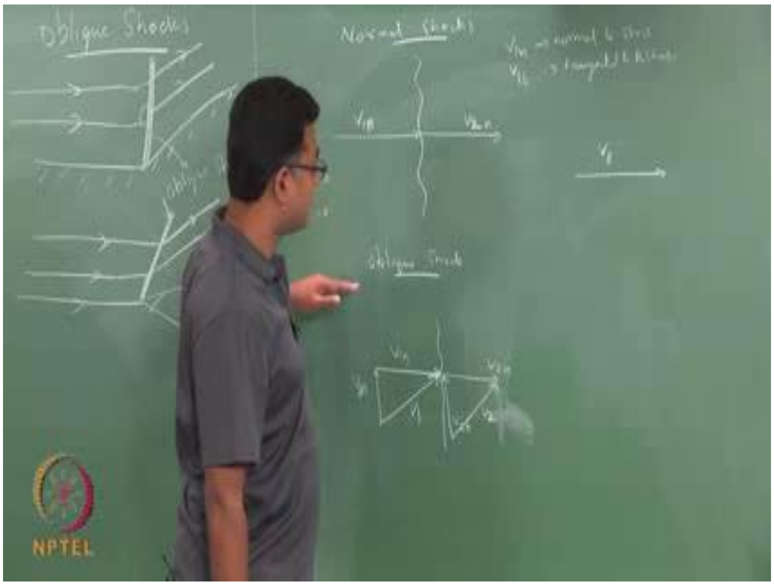
Fundamentals of Gas Dynamics
Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week – 10
Lecture – 38
Oblique shocks

Till now we were discussing about shocker that is normal to the fluid direction. Now, we are going to discuss something that is the shock wave that is at an angle with the flow direction.

(Refer Slide Time: 00:34)



So, for which are typically called as oblique shocks. If we have systems like this where there is a sudden ramp facing the flow, if the flow is coming this direction, the flow needs to take its deviation in such a scenario you would face something called shock wave which is slightly at an angle. So, there is an angle between the shock wave that is standing here with the flow direction. If these are the stream lines then the stream lines are at an angle with the shock waves and the flow is also deviated a bit. So, we should be talking about the angle of the shock as well as the deflection it creates.

There are 2 angles involved here or if we have a (Refer Time: 02:04) that is facing. So, I have a stream line that is coming and then it deflects, it is deflected this way. We are going to discuss these kinds of shock waves, these are again oblique shocks. We are going to talk about such kind of waves for which we will go back to the first set of shock waves we know, which is our normal shock. We will revisit normal shock and then change that to oblique shock. So, in normal shock, we had a shock and then I had a velocity that comes here and goes out. Now we call this as let me call this as V 1 and V 2 instead of our term x and y to differentiate that it is a oblique shock. So, I have a velocity V 1 and V 2.

Now, this is a velocity component that is normal to the shock wave, whereas my tangential velocity is 0. So, I can write this as, this is as good as writing this as the normal component of velocity V 1 and normal component of velocity V 2 with V 1 tangential is 0, V 2 tangential is 0 and that is my normal shock. Now to this, I am going to add a tangential component. So, I have a shock wave, I have a V 1 n, I have a V 2 n. I am going to add a tangential component, tangential to the shock wave, this normal to the shock and tangential to the shock, V 1 n s normal to the shock, V 2 i s tangential to the shock. I am going to add V 1 t here and V 2 t here or here let me put it here. So, this is my V 2 t. So, if I have V 1 t add it to this particular component then the resultant velocity would be at an angle like this and here again I have something at an angle V 2.

We will show that this V 1 should be equal to V 2 for oblique shock waves; V 1 t should be equal to V 2 t. We will show that analytically or we will derive that set, but this is what it seems. So, I have a velocity that is coming at an angle, hitting a shock wave and this thing. This I can redraw in this particular fashion. So, if I take my V 2 to be this way or V 1 to be horizontal to the plain then that is at an angle with the shock wave. So, the shock is this way.

