Fundamentals of Gas Dynamics
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Week - 11
Lecture – 46
Discussion on Prandtl-Meyer Expansion

So, we will continue with the tutorials on Prandtl Meyer expansion.

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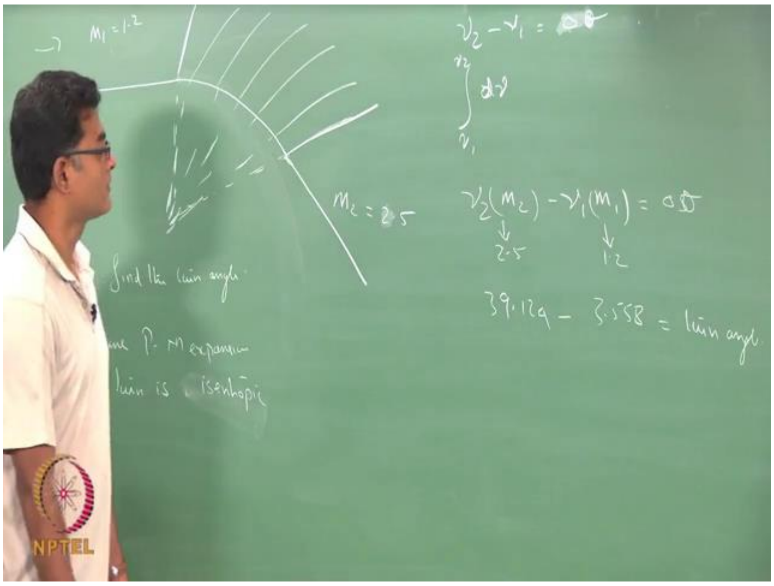

So, I have a flow, and it turns smoothly to some angle. So, this is a smooth curve. So, it is. So, let us take m 1 to the 1.2, and m 2 to be say 2.7, find the turn angle. So, we assume Prandtl Meyer expansion here; meaning the turn is, without entropy change or turn is isentropic. So, there would be a Mach wave here, and then there would be another mach wave here. So, there would be series of mach wave or eliminating from an imaginary point somewhere here. So, you make a small turn at every location. So, the turn angle is very small at each of these mach waves.

Now, from whatever we are learnt, the final mach number, because we have integrated the nu. So, the final values going to give are the important quantities to get your turn angle, because you have integrated from nu 1 to nu 2 d nu, and I do not care what is what happens in between. So, all I bother is the first 1 and the last 1. The first one and the last one depends on the mach number at the first 1 and last. So, all I get need to find is nu 2 and nu 1, to get the delta s, delta theta.

So, this is associated with 2.5, this is associated with 1.2 for gamma equals 1.4. So, it is a pretty straight forward question. So, all you need to do is, take a third quantity; 1.2 is something which we are just now seen to be 3.558 minus 2.5 is, 39.124, is my turn angle. So, now, the same question if I have a scenario like this, which is the flow that is going inside the surfers like that, and would have a compression wave m 1 is 2.5, the turn angle is same as what we obtained earlier which is, 35.6, what would be my m 2.

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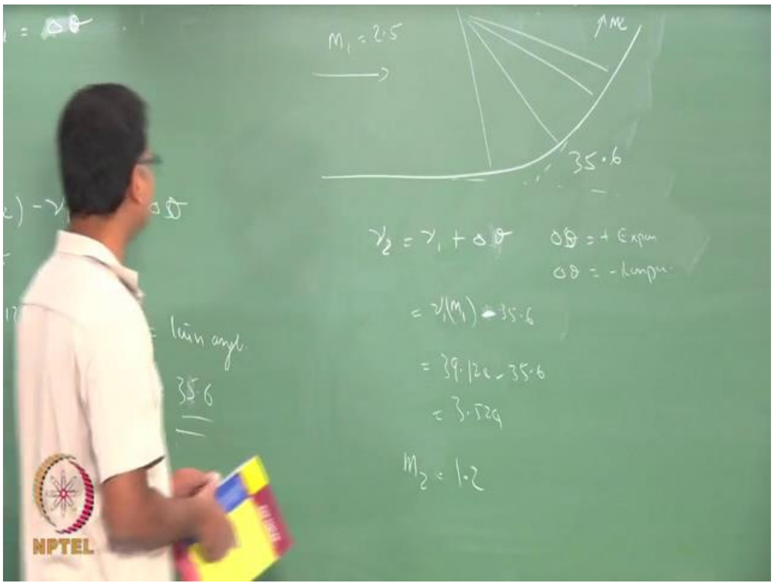

