Fundamentals of Gas Dynamics

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Week – 11
Lecture – 43
Discussion on Reflection of Oblique Shocks

We will do something problems related to Oblique Shock interaction and Shock Reflections. So, one problem I have right now with some this is.

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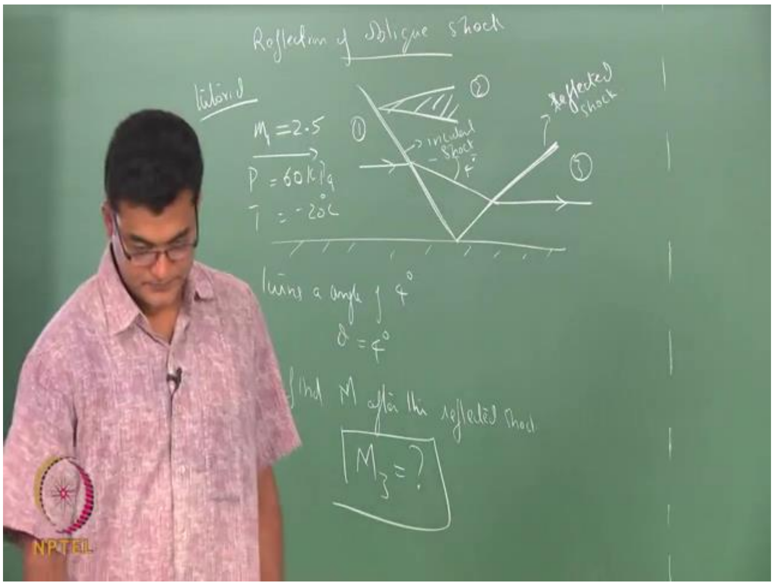

We are going talk on reflection of oblique shock we have a tutorial question right now. So, I have a flat plate there is a mach number which is greater than 1, there is an oblique shock that is fitting here probably may due to some wedge that is somewhere here. So, it creates an oblique shock it comes reflects at the surface and then there is reflected shock, reflected oblique shock. The wave comes here take a turn and then again turns it along with the flow. So, the values I have here is m equals 2.5. So, M 1 is 2.5 and turns an angle of 4 degrees. So, your theta is 4 degrees is what it means.

Find the mach after the reflected shock? So, this is my incident shock and this is my reflected shock. So, it comes and turns by 4 degrees, this is my 4 degrees. So, the fluid comes and because of the shock it turns by 4 degrees and then gets back to the flow direction that is parallel to the surface. So, what is the mach number after this? So, if I call this 1, region 2 and region 3 the question is to find M 3.

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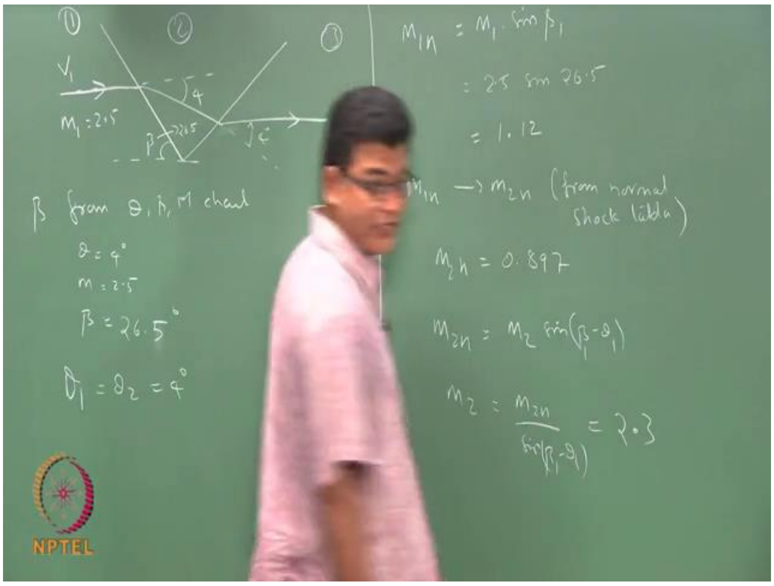

So, if draw the first shock, the fluid comes and make a turn this is 4 and M 1 is 2.5. So, I can find the beta from theta beta M chart. So, for theta equals 4, M equals 2.5 I would get a beta which from the chart if I look for 2.5 and theta equals 4 degrees I would get an M beta to be around 0.3.