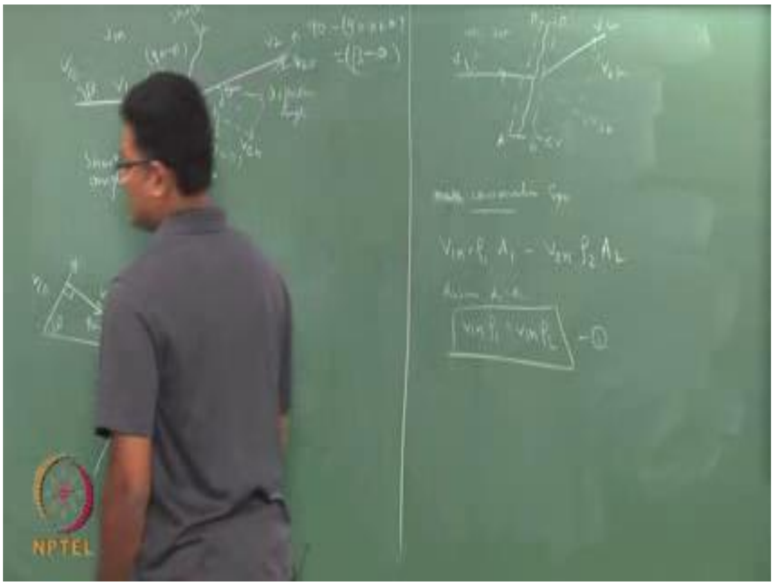
Then I have a V 2. So, I can redraw the same picture in that particular fashion. I will just rotate in my angle rotated my angle in this coordinates by this angle and this is my oblique shock. So, all I have done is added a tangential component to what we have already studied that is all I have done. Now, some notations which we are going to follow, beta is my shock angle, theta is my deflection angle and obviously t is for tangential, we have already written tangential component of velocity, n is the normal to the shock. So, what is beta and what is my theta?

So, if I have a flow then I have a shock wave that is here, the shock angle is this angle this is my V. So, this is my V 1 and this is my V 1, this is my beta or the angle which the flow makes with the shock is my beta and the deflection angle is. So, if my velocity is deflected this way. So, from V which is going this way my velocity is deflected this much that is my theta. So, this is my shock, beta is my shock angle and the velocity here that is deflected is my velocity deflection angle. So, we will fix this notation for the few lectures that is to follow. This is the notation then if normal is this V 1 n tangential is parallel to the shock. So, this is my V 1 t then this angle is my beta and here the normal is V 2 n, tangential is that is along the shock waves. So, that is my V 2 t.

So, this angle would be 90 minus beta. So this angle is 90 minus beta which is same as this angle. So, this is 90 minus beta. So, this would be 90 minus 90 minus beta plus theta is it fine this is theta. This angle would be this angle which is 90 minus beta. So, this and this should add up to 90. So, this is beta minus theta. So, if I redraw this, I have the shock I have V 1 and it is deflected to V 2. So, the defection angle is theta this is my beta. So, the tangential component is V 1 t. So, beta with V 1 then I have a normal, this is 90 degrees this is my V 1 n. So, this is 90 minus beta. So, the normal we can extend it here. So, that is my V 2 n then I have a tangent that is coming here that is my V 2 t, this is 90 degrees. This angle is same as this angle which is 90 minus beta and this angle is 90 minus this angle put together that would be beta minus theta. Now, we know all the components and all the angles.

So, this figure we have to familiarize and try to do the problems. The important angles are this beta which is the angle between the fluid velocity direction and the shock angle and this deflection angle is the velocity which is been deflected by the shock. So, if there was no shock the velocity would have been going straight. Now, it is deflected by theta that is your deflection angle. So, these 2 angles you have to be very clear and everything else will follow from there. Tangential velocity is parallel to your shock, normal velocity is normal to your shock that completes your velocity triangle from which you can find all this. Now, once we have this we will apply our continuity equation and energy equation across the shock like we have done for the normal shock. So, I have a normal shock.

(Refer Slide Time: 15:14)



I take a control volume here. Now, I have a flow that is coming here and then going this way and then I have the normal component V 1 n and the tangential component V 1 t likewise, you have a normal component here V 2 n and the tangential component V 2 t. Now, I am going to apply control volume approach or the mass conservation around this particular control volume that is around the shock assuming the thickness to be very small and there is no fluid that is going to the other direction. The only fluid that is crossing here is through the face of the. So, if I A and B, C and D the fluid is crossing through only A B and C D, there is no fluid that is crossing A C or B D.

If that is the case then I can apply my continuity equation or mass conservation equation which is V 1 n into rho 1 into A 1. So, the fluid that is crossing the area it is a normal velocity is same as V 2 n rho 2 A 2 assuming A 2 equal to A 1 assuming the control volume to be very thin, I can assume A 1 equal A 2, I have V 1 n rho 1 equal to V 2 n rho 2. So, if I have a normal shock with velocity V 1 n and then I can find their relation we have we have written for normal shock V x rho x equals V x rho y. So, that is my equation 1. Now, I will do the momentum apply the momentum equation to this control volume. So, I am going to rub this.

