Fundamentals of Gas Dynamics
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Week – 02
Lecture - 03
Control Volume Approach

In this lecture, we are going to discuss the governing equation. We will try to derive the governing equation using Control Volume Approach. So, what we are discussing is the fundamental physics that happens when a fluid is flowing, essentially the equations which we are derived in thermodynamics and fluid mechanics is what we are going to use here and we will use the control volume approach for deriving the fundamental laws which is conservation of mass, conservation of energy then essentially the second law of motion, the momentum conservation.

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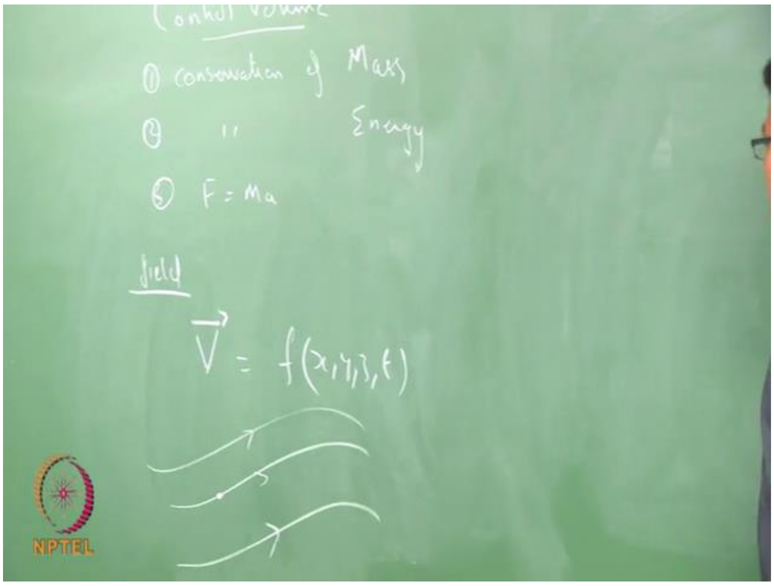

So, before we start this should understand what is a field or what is a 2 D or a 3 D flow? What dimensionality of the flow? So, your velocity V is a function of x, y, z and t. So, if I have a fluid flow in space, my velocity here is depending on the location of the fluid particle and the time at which you are measuring the velocity, likewise the other properties pressure, temperature, density everything would be some function of x, y, z and t. It need not be the same function, it can be different function it is a different function.

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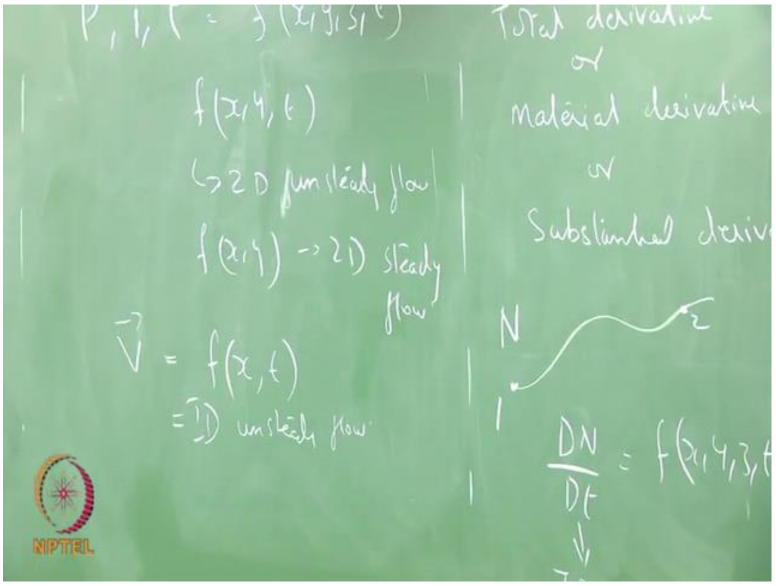

Now, if it is a function of these 2 variables, 2 dimension and time. Then this is a 2 dimensional on steady flow. If the time dependency is absent then this is a 2 D steady flow and if your velocity or any of those property is just a function of some direction, some time then this is a 1 D, 1 dimensional on steady flow. So, a fluid particle that moves from here to some other location, the velocity also changes due to the movement from point 1 to point 2 as well as the time at which it.

So, if I have a property that is associated with the particle that moves from point 1 to 2 then the rate of change of that property as we move along with the particle is called the total derivative or the material derivative or the substantial derivative of that property. So, it is the rate of change of that property if you move along with that particle. So, from point 1 to 2 some property N changes from 1 to 2 and this N depends on z and t.

