Fundamentals of Gas Dynamics
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Week – 10
Lecture – 41
Discussion on Oblique Shocks

We are going to discuss today some problems in Oblique Shock, using theta beta M chart.

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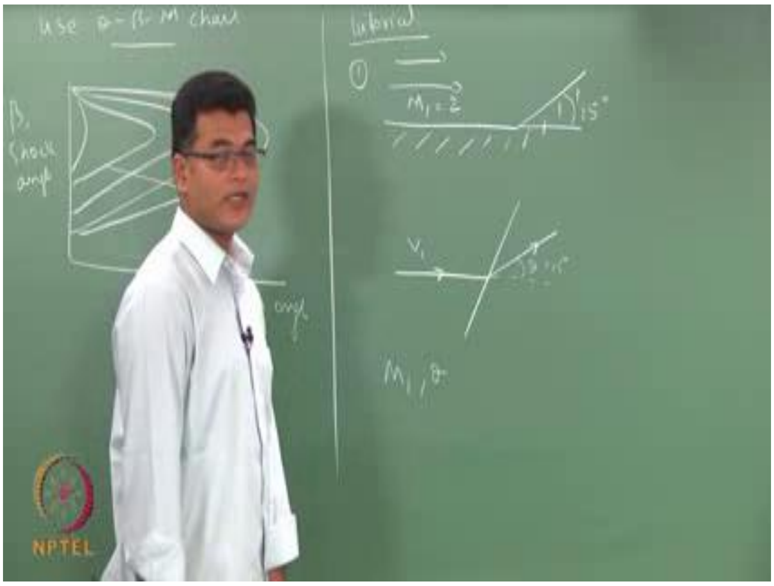

So, I will use theta beta M chart which is essentially your theta which is your deflection angle and beta which is your shock angle, and what is plotted here is for various mach numbers, increasing mach number and we leave this to compute for a given mach number what is the beta and theta if one of them is not.

So, first example I have here is I have a flow over a flat plate which is suddenly faces a ramp, thus rhombus say 15 degrees and the approaching mach number of the flow is 2. So, I have a mach number 2 approaching a ramp which is at a theta of 15 degrees. So, the velocity with which the oncoming flow then it faces an oblique shock then the velocity stream lines are deflected by theta degrees which is 15 degrees. So, I have M 1 and theta.

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The question is to find M 2 and P 2 by P 1, fine. So, what I will do is for this particular mach number, for this particular theta, what is my beta. So, how do I find M 2? If I know M 1 I should get M 1 n which requires beta and theta, if I know my M 1 n I can find M 2 n through theta beta M chart then, this is from normal shock, from this relation I can go back to my M 2 and if I know my M 2 I can find these ratios. So, now, given M 1 and theta what is my beta, for that I have to use the theta beta chart. So, what I do here is.

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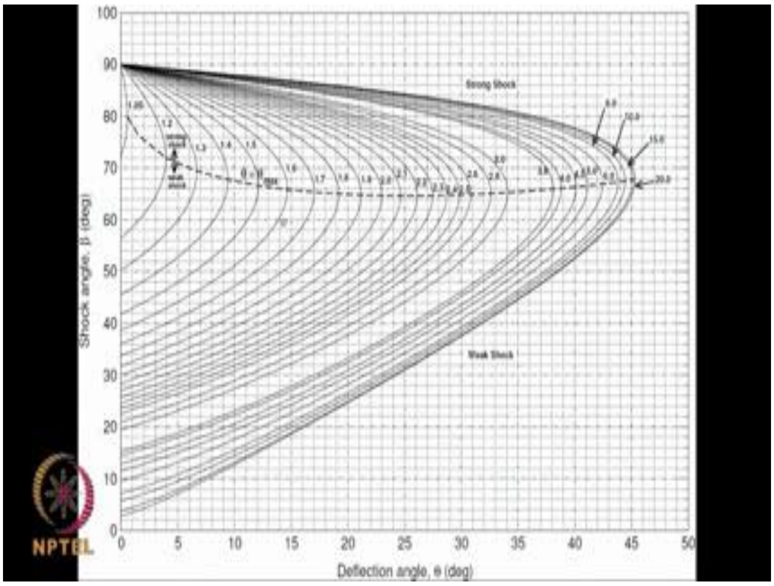

I look at the chart for mach number 2, I look for theta equals 15 degrees, so theta is 15 somewhere here and this point is my mach number 2 and theta equals 15 degrees which corresponds to something around 45 degrees, if you look at this thing, this is my point. So, I will I take a line from 15 degrees and when it intersect the contour of M equals 2 and take the y projection that should be around 45 degrees.

