**Fundamentals of Gas Dynamics**
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**Week – 11**
**Lecture – 45**
**Prandtl-Meyer flow (cont\'d)**

In the last class, we had seen how a flow can be turned isentropically without having an oblique shock or the turning compression waves. Now, we have derived rigorously how the entropy is 0. Now, we are going to extend that and see how we can turn on the flow which is what we call as the expansion turns and we see how the waves have been generated. So, what I would do here is when you have a turn like this.

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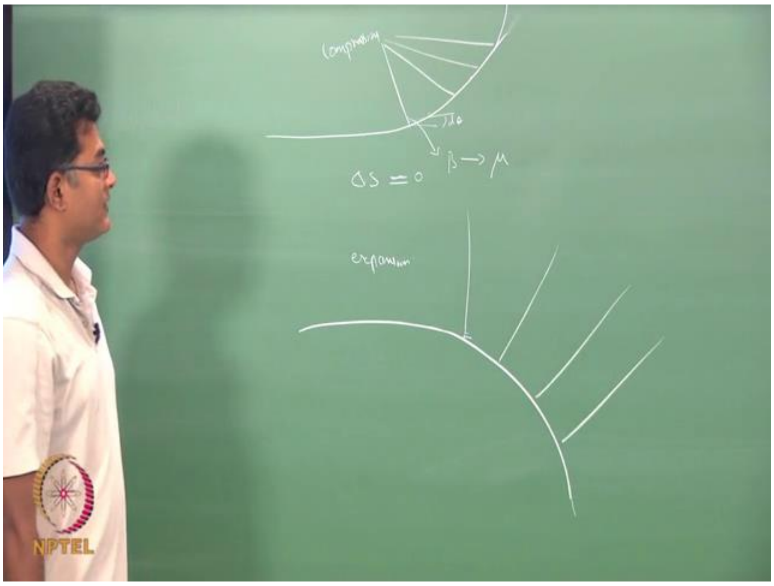


We had seen when the flow makes very small turn d theta at each of these location, we have derived how the entropy is going to 0, entropy approach to 0 as d theta goes to 0 and the shock angle approaches to your mach angle, this is what we had seen. Now, what we are going to see is if something like this happens and at every location there is a small turn d theta what happens to this flow. So, this is something; this is a compression, this is an expansion.

So, we will start by drawing the velocity diagram and see what exactly is happening in these 2 cases. So, we will instead of drawing these things, we will draw a small turn with 1 wave. So, I have a flow with a very small turn d theta.

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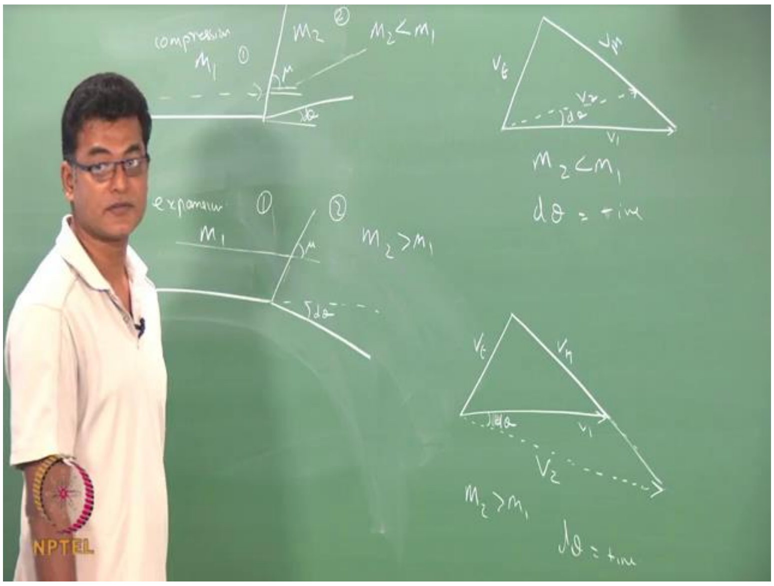


So, I have a wave that is approaching my mach angle, this is mu. So, the turning is very small turn angle which is d theta, likewise I have turned the other way round to this angle as a very small term d theta and I have a wave here, this is my mu. So, this is compression, this is expansion here. My M 1 M 2 and my M 2 is less than M 1, here I have M 1; my M 2 is greater than M 1 because it is an expansion process.