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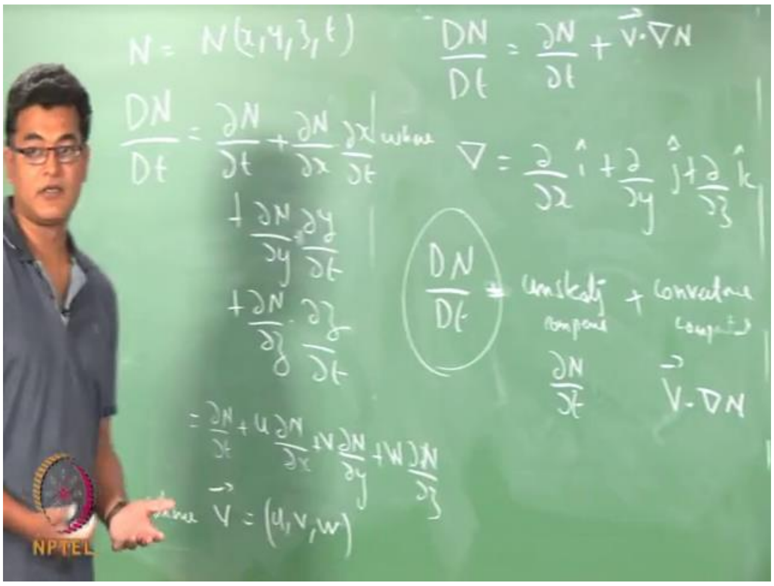

So, if your N depends on x, ,y, z and t then your total derivative which is denoted by the capital D is partial D N by D t plus D t plus dou N by dou y plus dou y by dou t plus dou N by duo z into dou z by dou t, which is dou N by dou t plus u into dou N by dou x plus V into dou N by dou y plus w into dou N by dou z where your velocity vector has components u, V and w in the x, y and z direction.

So, if I rewrite that pictorially, I can write it like this where del is dou by dou x by i plus dou by dou y j plus dou by dou z k in the cartesian coordinates. So, that is the expansion of your material derivative. Now, let us look at what exactly it means. So, your D N by D t is a combination of 2 terms; one is the unsteady component meaning the property that changes from time t to time t plus delta t. So, N is a function of time also. So, this N that is measured at some location is going to change or it will be different, but if you measure it at a different point of time that is this part D N by D t.

Second, the second term on the right hand side is something called a convective component or convective change of the changes of N due to the convection of the particle. So, if I measure property N at location 1 and I measure property at location 2 at the same instance of time then there will be a difference because f N is a function of x, y, z as our the properties of function and space its going be different in space.

So, at the same instance of time you are measuring N and that difference because the particle is going to move from point 1 to 2 and you are moving along with the particle you feel a difference in property N and that is due to this particular term which is V dot grad of N. So, N can be any property. Now, we are going to evaluate this property or this derivative the rate of change of property N for a given control volume. So, let us look at what is a control if I have a fluid flow.

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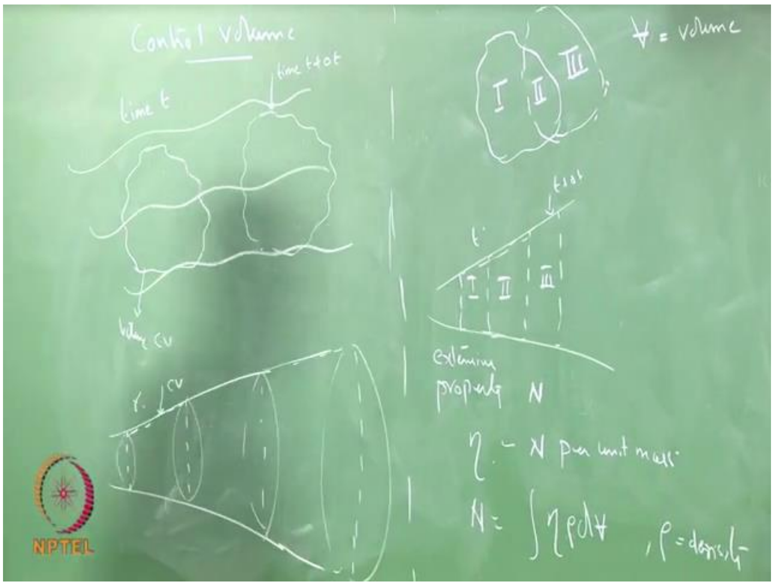

I consider a mass at time t which occupies a volume, which I define it as my control volume. So, this consists of several particles which contribute to the mass of the control volume. Now, I consider at a different point of time what happened to the mass that has inside this particular control volume. So, this control volume changes to this particular volume at time t plus delta t the particles that is, constituting or contributing to the mass is carried to this particular volume. So, the volume might have changed the mass is the same mass for the particles are the same particles.

