Fundamentals of Gas Dynamics

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Week – 10
Lecture – 39
Oblique shocks relations

We will continue with the oblique shock discussion. If you have an oblique shock like this and the velocity V 1, the shock angle is beta and the velocity is deflected like this. So, this would be my deflection angle.

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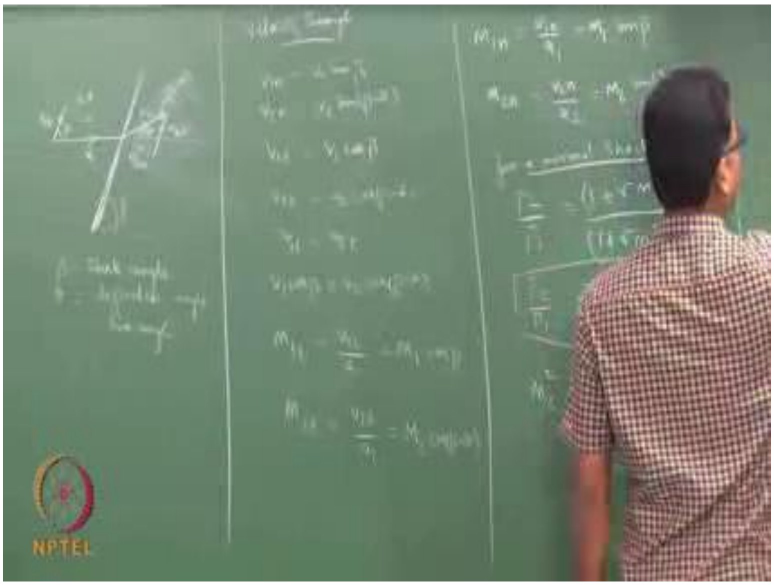

The normal velocity before the shock is this which is V 1 n and the tangential component is this V 1 t and we have also seen that V 1 t is same as V 2 t. So, this is V 2, this is theta, this is V 1 V 2 t and this V 2 m. So, this angle is beta minus theta, this angle is beta is the shock angle, the angle between the velocity V 1 and the shock and theta is the deflection angle because it is deflected by theta degree the velocity there are certain velocities deflected by theta degree. So, this is deflection angle or the turn angle because it is turned by theta degrees.

And from the velocity triangle we get V 1 n to be V 1 sin beta, V 2 n to be V 2 sin beta minus theta and V 1 t is V 1 cos beta, V 2 t is V 2 cos beta minus theta. So, if you take this angle sin of beta and sin of beta by 2 and cos of beta and cos of beta minus 2 you would get these components. We also know that V 1 t is V 2 t and hence we can write V 1 cos beta is V 2 cos beta minus theta and we have also seen M 1 t is nothing, but V 1 t by a 1 which is M 1 cos beta M 2 t is V 2 t by a 2 which is M 2 into cos beta minus theta, likewise your M 1 n, the mach number based on velocity normal velocity before the shock is this. So, this would be M 1 into sin beta.

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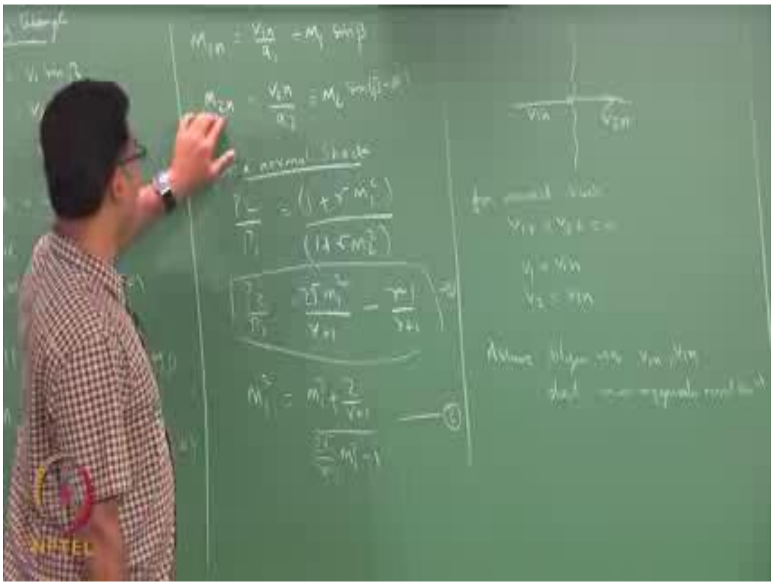