F equals around the control surface V to d a. Now, I have 2 components tangential of this and normal component of. So, the tangential component and normal component will differ when this would be my normal f normal and f tangential. So, f sigma f tangential equals integral V tangential rho V d a, tangential sigma is 0 we are not considering any tangential force here. So, this is 0 we can apply v tangential rho into this would be the mass. We have written there rho 1 V 1 n into d a 1, so this would be I can write V equal to V 2 t 2 rho 2 V 2 n into A 2 n A 2 and A 2 cancels out. So, you end up with this equation rho 1 V 1 n V 1 t is equal to rho 2 V 2 n V 2 t this is my tangential component 2.

So, for the normal component f n normal, it is going to have the static pressure involved. If this is my shock there is pressure at on the control volume. So, the force is pressure into area where we have assumed area to be same. So, my sigma f normal is p 2 A 2 minus p 1 A 1, which is equal to rho 1 V 1 n V 2 V 2 n minus rho 1 rho 1 V 1 n into V 1 n. So, this is mass into velocity, mass into velocity. So, I would get relation like this p 1 rho 1 V 1 n square p 2 p 2 plus rho 2 V 2 n square, there is a sign that is missing here.

Student: Right hand side.

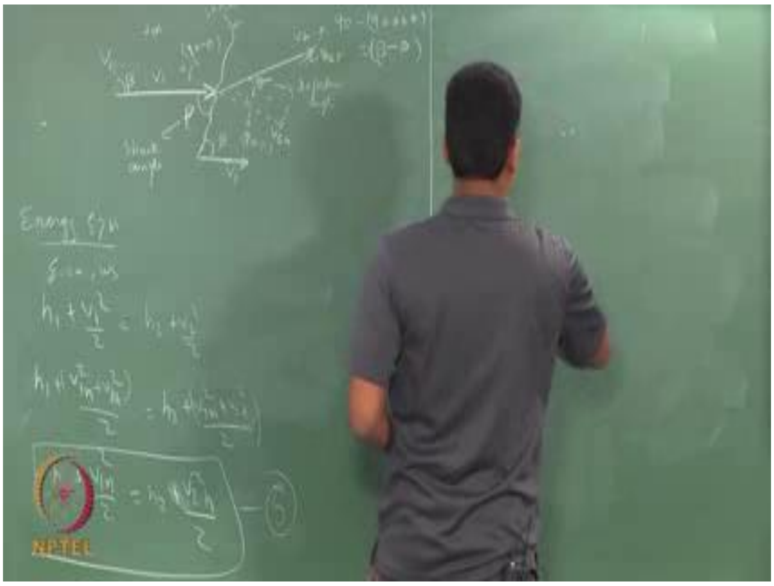
Right hand side, this would be A 2, A 1 and this would be minus this should be plus. So, that would take it this side this that side that and cancel out A 1 and A 2. So, I end up with this equation which is my equation 3 So, I will write down equations here rho 1 V 1 n V 1 t equals rho 2 V 1 n V 2 t that is my equation 2, and V 1 n rho 1 equals V 2 n rho 2 that is my mass conservation which is my equation 3. Now, if I substitute 3 in 2, I would get V 1 t equal V 2 t this is a very.

Student: (Refer Time: 24:24) equation.

This is equation 1. So, equation substitutes equation 1 in equation 2. Substitute equation 1 in equation 2 I would get; I can cancel out these 2 terms. So, I would get v equal V 1 t equals V 2 t which means the tangential component or the oblique shocks are is equal is identically same in magnitude. So, this deflection tangential component which we had drawn here would be modified such that V 1 t is same as V 2 t that is very powerful information we have. So, the tangential component before the shock and tangential component after the shock remains the same, your v normal and V 1 or the resultant velocity and the normal velocity changes whereas, the tangential component remains the same.

So, now we will proceed further now what we will do is we will apply our energy equation. So, I will leave this figure I will apply our.