If you have a difficulty in imagining this arbitration shaped volume, consider a divergent pipe. So, you have a volume here, this would be circular shaped. So, this is my control volume at time t after sometime at time say t plus t 1, time t 2 I have a different control volume. So, the mass that is associated with the particles inside this control volume is the same to the mass as carried, but the control volume has changed.

Now, we are going to look at a time where this delta t is small which means. If this delta t is small, my arbitrary control volume would have moved to a distance such a way that there is an overlap of region. So, if delta t is very small this control volume would have moved a little bit and that will overlap with the other volume. If you look at this way my control volume here at time t is now moved to this time t plus delta t. So, essentially I have region 1 and an overlap region 2 and the new region 3. Here, again 1, overlap region 2, 3. So, associated with this mass there is a property N. So, it depends on the mass of the fluid that we are considering, it is an extensive property.

Now, I define introduce a new variable eta which is property n per unit mass or the density of the property N. So, this is my control volume or in a given mass, your N is now defined as integral eta D V, D volume; V stroke is where rho is the density of the fluid. So, essentially eta into D M and density into volume is here D M and this is integrated over the volume. Our aim is to find the total derivative D N by D t, the rate of change of this property when it moves from time t to time t plus delta t, what is the rate of change of that particular property?

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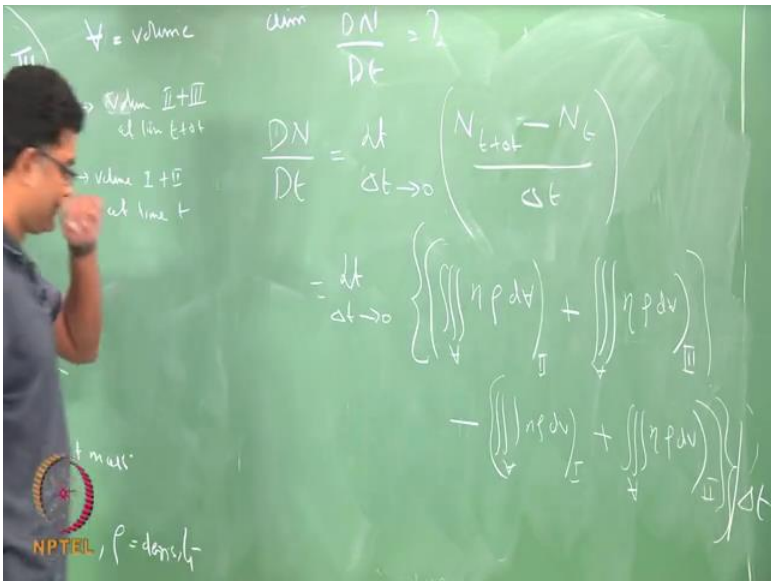

Now, from the fundamental calculus I can define D N by D t to be limit delta t tending to 0, N at t plus delta t minus N at t divided by delta t. Now, N at t plus t is limit delta t tending to 0, N at t plus delta t. So, at t plus delta t your control your volume is the region 2 and 3 at time t plus delta t, volume 1 and 2 is at time t. So, your property N with this definition, control volume; integral over the volume eta rho D V for the region 2 plus eta rho D V for the region 3 minus same thing, for region 1 plus for the region 2 divided by delta t. So, I rewrite that limit delta t D N by D t. So, this is at t plus delta t, this is at t.