Student: 5.

26.5. So, my beta I would get it as 26.5. So, this is reflected by another shock which turns the flow direction parallel to this surface. So, this turn angle is also 4 degree if it wants to have a line parallel to v 1. So, your theta 1 is same as your theta 2 is 4 degrees. Now you can get from region 1, region 2 and region 3, I can get M 1 n which is nothing but M 1 into sin beta 1 which is 2.5 sin 26.5, 1.12.

Student: 1.12.

1.12, if I know M 1 n, I can get m 2 n from normal shock tables, so my m 2 n.

Student: (Refer Time: 07:02) 0.897.

0.897, now if I know m 2 n, we can get m 2 from sin beta minus theta; this is beta 1 minus theta 1. So, this is m 2 n.

Student: (Refer Time: 07:33).

2.3.

Student: 2.34 (Refer Time: 07:43) table.

So, this is approximately 2.3. So, what did I get now?

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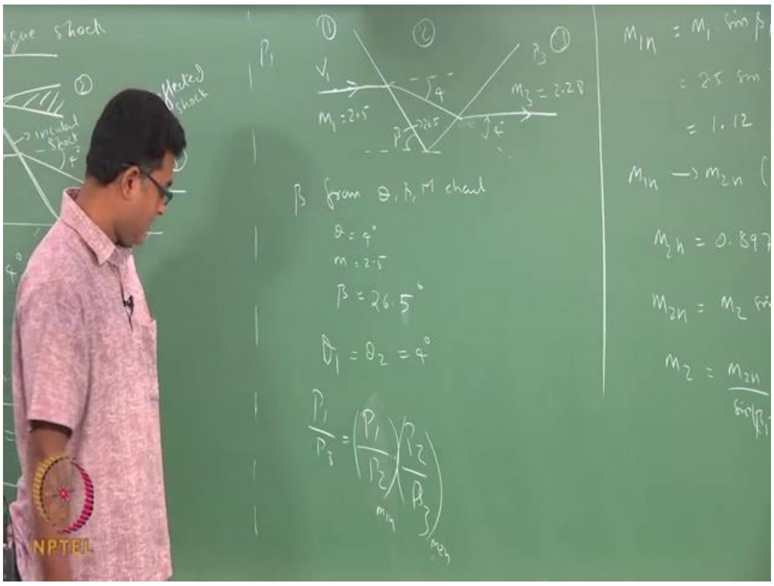

So, I have a shock v 1, then it turns this angle as 4, this angle as 26.6, M 2 is 2.3, M 1 is 2.5 and the m 1 n and I got m 2 n. Now this velocity is going and hitting shock wave which turns direction parallel to the surface. So, this velocity turn by 4 degrees, this is the second shock wave. So, this is the reflected shock wave, this is the incident shock, this is the reflected shock. So, this M 2 is now 2.3 turned by 4 degrees to this. So, if I want to find M 2 is 2.3, theta 2 is 4 degrees, I do not know this beta 2. So, how do I get beta 2? I look at theta beta M chart to get beta 2.

Student: (Refer Time: 09:46).

Which is 28 degrees, this angle is 28 degrees. So, I look for this M 2 and this theta I get a beta value which is 28 degrees. So, my M 2 n that is going to hit this, hit this second shock would be M 2 into sin beta 2, M 2 of the reflected.

Student: (Refer Time: 10:19).

Beta to this would beta 2 the angle with which it comes and hit the shock and angle between the velocity vector and the shock is your beta 2. So, M 2 n is normal to this. So, M 2 into sin beta 2 would be your M 2 n which is.

Student: 1.0 (Refer Time: 10:59)

1.08, which would give me M 3 n from shock tables one point.

Student: (Refer Time: 11:15).

0.928 from shock tables, now I have M 3 n which is 0.928. If I have M 3 n, my M 3 into sin beta 2, minus theta 2, my M 3 is.

Student: 2.2.

0.928 divided by sin beta 2 is 28 degrees minus 4 degrees. So, this would be.

Student: 2.28.