So, what would be my m 2? So, what I know is, mu 2 is nu 1 plus delta theta, and delta theta is positive for expansion, negative for compression. So, what I have is, nu 1 is associated with m 1 which is 2.5 plus delta theta is 35.6 minus. So, nu 1 is 39.124 minus 35.6, you would get 3.524. So, the mach number associated with nu of, this is 1.2. So, question 3; I have a turn which is 10 degrees, m 1 2.5 this is case 1.

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Case 2, again the turn as angle 10 degrees 1 at the shock turn, the other one is a smooth turn m 1 is 2.5. Find mach number after the turn, for both the case. So, in the first case we have a shock, in the second case we have a Prandtl Meyer compression.

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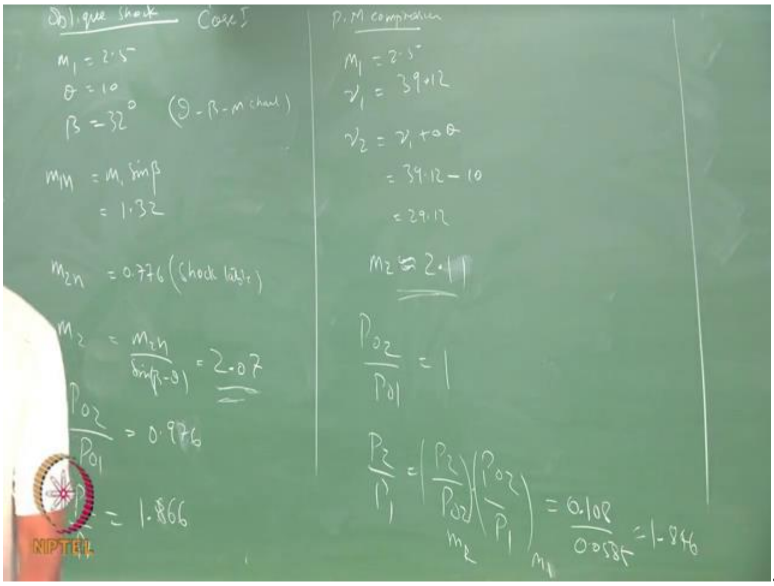

So, oblique shock m 1 is 2.5 a theta is 10. So, my beta from d beta theta beta chart 32.32 degrees, and I can find my m 1 n, which is m 1 into sin beta, then I would find m 2 n from the shock tables, normal shock tables, and I get m 2 as m 2 n divided by sin beta minus theta, which is 1.332 and the 0.776 and my m 2 is, 2.07 for Prandtl Meyer compression my m 1 is 2.5. So, my nu 1 is 39.12 nu 2 is nu 1 plus delta theta. So, 99.12 minus 10 degrees, this is 29.12, which in the table would give me m 2 is approximately 2.08.

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Approximately 2.11, the mach number is not very different from what you get from this. So, your P 02 by P 01 for this particular mach number 1.2. If you look at the tables; 1.32 P 0 P 02 by P 0, this thing is 0.9993, and P 2 by P 1 is going to be 1.5.

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1.32, 0.996 and this is 0.8 something. Whereas, what would be the P 02 by P 01 here and P 2 by P 1. This is going to be 1 because it is an isentropic flow you know the stagnation pressure is going to be the same, and P 2 by P 1 will depend on your mach number, which is P 2 by P 02 into P 02 by P 1. So, this will depend on your mach number m 2. This will depend on your mach number m 2. So, this is going to be P 02 by P 0 look at the isentropic tables f or m 2, which is 2.11, P by P 02.11 is 0.08, 0.108, and for m 1 which is 2.5, 2.5 P 0 by P by P 0 is 0.55, 0.0585. So, this is.