M 2 n is V 2 n by a 2 which is M 2 into sin beta minus theta. So, those are the components of mach number based on based on components of velocity. So, for a normal shock I can write P 2 by P 1 as 1 plus gamma, M 1 square divided by 1 plus gamma M 2 square or I can also write this as 2 M 1 M square by gamma plus 1 minus gamma minus 1 by gamma plus 1, if I substitute M 2 square equals M 1 square into 2 by gamma plus 1 plus minus plus divided by 2 gamma by gamma minus 1 M 1 square minus 1 square M 1 and M 2 are the mach numbers before and after the shock. So, in this equation if I substitute M 2 in terms of M 1 I would end up with the relation that is like this.

So, your pressure before and after the shock in the normal shock is this, but here if I draw this in a normal shock what I have here is V 1 n and V 2 n, and for normal shock our components V 2 equals V 1 t equals V 2 t equals 0 and your V 1 is V 1 n and V 2 is V 2 n. So, you could assume oblique shock with V 1 n and V 2 n as your velocities across an equivalent normal shock. So, if I have my V 1 n, I can get shock relation to get V 2 n. So, if I have M 1 n, I can get an equivalent this thing. So, instead what I would do is I will replace M 1 with M 1 n, M 2 with M 2 n and I get my shock relations across my.

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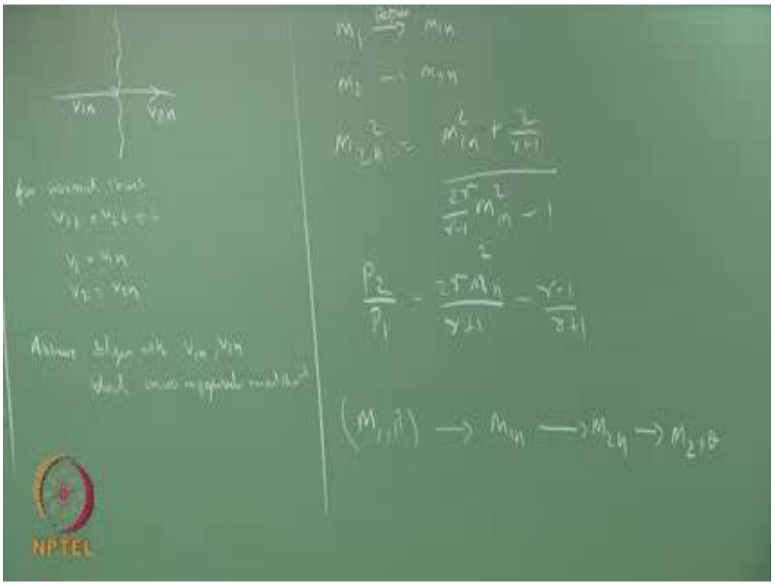

Now, use normal shock relations to get the mach numbers before and after this equation which I have here. So, this is 1, this is 2, I can relate M 2 n square equals M 1 n square plus 2 by gamma plus 1 divided by 2 gamma by gamma minus 1 in M 1 n square minus 1, likewise my P 2 by P 1 also can be related to my normal component. So, if I know my normal component of the velocity, I can get the mach number based on the normal component I get the pressure relation related to that or my mach number after the normal component of the velocity, after the shock and if I get that, once I get M 2 I can substitute in this equation M to n which is related to M to n this thing beta and theta and this also in beta and theta, we can find beta and theta essentially.

So, the idea is if I have M 1 and beta I can compute my M 1 n. If I know my M 1 n I can get my M 2 n from shock relation. If I know M 2 n I can find my theta or M 2 and theta we will see how we can do this by getting a relation between M theta and beta.

