plus v 2 square by 2 plus g z 2.

So, this is one form of, another form of energy equation, which I can write like this; plus sigma h plus v square by 2 plus g z or in this particular form where I can write it for two sections. So, essentially this is in and out and energy associated with the fluid that is coming in and that is the energy associated with the fluid that is going out, fine.

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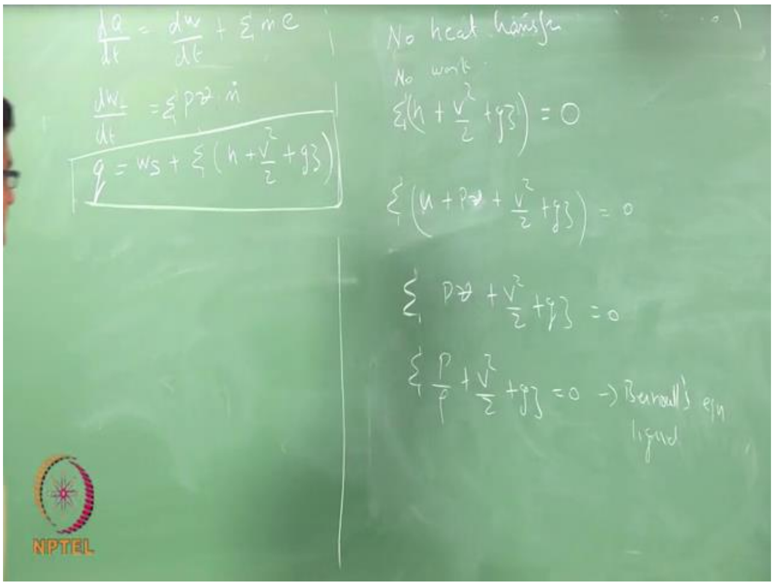

So, if the fluid adiabatic, so no heat transfer, no work, then you have h 1 equals 0 or from the original equation, sigma internal energy plus P v plus, equal 0. Now, if I consider there is no change in internal energy, I can write, say, as in the case of the liquid flow I can write this, which is P by rho plus v square by 2 plus g z equals 0, which is your Bernoulli's equation for, for liquids or with, when there is no internal energy change.

So, we have derived the mass conservation equation, energy conservation equation and momentum conservation equation. It is combination of Reynold's transport equation with the laws of motion and 2nd law of thermodynamics. So, we will be using some of these equations for further discussion. In our course we will, for the time being we will restrict ourselves to 1D flows and, and we will see, we will do some examples and we will see how this can be applied to some of the applications.