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1.846. Now we can extend this things to your under expanded nozzle, in the slide which I have shown here.

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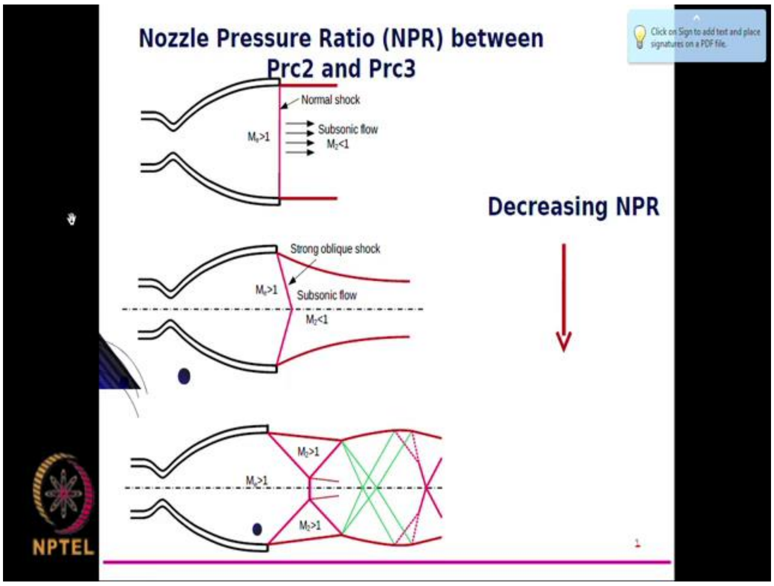

If I take the first picture, there is a normal shock that is sitting at the exit which is the case, where pb same as pb critical 2. Now if I decrease my pb further the shock is push down stream, then you have a scenario where a the shock reflects with another shock, and also form something called the macro flexion.

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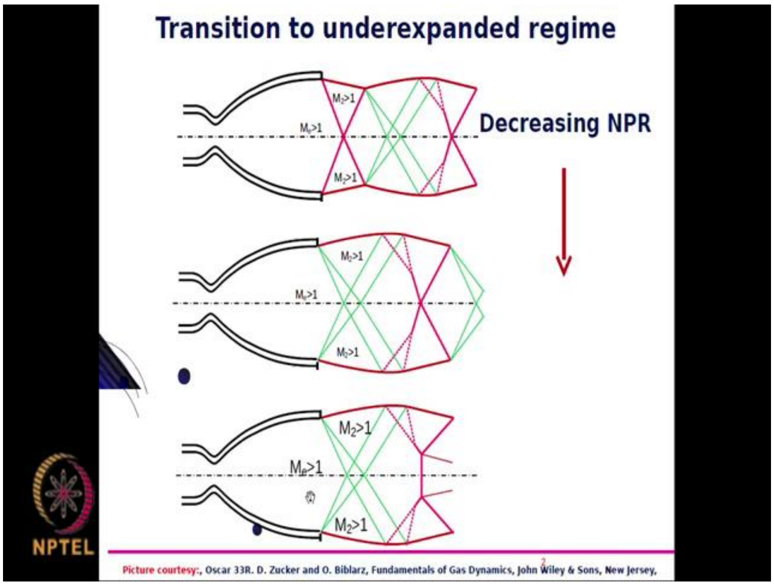

And if I decrease it further, then you move to the region called the under expanded nozzle. So, in the under expanded nozzle the flow would try to expand further, and that is what is shown in the second picture here. So, here you have an expansion fan. So, these expansion fans are formed at the edge of the nozzle, where there is a sudden expansion, the flow 10ds to turn. Turn this way, because of the expansion and then need us to come back to the other direction, to equalise the pressure. So, it forms structure like this, where you have expansion fan at the tip of the nozzle in. Where as in an under expanded in a over expanded case you have shock at the tip of the nozzle.