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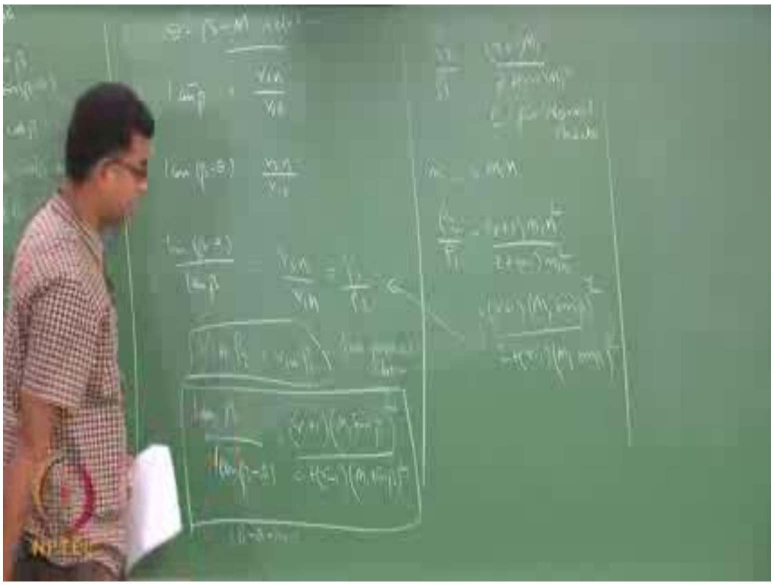

So, that is our next exercise, we will try to derive something called theta beta M relation. So, I would; my tan beta is 1 n by V 1 t from these triangles. So, I get V 1 n by V 2 n is tan beta, likewise tan beta minus theta is this thing V 2 n divided by V 1 t. So, if divide these 2, tan beta minus theta divided by tan beta is V 2 n by V 1 n which is also my rho 1 by rho 2 from the continuity equation, which we have seen some classes previously from previous lectures on continuity equation around the oblique shock.

So, rho 1 by rho 2 across the shock is rho 2 by rho 1, across the normal shock is M 1 square divided by 2 plus gamma minus 1 M 1 square for normal shock. I use this relation and replace M 1 with M 1 n. So, my rho 2 by rho 1 is gamma plus 1 divided by M 1 n square divided by 2 plus gamma minus 1 M 1 n square and M 1 n is nothing, but gamma plus 1 M 1 n is nothing, but M 1 sin beta divided by 2 plus gamma minus 1 M 1 sin beta the whole square. So, if I substitute this here I would get tan beta minus theta divided by tan beta equals gamma plus 1 into M 1 sin beta the whole square divided by 2 plus gamma minus 1 in the M 1 sin beta the whole square.

So, we will reduce this equation. This equation is relation between theta beta and M 1, we will reduce this equation further, but this in this equation it is just beta, theta, gamma and M there is nothing that is. So, I can reduce that equation to; this is rho 2 by rho 1 what I have here is rho 1 by rho 2. So, this would be inverse of that, this will be tan beta by beta minus theta. So, this is rho 2 by rho 1 this is rho 1 by rho two. So, I have to take inverse of that.

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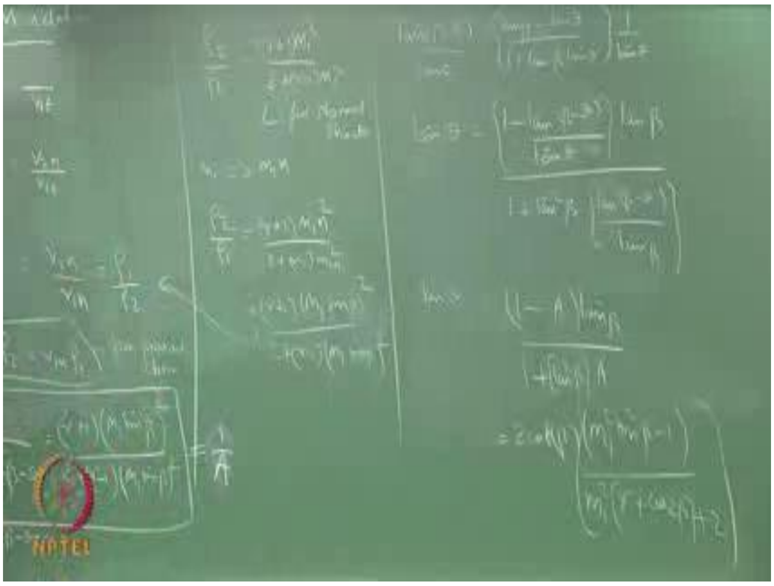

So, if I use the trigonometric relation beta and you reduce this further, I can get a relation like this. So, I take this relation divided by tan theta into 1 by tan theta and I reduce that to this particular form, and this is what we have right now here as my say, let us take this as A. So, my tan theta is essentially 1 minus A into tan beta divided by 1 plus tan square beta into A. So, I substitute A from this relation which is the inverse of this which is 1 by A which equals 1 by A. So, I substitute this quantity there would get this relation which I can modify to get this form 2 cot beta into M 1 square sin square beta minus 1 divided by M 1 square gamma plus cos 2 beta plus 2.