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So, from theta beta M chart my beta is approximately 45 degrees. So, if I know the beta I can compute my M 1 n as M 1 sin beta. So, this is 2 sin 45.

Student: (Refer Time: 05:10).

Yeah. So, this would be 1 point around.

Student: (Refer Time: 05:22).

One point?

Student: 4 1 (Refer Time: 05:24).

It is fine. So, around 1.4, the shock is facing a normal component of the velocity with the mach number 1.41 and from normal shock table your M 2 n is. So, I look at the normal shock table for gamma equals 1.4, M x is now 1.4, around 1.4 my M y is 0.74 which is my M 2 n. If I know my M 2 n the pressure ratio M 1 n also gives me my P 2 by P 1 which is across the shock which is 2.120, and my T 2 by T 1 also can be obtained from this which is 1.255. And my M 2 is M 2 n divided by sin beta minus theta which is 0.74 divided by sin 45 minus 15, 1.48. So, what we have done here is.

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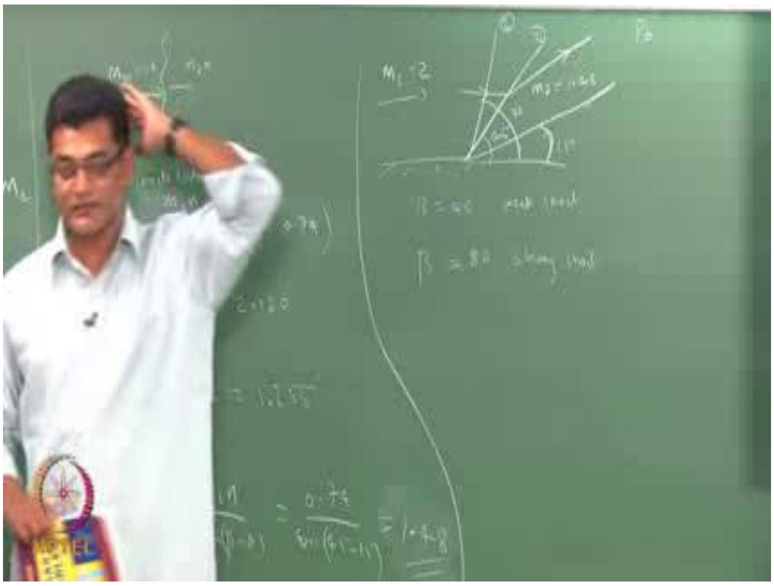

I have this shock, this angle is 15 degrees, this angle is found to be 45 for a mach number of 2 and the velocity or stream lines are deflected by 15 degrees. The M 2 here we have found to be 1.48. There is also another solution possible because here we have taken beta to be 45, you can look at the chart there is also another beta available which is going to be the so called strong solution. So, beta 15 is going to give you around 80 degrees. So, this is a weak shock, beta approximately 80 degrees is the strong shock. So, whether the flow takes weak solution or strong solution depends on what you have here as your back pressure. So, it can be either this or another shock like this it can be either this or this. So, 1 is the weak solution 2 is the strong solution, and which one is being decided, by your back pressure, alright.

Now, the same problem if I increase my beta. So, M equals 2, I am now going to increase 15 to say 30 degrees.

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So, let us see what happens. So, this 15 is increased to 30 degrees, again the procedure is same except that now I have a different beta, I look at the table. Now, my theta is 30 degrees, I look for a cross section where my mach number is 2. Now I see that for 30 degrees there is no mach number line that can intersect which means this theta is greater than theta max for M 1 equals 2. So, I would have a shock that is not touching this. So, I would have a shock that is a something like this which is not to that something which we are not going to analyze in this course.

Now, let us consider when theta is 20 degrees, mach number is the same. So, there will be one public shock that is here. So, the procedure is same again we look for beta. So, I take a mach number 20 and I look for a line mach number deflection 20, mach number 2 which intersect at some point here 52 degrees.

Student: (Refer Time: 12:10) 54.

54 degrees, around 54 degrees, we look for this particular line which intersects the mark 2 line. So, essentially what you have done is you have taken this line which is 20 degrees, you drop try to find this particular point where this mach number is 2 and you have taken the beta here and that beta from the chart we found to be approximately 54

degrees. So, when it is 54 all these values change.