You could try to evaluate what happens if you know the pressure at the exit of the nozzle, and some pressure at the other location, you could see the deflection angle and see how the flow is turning in a direction which is not wondered, and it to bring it back to the position where, the flow direction is same as your access nozzle access. You need some kind of deflection happening with expansion fan as well as shock and the combination of those. So, we will try to do 1 problem related to that and see how it is being deflected. So, what I am going to do is, I have an under expanded nozzle.

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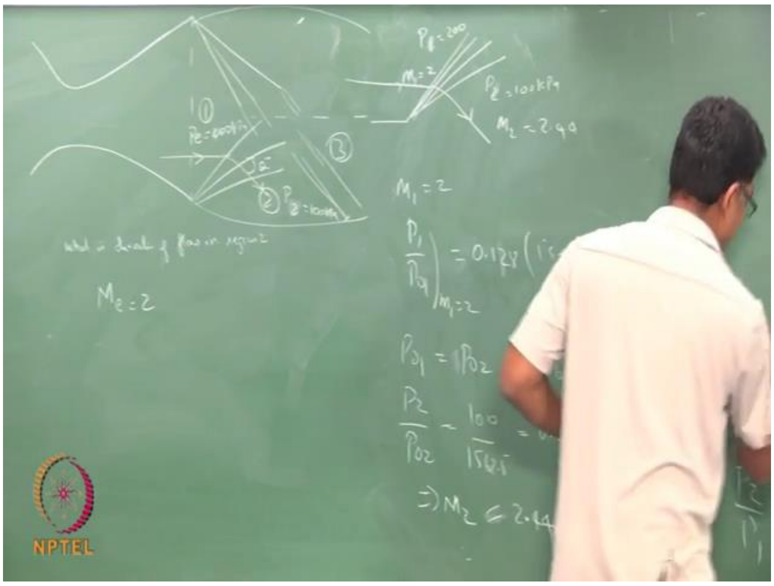

So, under expansion means if I have some kind of expansion happening after the nozzle, it is under expanded, and it is trying to expand it further 1s it goes out of the nozzle. So, this is region 1, this is region 2. Assuming symmetry there will be some other expansion happening here, till the region 3 and so on. So, we will do a small problem in which this pressure is 2 hundred, say kilo Pascal, and this is let us say 100 kilopascal. What is the direction of flow in region 2? So, instead of going it straight outside the nozzle, it does deflect to some angle.

So, what is the direction, what is the angle at which it is directing? So, the fluid comes and then it is deflected this way. So, what is this theta is the question. So, the given quantities are the pressures, and mach number at the exit is 2. Now you have to assume few things, because the mach numbers are changing on along this region. If you want to do the 1 d analysis what we have been doing. I will have to assume mach number 2, to be same everywhere, and then in the region, this particular region I have to assume mach number to be the same, and then only I can do this particular problem, if I want to stick with 1 d relation; otherwise you have to solve the full set of this are called momentum equation, which is our Navier-stokes equation.

So, what I will do is. So, I have an oblique shock, oblique expansion fan here m 1 as 2. This pressure is 200, this pressure is 100 kilo Pascal flow comes, and it makes a turn. What is the direction of we turn. So, P 0 we can evaluate. I can find pe by or pe 1 by P 01, from the isentropic tables or from the relation that which we should have by hearted, which is.

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0.138, from which you can P 01, now this is an expansion Prandtl Meyer expansion; that is an isentropic flow. So, your P 01 is same as your P 02. This is 200; 1 5 6 kilo Pascal. Now P 2 is given. So, P 2 by P 02 is 100 by 1 5 6 2.5, which would give me a m 2.

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6 4, from which I will get an m 2, which is?

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Around 2.44, now I know my m 2 to be 2.44, if I know my m 1 and m 2 you can always get the angle at which you it turned. So, the angle at which it turned is, nu 2 minus nu 1 equals delta theta.