This is clear when you draw your velocity diagram. So, I have a V tangential, this is my V 1. So, before the wave is 1 after the wave is 2. So, I was cautious enough to say not shock here just a wave. So, before the wave is 1 after the wave is 2. So, I have a V tangential and then the resultant velocity and if I draw 90 degrees from there, this is my V normal, V 1 normal. So, in a compression wave the velocity would be because the turn angle is this way, the velocity resultant velocity V 2 is less than V 1. Whereas, when you come to this kind of turn. I have a V tangential V 1, 90 degrees from here is my V n, this is my V tangential and the turn is the other way round. So, I have a small d theta.

This velocity V 2 is larger than V 1 hence you are going to have M 2 greater than M 1, here M 2 less than M 1. So, there is an always increase in your velocity when you make a turn like this. So, in a compression wave this is typically assumed negative and this is assumed positive the importance of which we will see it later. For the time being is just the modulus of d theta in 1 case, we have an expansion another case we have a compression.

Now, we look at this particular triangle and try to see what we can get? How we can get this turn angle? If I know the mach number before and after the wave, can I get a turn angle is what we are going to derive, which is what we call as Prandtl-Meyer function.

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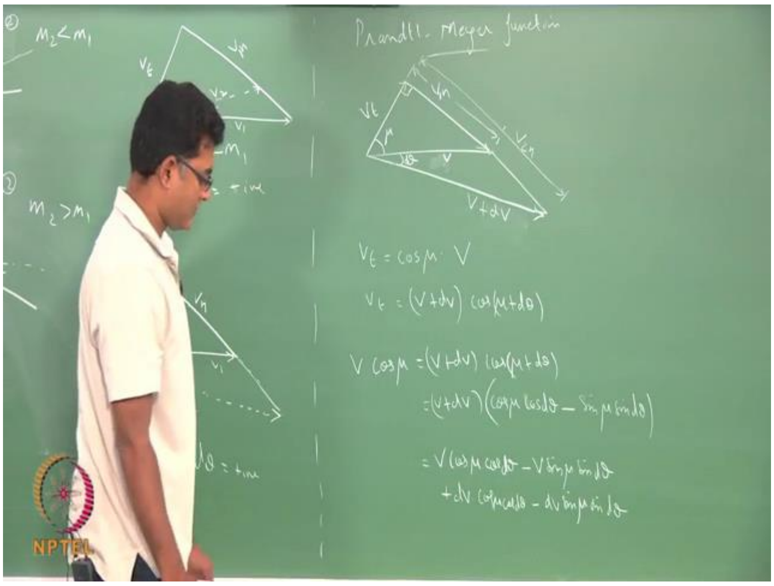


So, the difference in Prandtl-Meyer function before and after the wave will give you the turn angle. So, we are going to derive this. We look at this particular triangle I will draw that here and then try to deduce what is the Prandtl-Meyer function V tangential V n, this is V 1 and I make a small turn d theta. I get V 2 this is my wave angle or the shock angle as we have described in our oblique shock which I call it as mu because that is now our mach angle and this is d theta not theta.

So, let me check the notations. I would write V 2 as V plus d V and V is V 1 is V this is my 90 degrees. This is V 1 normal and this whole length is V 2 normal, V tangential is the same as we had seen in the oblique shock. Now, looking at this triangle, V 1 is before the wave or V is before the wave, V plus d V is after the wave. Now, from looking at this triangle we can write V 3 is cos mu of your V. So, it is just the cos of this angle, V t is also this angle and component under this. It is V plus d V into cos mu plus d theta. So, cos mu plus d theta is in multiplied by this is here V t, these are same condiment.

So, V cos mu equals d V cos mu plus d theta which I expand as the following cos mu cos d theta minus sin mu sin d theta from our basic tachometric relation, I expand this further V cos mu cos d theta minus V sin mu sin d theta plus d V into cos mu cos d theta minus d V sin mu sin d theta, I just expanded this brackets. Now, I will assume d theta is very small.