2.28, all taking the weak shock solution; so, this beta and beta 1 are from the weak solution. So, I have reflected angle and then this M 2, M 3 is 2.28. So, from the problem if you look at it I have M 1 is 2.5 comes turns by for 4 degrees and then turns again to M 3, M 3 is calculated to be 2.28. If you know these values M 1, M 2 and M 3 you can find the corresponding pressure ratio. So, if I know pressure P 1 I can find P 3 from M 1 n and M 2 n and then M 2 n and M 3 n. So, the pressure across this shock pressure across the shock is known the ratios are known. So, I can find P 1 by P 3 to be P 3 by P 2; P 1 by P 2 into P 2 by P 3.

So, this is across the first incident shock, this is across the reflected shock. From M 1 n and from M 2 n of the reflected shock, if you know this then P 1 is given, you can P 3.

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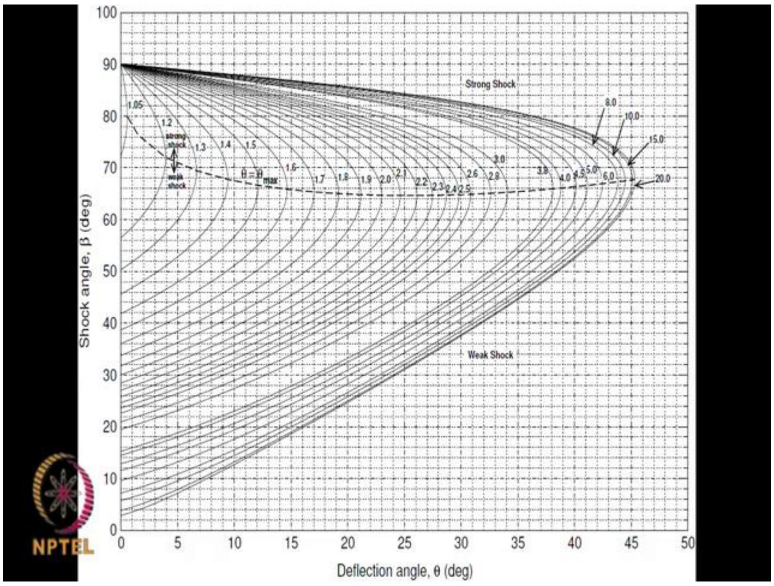

So, that is this particular problem. Similarly, you can do another tutorial, my flat plate then suddenly encounters a ramp and there is top wall here.

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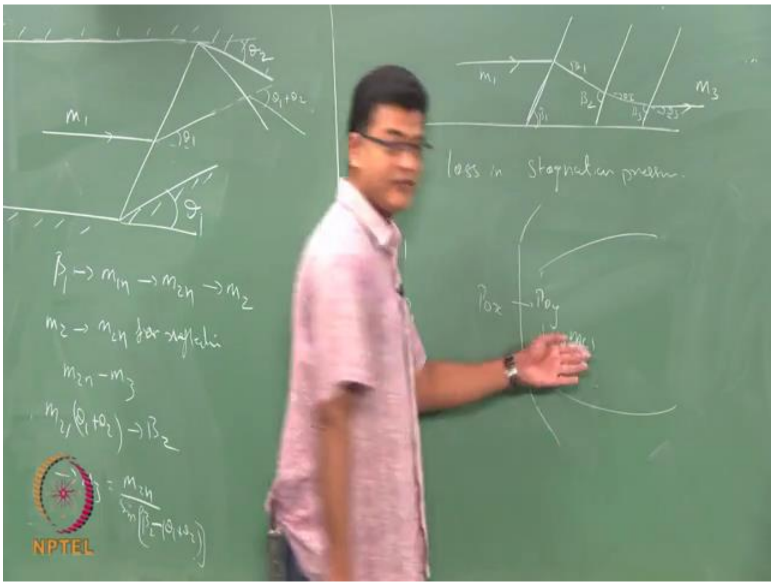

So, this is y some theta I know my M 1. So, it creates oblique shock, it goes and hits the top wall and it reflects. So, this velocity vector turns by theta degrees goes and hits the second reflected shock and then again turns back to the line parallel to your surface. So, this is not normal to this. So, it will at some angle and then turns. So, I have M 1 coming and hitting M and the resultant mach number is M 2 and then I get M 3. This is similar to problem what we have done; it is theta reflected value is the same. So, this deflection is the same as what you have here. So, the same deflected value from which you can find from the same procedure what we have done, it is exactly the same problem except that what are we are done here it is on the top side, it is same problem.