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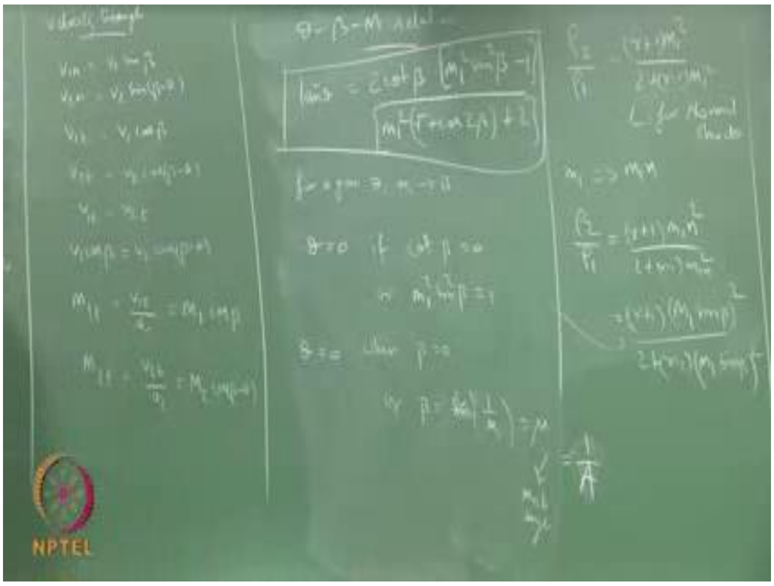

I will right that equation here again. So, I get a relation of tan theta in beta and M alone, which would be 2 cot beta into M 1 square sin square beta minus 1 divided by M 1 square gamma plus cos 2 beta plus 2 in this. So, this is our so called theta beta relation for a given theta and M find your beta is the; if know my beta and M 1 I can get a theta or if know my theta and M I can get a beta. Now, if you look at this equation, theta is 0 if cot beta is 0 or M 1 square sin square beta equals 1 or the theta is 0, when beta is 0 or your beta is 1 by sin inverse 1 by M 1 which is also your mach angle.

So, you have no deflection this would if deflection angle is 0, when beta is 0 or mach angle is your beta is same as your mach angle. So, theta is the velocity the angle at which the velocity is deflected. So, this deflection is 0, the beta is 0. So, beta is 0 means beta is 90 pi by 2 beta is pi by 2. So, when beta is pi by 2 you have no deflection or when beta is equal to your mach angle there is no deflection, it means if I have a normal shock my beta equals pi by 2.

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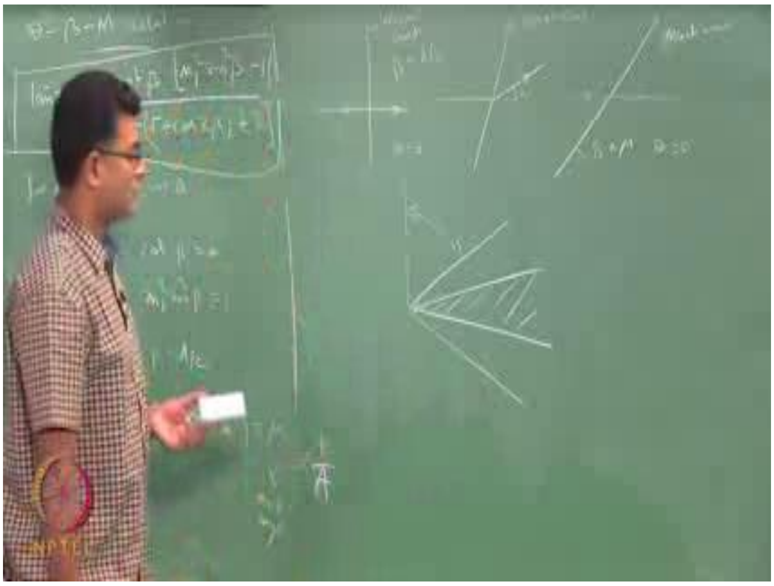

So, when I have a normal shock, there is no deflection in my velocity component. So, theta is 0 in this case for a normal shock or when I have a shock angle which is same as my mach angle then again I have no deflection in my velocity component.