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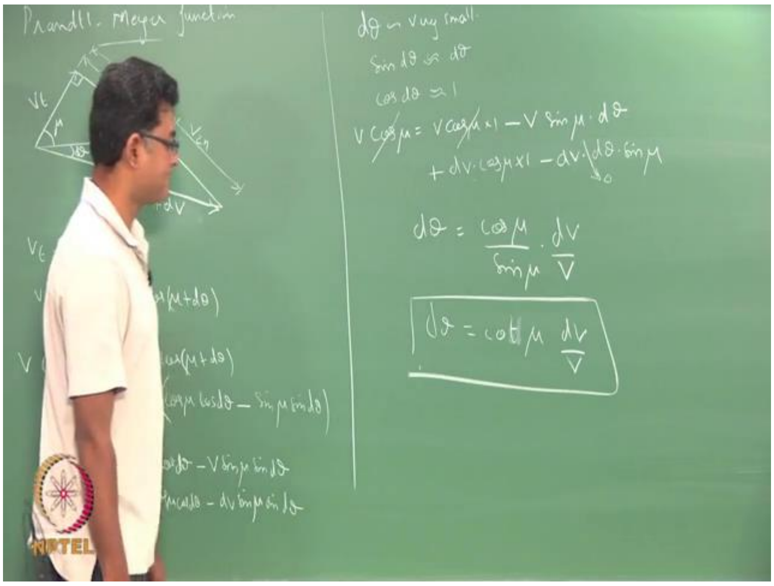


We have already assumed that when we decided this angle is mu. So, if I assume this my sin d theta is d theta and cos d theta is 1 and if I substitute that in here I would get V cos mu equals V cos mu into 1, my d theta is 1 minus V sin mu into d theta plus d V into cos mu into 1 minus d V into d theta into sin mu. So, I can cancel this 2 quantities and d V into d theta approaches 0 because both are d V and d theta are small quantities. So, multiplication of that would go to 0. So, we are left with these 2 quantities and you would end up with this particular relation d theta equals cos mu by sin mu into d V by V. So, I just taken d theta here and rearranged these terms these 2 terms. This is nothing, but cot mu and d V by V.

Now, we also know because mu is our mach angle, you have assumed this theta to be very small and this mu; this shock angle or the wave angle approaches mach angle.

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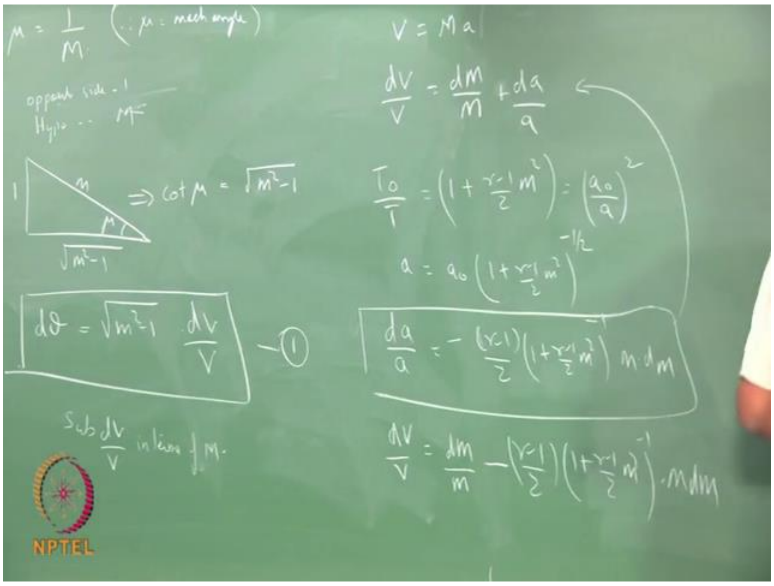


So, I can write sin mu equals 1 by n which essentials means, your opposite side is 1 and your hypotenuse is M. So, if I draw a triangle with hypotenuse M opposite side 1 and this is my angle mu, this is M square minus 1 which essentially means my cot mu is root of M square minus 1, I substitute this in that equation for d theta. So, my d theta is cot mu which is now M square minus 1 into d V by V. Now, I am going to substitute d V by V in terms of mach number. So, that is all what I am going to do. So, if I do that I will get the Prandtl-Meyer, so called Prandtl-Meyer function.

So, I go back to the continuity equation, differentiate that to get d V by V from that equation and we start from this particular relation where velocity is mach number into the velocity of sound I take globe and differentiate. So, I would get d V by V equals d M by M equals plus d A by A from isentropic relation between the stagnation quantity at the state and the static quantity at the state T 0 by T. I know it as 1 plus gamma minus 1 by 2 M square which is also my 0 by A square. So, I differentiate this and get d A by A and substitute it here. So, I get plus minus 1 by 2. So, I get d A by A as minus 1 gamma minus 1 by 2 into 1 plus gamma minus 1 by 2 M square power minus 1 into M d M. So, I have differentiated this term and I substitute that in here. So, I would get d V by V to be d M minus M minus this quantity which is gamma minus 1 by 2 into 1 plus gamma minus 1 by 2 M square to the power minus 1 into M d M.