So, you can evaluate M 2 as M 3 as what we have done before. Now, if the top surface is also at some angle. So, I will not evaluate these problems, I am just demonstrating that it can be done the same kind of analysis. So, I have a surface at say let us put it as theta 1, it generates oblique shock. So, this is a surface and this top surface also turns by some angle theta 2, there is a shock that comes and at this point itself the surface turns and because of which there is another shock wave that is here. So, I have b 1 or M 1 approaching this first shock, then it turns by theta 1, it goes and hits this surface then it turns theta 1 and theta 2. So, the turning angle here is going to be theta 1 plus theta 2.

So, if you realize that this angle is theta 1 plus theta 2 the problem is same as what we had done before - find beta, find the normal one, find the shock. So, find beta 1 from which I would get m n 1, from which I will get m n 2, from which I would get m n 2 and the movement I get m n 2, I get m n 2 n for the reflected shock and m n 2, I would get m n 3, from m n 2; from M 2 and theta 1 plus theta 2 I would get beta 2 and I would easily get M 3 n based on m 2 n divided by sin beta 2 minus theta 1 plus theta 2. So, I would get my values at M 2 n. So, the procedure is the same thing as we have done before, just that you have to realize this angle is now turned for theta 2 also.

Now, we are going to discuss something called series of shock wave. So, if I have series of shock wave what would be the mach number after that; so, if I have a plate, I have a first shock, then I have a another shock, then I have (Refer Time: 20:44) shock. So, there are the angles of which are different. So, I have the M 1 here. So, it is turned by some angle theta 1. So, the shock angle this is beta 1, then this would be my beta 2, it goes and hits another shock wave with beta 3 and the turn angle is again different theta 2 and this again say suppose it turns back to the original position this would be my theta 3. So, I have series of solution. What would be my loss in stagnation pressure instead of having it as one strong wave?

So, if I can have M 1 reduced to M 3 by single wave, M 1 reducing to M 3 by series of shock waves, what would be the loss in stagnation pressure? So, this is something which typically is known as the phenomena or the technique of pressure recovery. So, you want to reduce your mach number form M 1 to M 3, but you do not want to lose your stagnation pressure. So, instead of having one single strong shock wave you reduce that to series of oblique shock wave retain your stagnation pressure as much as possible, all the way reducing M 1 to M 3. So, that is the technical typical technique here. So, you can put some numerical values and see how it reduces. So, this happens typically (Refer Time: 23:02) in gas turbine engine where I have an inlet.

So, if my oncoming mach number is greater than 1 there will be a strong shock sitting here and this is one single shock. So, this would be P 0 x which reduce to P 0 y and this mach number behind the shock would subsonic and you have a subsonic flow inside the diffuser.

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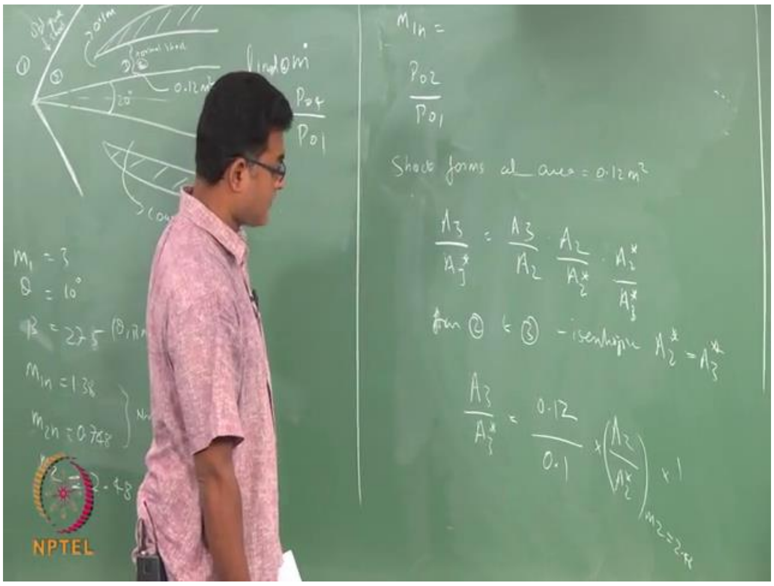

Instead of this, the usual technique is to; if I have engine here very high mach number n is coming. So, instead of having a strong shock here I would attach something like this in front of the diffuser and inlet. So, I would have a shock here, I have oblique shock here. Now oblique shock will have a supersonic mach number behind the wave. So, that would create another shock here. So, instead of one strong shock now I have two shocks here. So, the pressure jump, the pressure P 0 x to P 0 y is done in a series of shocks. So, your P 0 you can reduce it very minimal value or you can retain your P 0 to as much as possible even when reducing your mach number to something subsonic.