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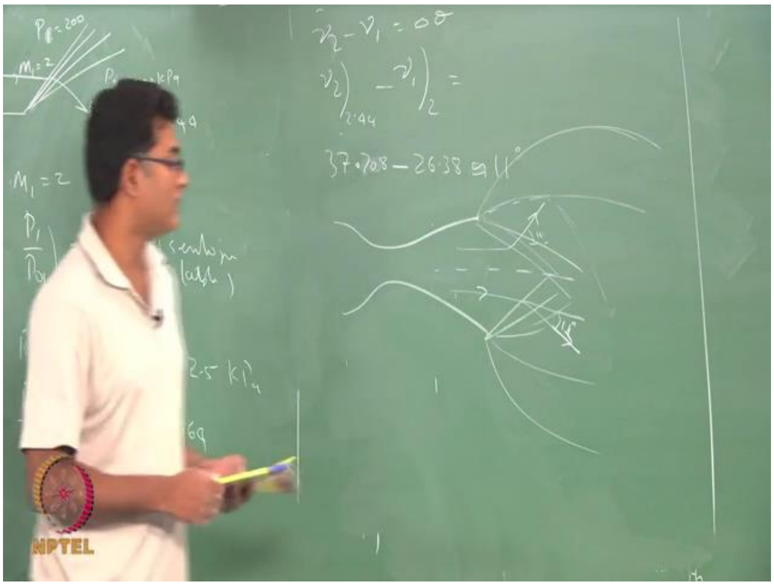

So, nu 2 for 2.44 minus nu 1 for 2 from the Prandtl Meyer expansion, how much?

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I want 11 degrees, in the gate. So, the velocity vector, as it encounters the expansion fan, turned a degree of angle of eleven degrees to this. Here also presuming at the centre line. Here also the flow would turn eleven degrees. So, it is going out in the, and that is 1 reason why most of the under expanded gates have you know, get with much larger than the nozzle diameter, and because of the periphery of the jet that is coming out there will be a deflection from here, and you will have series of expansions fans and shocks. Now we will end the discussion on Prandtl Meyer function with 1 simple problem on external flow, which is a flow faster, flat plate at some angle, at some angle of attack.

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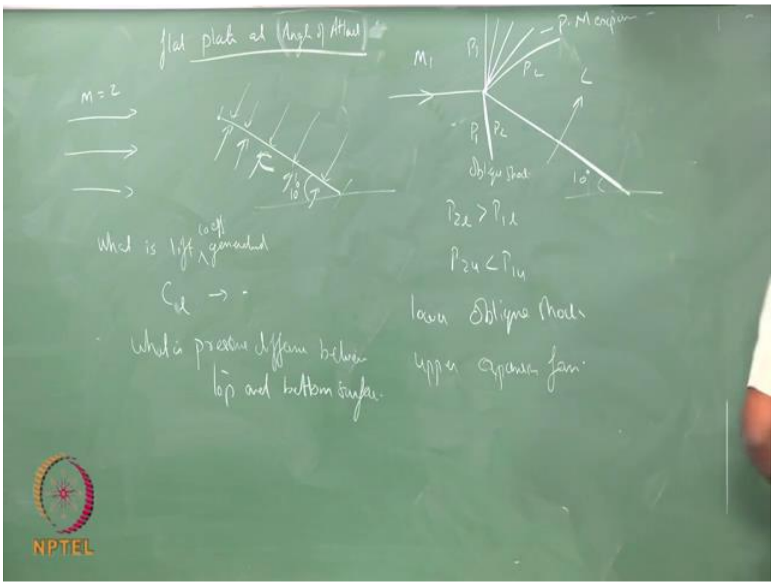

So, I have a flat plate, this is the flow direction, flat plate length is c, and it is facing a mach of say 2. So, what is the lift generator, or the lift coefficient, you are defined with cl, coefficient of lift. So, what is lift? Lift is nothing, but the pressure difference edges acting on the top and bottom of the plate. So, related question will be what is the pressure difference between top and bottom surface.