Which after some rearrangement, after some algebra I can get d V by V to be 1 plus 1 plus gamma minus 1, 2 M square into d M by M.

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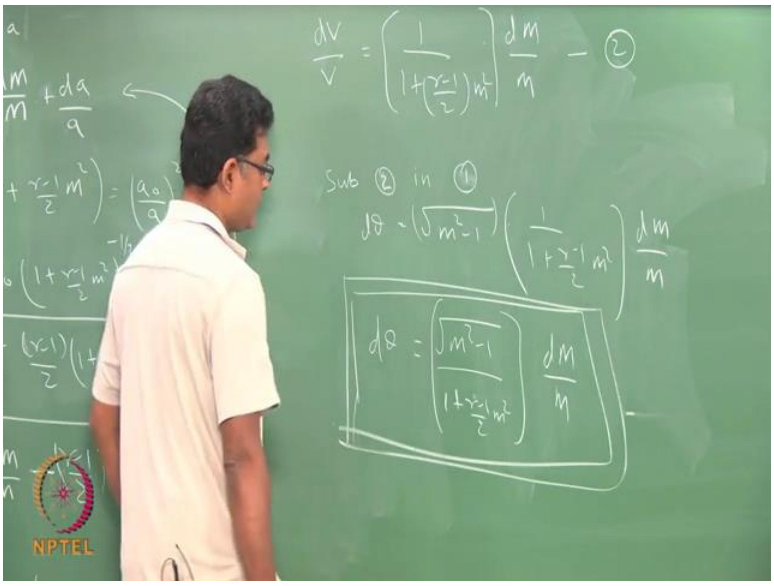


This is my equation 2 and I substitute 1 in 2 which in my d theta is root of 1 minus M square into d V by V is 1 by 1 plus gamma minus 1 by 2 M square into d M by M. So, I undergo the relation like this which is substitute 2 in 1, get d M by M. So, the integration of this is your, so called Prandtl-Meyer function.

So, all I do here is that particular one, this is the final equation. You integrate this you would get the Prandtl-Meyer function. So, what is important there is your d theta it is function of mach number and your gamma. So, I call this as d nu function of mach number and gamma.

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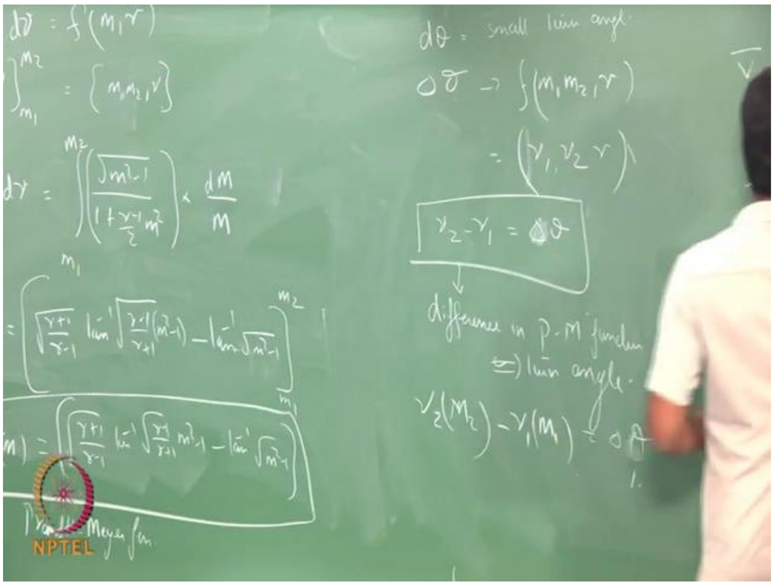


So, your nu integrated over 2 mach numbers would be a constant which will be a function of M 1, M 2 and gamma that is the idea. So, if I integrate this quantity between nu and to nu 2 d nu as integral M 1 to M 2 root of M square minus 1 divided by 1 plus gamma minus 1 by 2 M square into d M by M. So, this can be between nu and 2 nu, 2 which is I am writing this from whatever been done I am copying it from the notes, tan inverse gamma minus 1 by gamma plus 1 into M square minus 1 minus tan inverse root of M square minus 1 between the limits M 1 and M 2.