Student: 1.62.

1.62, M 1 n is 1.62 which means our M 1 n here is 1.62 instead of the previous value. From the shock tables again from 1.62, I will get M y as 0.663 which also gives me a pressure ratio of 2.8 and the temperature ratio of 402 which gives me an M 2 of 0.662 and the angle is now 45 degrees, instead of 45 degrees it is 54 degrees minus 20 degrees that is going to give me a new mach number.

Student: 1.19.

1.19. So, now, when increase my deflection angle from 15 degrees to 20 degrees, now I have a shock with a beta of 54 degrees instead of a 45 degrees and my M 2 is 1.19 instead of 1.48 previously. So, when I increase my deflection angle for the same mach number, the mach number after the deflection is going to be less which means that if I increase my theta I am going get stronger and stronger shock. So, that I am going to get a mach number which is smaller and smaller.

Same way, if you look at your pressure ratio you would realize that this is more than what is previously obtained. So, if you increase your deflection angle you are going to get a stronger and stronger shock that is one message from this particular set of problem we do. Now the same set of problem or same kind of problem, if you keep increasing your mach number for a given deflection angle see what happens. Here again we have 2 betas - we have a strong solution and a weak solution. So, this is weak shock and beta 70

Student: (Refer Time: 16:03).

74 is our strong shock. So, in the problem if you are asked to find the deflection angle it is just the weak solution, it does not be strong solution. So, this is our so called default option, you find the weak solution unless otherwise mentioned.

Now, what we do is we can increase our mach number and see what happens to the shock after the deflection. So, for the same mach number, consider the previous theta of 15 degrees. Now we will consider the value to be 20 degrees, and mach number is increased from 2 to 2.5, 2 to 2.5. So, your angle is going to be now different. So, instead of 54, I go and look at for a deflection angle of 20 degrees, look at the intersection line with 2.5 mach, that is going to be around 42, around 42 degrees which means my M 1 n is now modified and I would get a new M 1 n. So, this is 2.5.

Student: 1.67.

1.67. So, my M 1 n is 1.67. So, my M 1 n is 1.67 and M x is 1.67. So, I look at the shock tables 1.67 around M y is going to be around the 0.648. So, my P 1 by P 2 is also modified and T 1 by T 2 is also modified its something around the 3.12 and T 1 by T 2 is 1.44. Likewise your M 2 is also modified; M 2 n is now 0.648, sin beta is 42 minus 20 this is around.

Student: 1.73.

1.73. So, your M 2 is now, I am going to draw that thing here.

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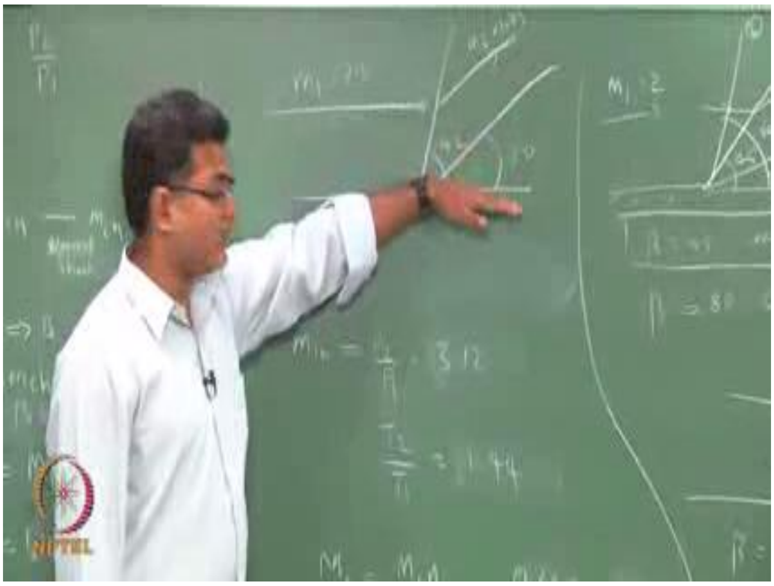

So, this is 20 degrees I have M 1 of 2.5, M 2 of 1.73 and beta is 42 degrees. So, the M 2 is increasing as you increase your mach number for a given theta and the shock is weaker than what you had before, and the same mach number for a different shock angle, I mean the different deflection angle you would get a stronger shock if you in keep increasing your deflection angle.