So, if I have a shock angle between these 2 somewhere like this, something like this I have a shock angle. So, this is mach wave this is oblique shock. So, all your oblique shock is between the normal shock solution and the mach solution. So, if I have wedge like this, the mach angle may be something like this and the normal shock is something like this. So, all your oblique shock solution lies between these two values; between your shock angle and this thing. So, all your theta values should be within this, if my theta is some finite value. Now, another inference that you obtain from this plot or this equation is from a plot of theta and beta for a given amp.

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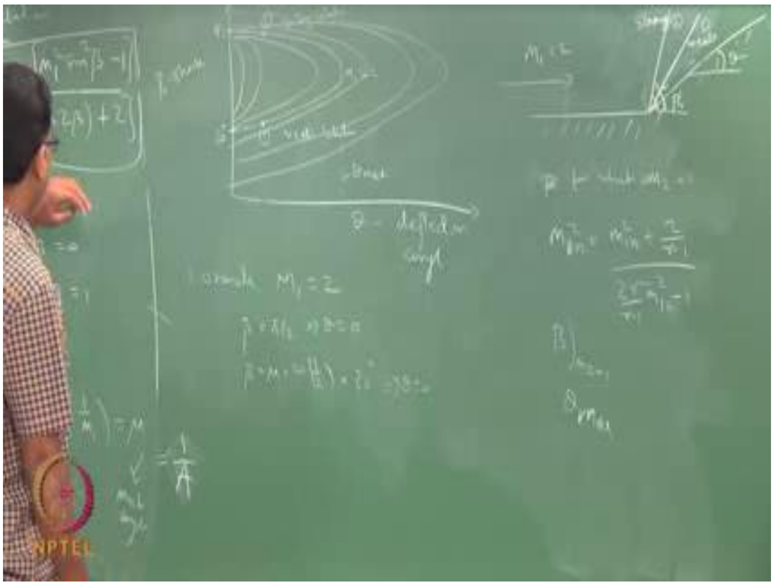

So, I have my theta here which is my deflection angle, beta is the shock angle. Let us consider M 1 to be 2. So, for a given M 1, for various theta how will the beta change is what I am going to plot, first is beta equals pi by 2 there is no theta and when beta equals mu which is sin inverse 1 by 2 which is your M 1 which is 30, I have theta equals 0 for theta equals 0 for M 1 I have a shock angle 30 degrees or pi by 2. So, this is my normal shock solution, this is my mach angle solution for a mach number of 2.

Now, if I change my theta, theta is something else I can compute my beta and I will see that will look something like this. So, if I change my beta for M 1 equals 2, I would have 2 betas, if I change to another theta again I would get 2 different beta. So, what does this mean I have a deflection here from by a plate that is kept at an angle. So, the flow is this way the flow comes and there is an oblique shock here and the flow gets deflected by theta degrees and this is my beta. So, I have a shock angle beta because of this deflection the streamlines have to be deflected this way. So, this angle is my theta, now this theta can be 1 of these 2 is what it says.

So, for M 1 of M 1 equals 2 for this particular deflection angle theta I can have 2 betas it would be either this or this. So, it could be either this or another value here. If I call this as beta 1 this could be beta 2. So, there would be 2 shock angles possible. So, to draw this little more neatly this would be this. So, this is first shock beta 1, this is the second shock beta 2. So, it is this is the first solution, this is the second solution. So, this would be my first solution, this would be my second solution. So, for a given theta and M 1 I can have 2 betas that is what you get from this equation which is plotted here. So, if you write a program which can input M 1 and theta it will compute 2 values of beta.

So, the if you know beta I can find my M 2 n, which is M 2 n is M 2 by sin beta minus theta. So, if know my b M 1 n I can get use the shock angle to shock relation normal shock relation to get my M 2 M 2 n and compute my M 2 from this particular relation. So, essentially I have 2 solutions; one is this one, one is this one. So, this particular 1 is called the weak solution, this is called the strong solution or strong shock. So, this is my strong shock, this is my weak shock strong shock weak shock. So, in a strong shock the mach number after the shock is less than 1 in a weak shock the solution the mach number after the shock is can be supersonic, it is supersonic. So, which 1 is selected depends on these are called back pressure.