So, I have a quantity here between nu 1 and nu 2 in terms of M 1 and M 2 or I can write mu as a function of M, this particular quantity which is gamma plus 1 by gamma minus 1 tan inverse root of gamma minus 1 by gamma plus 1 M square minus 1 minus tan inverse M square minus 1. So, if I integrate this I would get this and I can use it between the limits nu 1 and nu 2 and this particular function is the so called Prandtl-Meyer function and if I know my nu 1, what I am talking about is my turn angle here.

So, my turn angle is d theta. So, what I have here is d theta is my small turn angle and if I have finite value of turn angle d theta that now, depends only on my delta M and delta M is a function of M 1 and which depends only on my M 1, M 2 and gamma which we have now defined as nu 1 and nu 2 and gamma. So, this function is nu 1 and nu 2 and gamma. So, my nu 2 minus nu 1 is my delta theta. So, this difference in this function is my turn angle. So, your difference in Prandtl-Meyer function is my turn angle that is the important thing and as we would have seen d T is minus for a compression and plus positive for a expansion.

So, your nu 2 is a function of M 2 and nu 1 is a function of M 1 which will give your d theta. So, if I know my turn angle or if I know my mach number after the turn and mach number before the turn I can get the turn angle. So, to demonstrate that or functions which is now is function of mach number alone nu.

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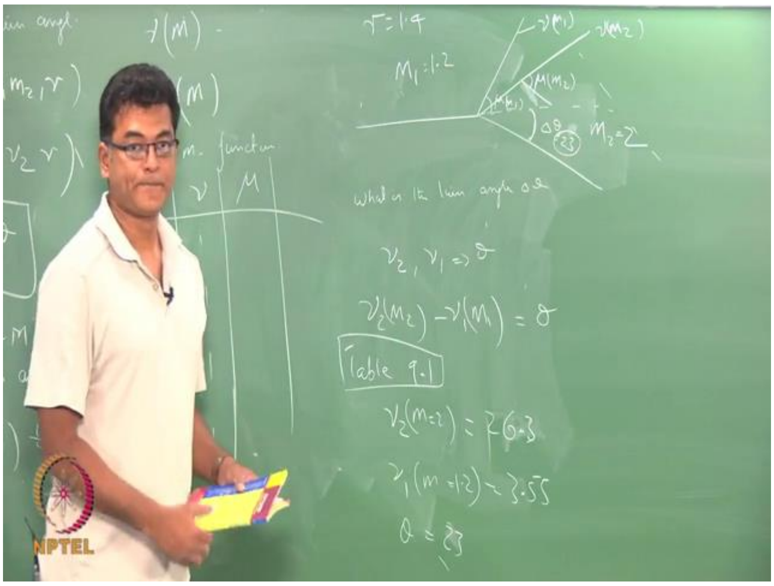


So, my mach function of alone, we also know that our mach angle is a function of mach number alone if I can tabulate this that is what you would see your gas table as your Prandtl-Meyer function. So, for a given mach number I can have mu and the mach angle. So, for mach numbers I would have different values of mu and mu and this is what you would see.

So, if I have A turn A finite value of 2 delta, theta is or my turn angle is which is, let us try to find the turn angle if this mach number is 1.2 and the final mach number is 2, M 2 is 2 what is the turn angle delta theta. So, how will you need to do is find nu 2? Find nu 1? Find the difference and that would be your turn angle theta or delta theta. So, your nu 2 function of M 2 minus nu 1 function of M 1 will give the turn angle theta. So, what does this really mean when I have mach number 1.2 it generates a mach wave, this mach wave is at an angle mu associated with this thing I have a nu which is a function of mach number alone and this generate an expansion wave with the first one is this and the last one would be nu function of M 2.

This will have an angle mu angle along with this direction. So, this my mu, so called shock angle or the wave angle here. So, shock angle is only for compression for an expansion we do not call it as shock angle. So, this wave angle is mu which is our mach angle again associated with M 2, this is associated with M 1. So, I have a wave angle mu function of M 1 associated with it. I have a Prandtl-Meyer function in nu, which is function of M 1 again nu with M 2 and mu with M 2. So, this difference in this particular function is your turn angle.

So, you can also find your mu 2 with your M 2, mu 1 with your M 1 and hence you can get all the angles that is associated with this particular turn. So, if you look at the tables nu 2 for mach number 2 is for gamma 1.41 in the gas tables book. So, nu 2 with mach M equals 2 is 28.6 and nu 1 with M equals 1.2 is sorry here 26.3 and the other one is 3.55. So, your delta theta or theta is this minus that will be around 23, which means this angle is 23. So, we will now do some exercise tutorials on Prandtl-Meyer functions.