Instead of these two shocks, I can have at another series of oblique shock. So, I have series of oblique shocks here. So, there will be one shock here, there will be another shock here, there will another shock here. So, at the series of shock I can reduce my mach number to subsonic mach number, even when keeping P 0 the same value. So, P 0 is the total; it is an indicative indication of how much work it can do. So, you want to maximize that and you want to gain compression inside the diffuser as much as possible. Now in compression shock wave, you are actually compressing your increasing your pressure. So, you want to do that here yet you do not want to lose your P 0 x or the stagnation pressure. So, you can do that numerical problem here.

So, let us try to work it out. So, I have one oblique shock here then somewhere here I have a normal shock. So, this is something called cowl region. So, that is where you have the fluid that is going through towards the compressor, the diffuser or the inlet of the gas turbine engine. So, you have oblique shock that is sitting here, wedge angle is given 10 degrees. So, this is 20 degrees. So, the wedge angles the beta or the theta or the turn angle is 10 degrees. So, I have M 1 is 3, theta equals 10 degrees. So, I can find M 2. So, I decide this region as region 1, region 2, region 3 and 4. 3 is before the shock, 4 is after the normal shock.

So, I can find M 2 for that I would get beta then I would gives M 1 n then I would compute my M 2 n then I would get my M 2, from the normal shock and the oblique shock tables. So, up to this is from theta beta M chart, this is from normal shock table, this is from the chart. So, if you the question is to find m dot and P 0 4 by P 0 1. So, the movement you have M 1 n you can find M 1 n, I can find P 02 by P 01 from A 3 by A 3 star meter square.

The area where the normal shock forms is 0.12 meter square. So, I can compute A 3 by A 3 star as A 3 by A 2 into A 2 by A 2 star into A 2 star by A 3 star. So, from A 2 to A 3 its isentropic flow, A 2 star and A 3 star are the same. So, from 2 to 3 is isentropic flow. So, my A 2 star is same as my A 3 star A 2 and A 3 are given A 3 is A 3 is 0.12 divided by A 2 is 0.1 into A 2 by A 2 star of associated with M 2 and this is 1. So, I come back and put the values here the M 2 is, what is M 2? 2.48. M 2 n is?

Student: 0.746.

0.746, and M 1 n?

Student: 1.385.

1.385. Beta?

Student: (Refer Time: 33:09).

27.8.

Student: (Refer Time: 33:11).

So, from beta I would get M 1 n and M 2 n, from the normal shock table from which I can compute M 2 as 2.48. So, this M 2 is 2.48. So, if I know this value I can find M 3.

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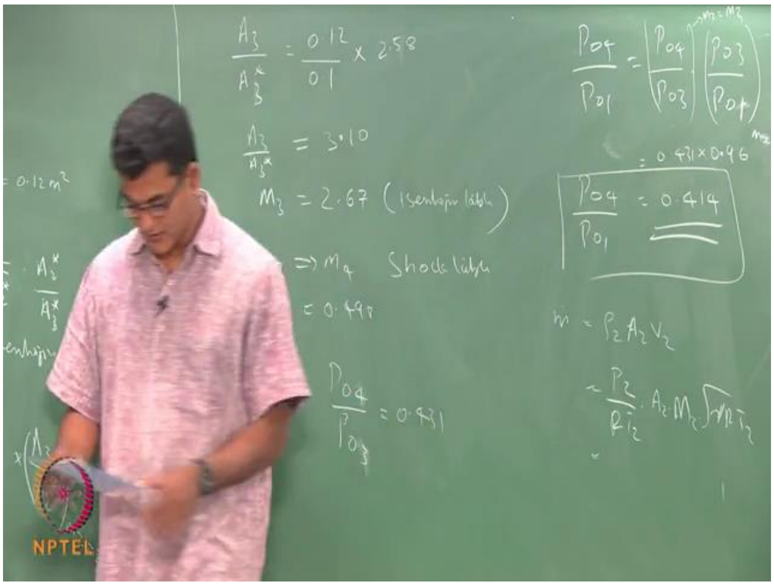

Star is into, what is this value? A 2 by A 2 star at 2.48 this would give me an M 3 from the isentropic tables.