So, the pressure available here will decide whether the shock should be weak or strong. Depending on the back pressure 1 of these solutions would be picked by the flow or I will write here strong or weak shock depends on your back pressure, but it has to be 1 of these 2, it is either angle or this angle nothing in between; now, if I keep increasing my theta. So, I keep increasing my theta there is a limit to which I can get these solutions. So, the maximum theta that is possible is my theta max. So, theta max is nothing, but for given M 1, how far I can go to get a solution get a beta from this solution from this equation. So, that equation gives me a beta and that. So, if the maximum theta that I can go to get a solution from this equation is the theta max that is also the angle at which when these 2 solutions are what, so that is my theta max and I can also find.

I can also find theta for which or beta for which my M 2 equals 1. So, what I do is I take that relation M 2 square equals M 1 square plus 2 by gamma minus 1 divided by 2 gamma by gamma minus 1 M 1 square minus 1. So, I substitute this would be n substitute the appropriate number for what would be the beta that would give me, M 2 equals 1 conditions here. So, there would be some beta which would give me equals 1. So, I have a theta max and I can also get this condition, where my M 2 is equal to 1. So, if I draw it for different mach numbers it may look something like this. Now, I can find the limit of this.

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I will draw this here. So, this is tending to infinity, I have a limiting value.

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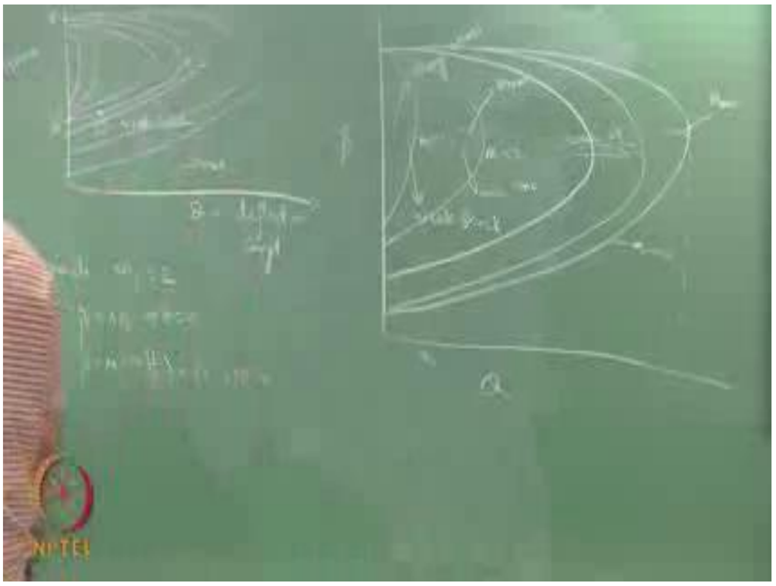

So, this is for M 1 equals 2, this is for M 1 equals 1 point something, this is for higher values, this is increasing M. So, I can get the tangent of this theta max; theta max for different M 1, if I connect if I take the locus of those points of theta max I would get a line like this.

So, this is my theta max line, likewise I can compute, I can get the angle at which beta at which you get M 2 equals 1 that would be my sonic line, it would be here. Limit M 1 tending to 0, tan theta and find the theta max for this. So, can tending to infinity. So, in the limit M 1 tending to infinity, how will the theta vary if something which you can compute from this? So, if M 1 is infinity then I would have a curve and I can also find the theta max for that particular case, where M 1 tending to infinity. So, theta max would also result in a limiting value which will look something like this, it goes from here and it goes.