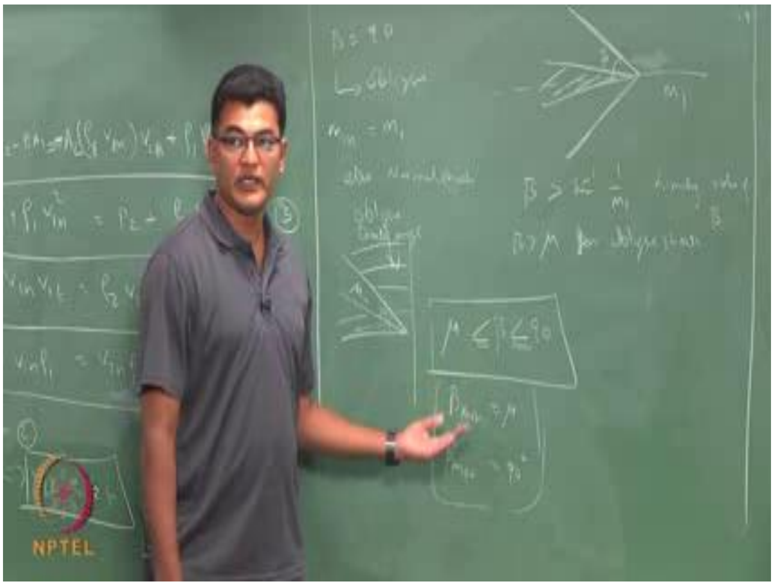
(Refer Slide Time: 25:41)



Energy equation energy equation with no heat and no shaft work your h 1 plus V 1 square by 2 equals h 2 plus V 2 square by 2, but V 1 is now the resultant of this. So, h 1 plus V 1 n plus V 2 V 1 square plus V V 1 v square by 2 equals h 2 plus V 2 n plus V 2 t square equals 2. So, this is my equation four V V 1 t equals V 2 t is my equation 4. So, now, if I substitute equation 4 here my V 1 t square is same as my V 2 square. So, I can cancel that out. So, I would end with this equation which is my equation 5.

So, my energy equation looks same as my normal shock equation with the component V 1 n. So, as if I get the component V 1 n then my energy equation I can use the normal shock tables or the normal all the information I can get from here and then I can substitute this also here the momentum equation also looks same with the normal component. So, I can use, essentially I can try to use the normal shock tables wherever possible that is what we are trying to get. So, if that is true then we can proceed further and define our mach number.

(Refer Slide Time: 28:23)



Mach number is V 1 by A 1, if I define mach number along the normal direction this is V 1 n by A 1 n. So, V 1 V 1 normal direction is this component of V 1. So, it is sin of deflection angle correct right, its sin beta its cos ninety minus beta which is sin beta V 1 by A 1. So, my m 1 n is nothing, but V 1 by A 1 into sin beta which is m 1 sin beta now from normal shock we know that this has to be greater than 1 to have a shock. So, my m 1 n should be greater than 1 for a shock or m 1 sin beta 1 should be greater than 1 or your sin beta should be 1 by m 1 or equal to greater than or equal, greater than 1 by this thing to have a shock which is beta is sin inverse I by m 1 to have a shock.

So, if I define my beta to be this then it has to be equal to or greater than to have a shock. So, it is greater than or equal to this to have a shock. Now, this is also this definition is also my mach angle which is mach angle mu. So, if i have a oblique shock. So, this is my shock angle the pardon.

Student: (Refer Time: 32:30).

On this side this would be my beta. So, this beta has a limiting value of my mach angle. So, I cannot have beta smaller than mach angle. So, beta should be greater than sin inverse 1 by m 1. So, it is always should be greater than m 1, m 1 is this. So, if my beta is smaller than this I cannot have a shock angle because m 1 n, m 1 n should be greater than one. So, if I use that condition, I would end up with this condition to have a limiting value, this is my limiting condition. So, beta should be greater than my mach angle for oblique shock I cannot have shock inside my mach column that is what it essentially means. So, this is when m equals 1. So, the limiting condition is m equals 1. So, when m equals 1 you anyways do not have a shock. So, mach angle is the coalition of all the waves that is generated by the sound waves. So, this surface when you have this condition is also called the mach wave because of that reason. So, it is a wave that is generated and it is also the shock waves that is, either way it is a limiting condition.

So, this is 1 limiting condition the other limiting condition is when beta equals ninety degrees. So, when beta is 90 degrees your oblique shock is m 1 n is same as your m 1 oblique shock is also your normal shock. So, the oblique shock angle goes from for example, if I have a shock somewhere here this goes from beta to 90 degrees. So, in between this region is my oblique shock angle. So, it has to be greater than mu and less than 90 degrees. So, beta lies between greater than mu and less than 90 degrees greater than or equal to this equal to 90 is normal shock. So, beta should be between this beta max is 90 degrees beta minimum is mu.

Next class, we will try to derive relation between beta, theta and your m; your mach number for a given mach number can we relate, can we get a unique value of beta and theta.