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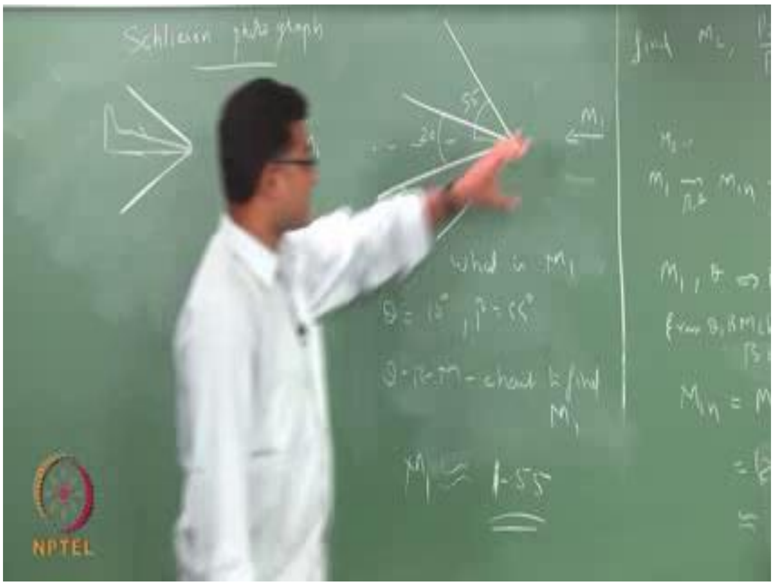

Schlieren photograph is something which would photograph the density gradients. So, if I have a flow with say some mach number, because of the body you would see some density variations here around the structure. So, the schlieren photograph is a technique in which you photograph the density gradients, I am not going into the details of this, but schlieren photograph you can see the density gradients.

If there is a shock here shock as we know has large density gradients. So, this angle or this shock is very clearly picked by a schlieren photograph. So, now, if I have a wedge of say, let us take wedge angle to be 20 degrees and some mach number, now I take a schlieren photograph assuming that this mach number is larger than 1 it produces a shock, oblique shock of this kind. So, I look at the photograph and I measure this angle to be something of let us take this to be 55 degrees. Question is what is M 1?

So, it is a very easy question, but you have to understand that your theta is 10 degrees and your beta is 55 degrees, the movement you know this you see your theta beta M chart to find your M. So, all I do is I go to the chart take the deflection angle 10 degrees and see where it crosses the value 55. So, it crosses somewhere here, so this is the point where my deflection angle is 10 degrees and shock angle 55. So, this is around mach number of 1.51 or 52 or something, approximately from the chart M 1 to be 1.55. So, if the mach number is not known you can get this angle.

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I have a wedge of some theta, M 1 is 2. So, I keep increasing my theta, theta is progressively increased. Again this is a steady case we are not doing this increase in time, but we are having a different wedge and appropriately finding the solution. So, it is a steady solution that we are looking at. So, theta is progressively increased.

So, what is the theta at which the oblique shock is no longer attached to the wedge? So, what do I mean by that? I increase my theta to some value, now the shock instead of attaching it does detached. It is no longer attached to the body. So, what is this theta? What is this theta? So, this theta at which this shock is no longer attached. So, what do you mean by that? You need to find what is the theta max for M 1 equals 2 that is all you need to do.

Student: Theta by 2 is equal to beta (Refer Time: 25:38).

That is OK, what is the theta at which the oblique shock at. So, we will find theta equals theta max by 2 or this one.

Student: (Refer Time: 25:53).

So.

Student: (Refer Time: 25:53) count from chart (Refer Time: 25:54) theta by 2.

Correct. So, you only need to find is theta max by 2. From chart you are going get theta by 2. So, the deflection angle is half of the wedge angle, theta by 2, because the flow comes here and is deflected this way. So, this would be theta by 2. So, all I need to find is go to the theta beta chart, theta beta chart, I have this M equals 2 curve find this angle which is half of the wedge angle. So, I look at M equals 2 curve find where it is turning or the maximum theta. So, I go to M equals 2 curve and what I see here is an angle of around max equal 23 degrees for M 1 equals 2.

Your wedge angle is twice this that is 46. So, when the wedge angle becomes 46 the shock is no longer attached to this because you do not have a solution to whatever we have derived which is oblique shock solution and at 45 degrees if at theta of greater than 46 degrees the shock would be slightly detached from the body.

In the next class we will try to do the reflection of shock and there is nothing much to discuss there, but we will do some numerical problem and see evaluate those things and then we go to the CD nozzles.