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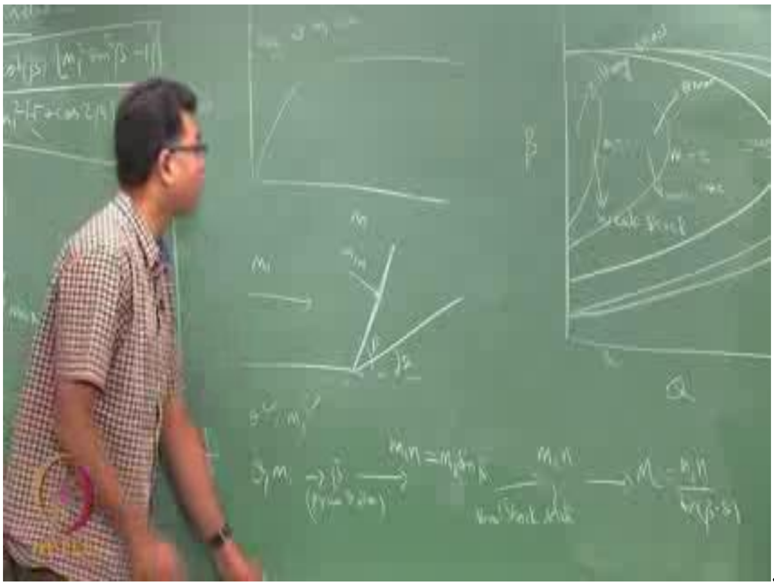
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You can find this value theta max at M 1 as M 1 tending to infinity. So, all this things you can get it from this particular equation. So, the point is if I have oblique shock say, for example, I have a shock here, this is my theta and this is my beta, this is my M 1, what I can do is I can find my M 1 n because I know 1 of this. So, let us assume that we know theta. So, this is a fabricated ramp. So, I am given theta and I am given M 1.

So, from these relation theta M 1, I know from which I can find beta from theta beta relation from theta beta M relation, I know beta. The moment I know beta I can find M 1 n M 1 n is nothing, but this M 1 divided by sin beta. So, if I have a ramp theta is given M 1 is given. So, from theta and M 1 using theta beta M relation I get the beta. The moment I get the beta I can find M 1 n, M 1 n is nothing, but M 1 by sin beta.

Student: (Refer Time: 39:24).

M 1 is M 1 sin beta correct, M 1 sin beta. The moment I get M 1 n I can get M 2 n from normal shock relation. The moment I get M 2 n I can get M 2, which is M 2 n divided by sin beta minus theta. So, if I have wedge, if I have a ramp like this or a wedge like this which.

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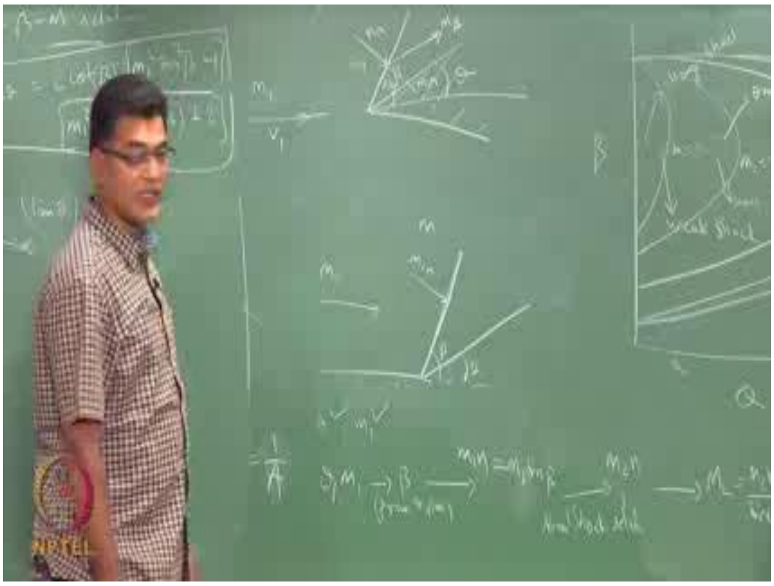

So, this angle theta which is facing the flow V 1, I know my M 1. The moment I know my M 1 and theta I can get the shock angle beta from theta beta curve. The moment I know my beta I can find the M 1 n. The moment I know my M 1 n I can find M 2 n from normal shock relation. Once, I know my n M 2 n I can find my M 2.

If I know the mach numbers before and after the shock I can find, literally find everything we know all the quantities across the shock for P 0 y, P 0 x by P 0 y or P 0 1 by P 0 2 all those things you can actually find it across the shock if you know the mach numbers, essentially that is what we need to do. Once the flow encounters an oblique shock, how will the property change, and how are we going to use the normal shock relations to find the properties after the shock.

Thank you, we will do the problems next class.