To do this problem we have to identify, that when you have a plate like this, when you have a flow that is approaching this particular thing. In the top surface there is a turn which is an expansion. So, there will be an expansion. At the bottom surface there is a compression. So, there will be oblique shock. So, this would be Prandtl Meyer expansion, this would be an oblique shock. Obviously, when there is a shock P 2 lower, is going to be greater than P 1 lower. Whereas your P 2 upper, is going to be less than your P 1 upper, because it is an expansion.

So; obviously, this pressure is going to be more and this pressure is going to be less, and hence there will be a lift that is generated, and the coefficient of that is what is being defined. So, all you need to find is the; P 2 and the lower surface and the P 2 at the upper surface and find the difference, and this angle is. So, the angle of attack is. So, let us take 10 degrees. So, this is 10 degrees. So, it is an easy problem, find you have m 1 given for the lower surface, oblique shock, upper surface expansion fan; m 1 is given. So, it is very easy to find the other quantities as we are done before.

Find for the lower surface yours theta beta m chart find m 1 n, from m 1 and theta find beta from theta beta m chart, find m 1 n find m 2 n from shock tables.

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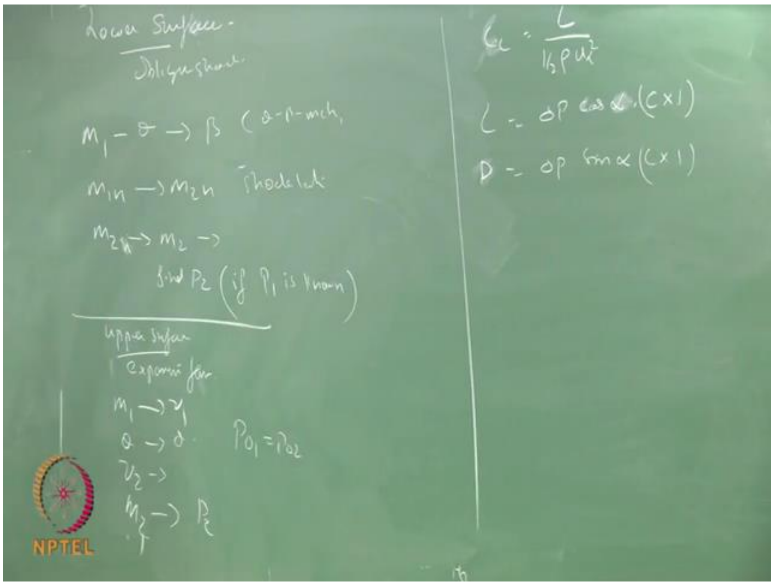

1s you have m 2 n find your m 2 from m 1 n, m 2 n. once you have m 2 find your pressure difference, pressure acting on this thing find P 2, if P 1 is known. Now for the upper surface, is expansion fan; m 1 is given, so you can find nu 1. Theta is given, so you can find nu 2, which will give you m 2, and from m 2 you can find your pressure difference P 2, because P 01 is same as P 02 for the Prandtl Meyer expansion. So, you can find your P 2. The difference between P 2 upper and P 2 lower would be your force that is generator multiplied by the angle, cos theta of this, so that would give you the lift. So, that is t he lift, and cl is nothing, but lift by 1 by 2 rho u infinite square, which you can write in terms of mach number.

So, your lift will be delta P cos, cos theta which is your angle of attack. So, I will use a different letter alpha. So, angle of attack is alpha equals 10 into the area, which is l into 1, which is the area that is across along the perpendicular to the board, and your drag force is delta op sin alpha c into 1. So, this is the force per unit area multiplied by area. So, this would be the lift force and drag force, and you can find the appropriate coefficients if you know the density and velocity.

So, that brings us to the end of the discussion on Prandtl Meyer expansion that also brings to the end of the course. So, what is discussing the course is the fundamental of gas dynamics, where we are discuss isentropic flow shocks and oblique shocks and expansion fans. So, this course is essentially for under graduation student, and we have skipped several advance topics, which is not meant for this particular course. So, whatever is been discuss is very fundamental, and how to problems using gas tables is what we have discussed in the course.

Thank you.