Student: 2.588.

2.58.

Student: (Refer Time: 34:18).

588, and this value is?

Student: 3 point (Refer Time: 34:26) 3.10.

3.10 and this corresponds to a mach number.

Student: 2.67.

2.67 from isentropic table; if you know M 3 I can find M 2 from shock a table which is, for this mach number there is a shock at 3 and 4, M 3 is 2.64 and the normal shock.

Student: (Refer Time: 35:20).

0.498, this will also give me a P 03 by P 04.

Student: (Refer Time: 35:33).

0.431, P 03 by P 03; P 04 by P 03 is 0.431. So, the question is to find P 04 by P 01, which is P 04 by P 03 into P 03 by P 01. So, this is associated with the shock relation which is 0.431 this is associated with. So, this is with M x equals M 3 and this is associated with the oblique shock.

Student: (Refer Time: 36:38).

M x equals M 1 n which is 1.38, M 1 n which 1.38 which is.

Student: 0.414.

0.414. So, that would be the ratios P 04 to P 01.

Student: Sir, final answer (Refer Time: 37:09).

Final answer is point, this is.

Student: (Refer Time: 37:17) 196.

So, let me see if. So, m dot you can compute from rho 2 A 2 V 2. So, A 2 is given V 2 and rho 2 you can compute from P 2 by rho T 2 into A 2 into M 2 into root of gamma R T 2. So, T is given, you have to know T 2 get the values. So, if you know your T 2 you can get your m dot. So, if I along with the inlet mach number if I give my P 1 and T 1, I can compute the final pressure and temperature and also the m dot that is going through the cowl. So, you could divide your own examples with whatever we had discussed here, series of oblique shocks going to give P 0 recovery, and the other problems which we had just now reflected and the other shock.

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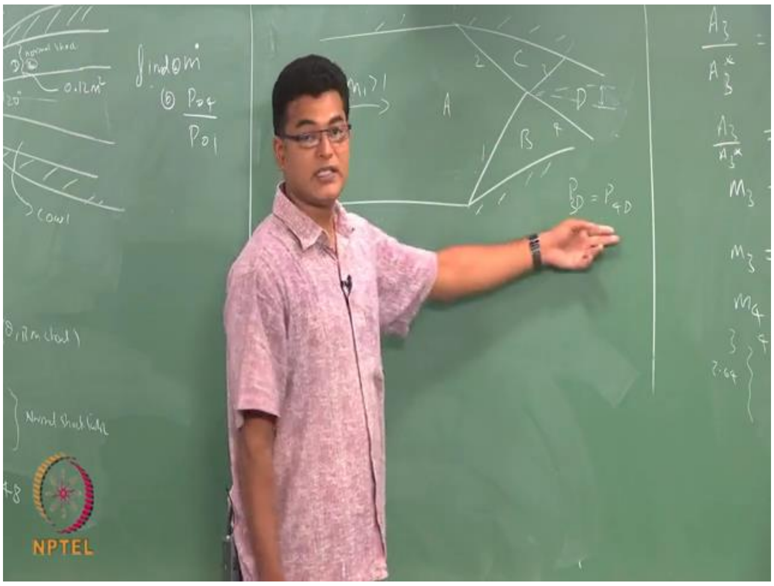

So, for the nozzle flow in an over expanded nozzle or if I have this shock, this kind of flows which will reflect to shock waves and then I have another set of shock waves. So, I have a flow here M 1 greater than 1, there is shock 1, shock 2, 3 and 4, this region a b c and d. So, first reflection is very straight forward you can do then the second reflection also you can somewhat some of the things are true, but then here you will have two slip lines.

So you have to do iteratively to get this particular region where you have the pressure in

D say for example, 3-D and pressure along 4-D are the same. Pressure due to the third shock pressure, due to the first shock in this slip line is same and the velocity is in the same direction. So, you have to iteratively do this. So, you we are now trying to do that numerical problem here, but it is possible to evaluate that if we have enough time or a computer with us.

That brings us to the end of Discussion with Oblique Shock Waves. In the next class onwards we will discuss something called the Expansion Fan and that would be our last topic of discussion.