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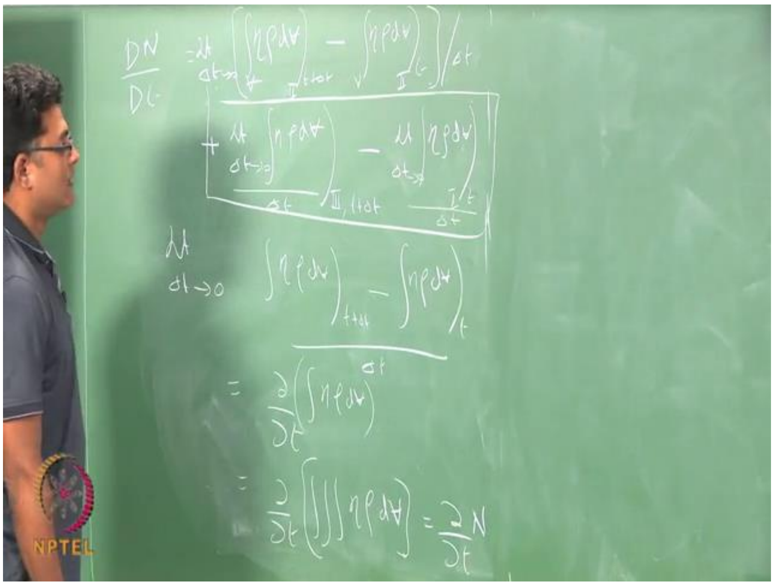

So, I take region 2 integral over the control volume eta rho D V at t plus delta t minus integral volume eta rho D V at t. These are triple integrals plus limit delta t, this is for region 2, this is for region 2; plus the region 3 integral eta rho D V integral this is at region 3 at time t plus delta t minus limit delta t integral eta rho D V for region 1 at time t.

So, the first part, limit delta t this, the first term limit delta tending to 0 integral eta rho D V at t plus delta t minus integral eta rho D V at t by delta t that is a my delta t, this is the definition of differential dou by dou t of this quantity which is integral eta rho D V and these are triple integrals. So, you could write this as N, which is dou by dou t of N. So, this term is the unsteady part of the material derivative of N.

Now, we will look at what happens to the other 2 terms. So, our aim is to evaluate these 2 terms. So, let us take the arbitrary surface we have already taken, take a small area, the normal to the area is in this direction and the velocity vector is in some other direction. Volume of fluid that is coming out of this area small area D A is V dot N into D A or simply V dot D A vector.

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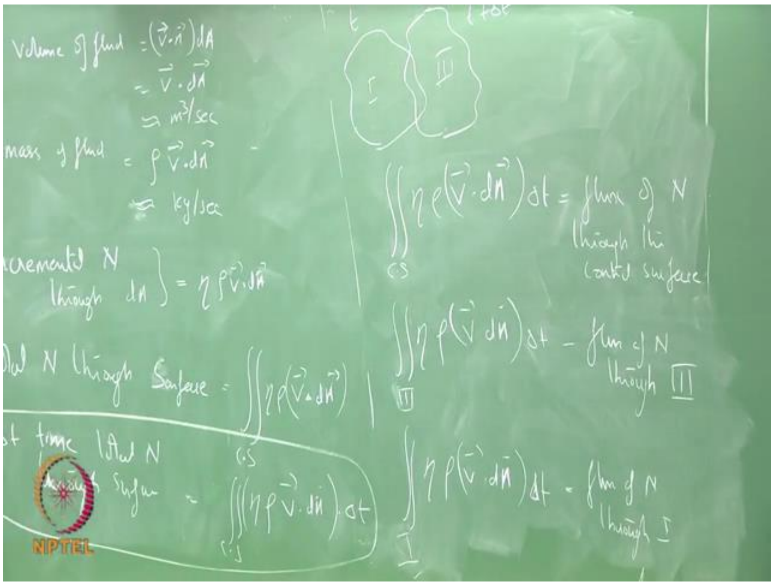

The mass that is coming out, the unit of this is meter cube per second, the volume flow rate that is coming out of this. So, the mass that is coming out of the small area D A is this quantity multiplied by your density. So, this would be kg per second. If that is the mass that is coming out of A then incremental quantity of N through the area D A is this quantity which is the D M which is the mass into eta which is defined as the quantity per mass. So, the total N through the entire surface is the integral of the double integral over the full control surface into this quantity eta rho dot D A, this is per second.

So, in delta t time, total N through the surfaces this quantity multiplied by delta t. Now, if you look at the control volume and the volume that has moved at delta t time, we can relate this to the following. So, I have control volume at time t then I have the volume at time t plus delta t. So, there is a property that is here at 1 and there is a property that is here at 3. So, the quantity of the total property N that is going through the surface at delta t time is this, what you see here and that is precisely what you have written here. So, if I take the material derivative, the material derivative is the change of fluid property that is happened when the fluid moves from 1 to or from this volume to this volume at t N t plus delta t.

So, this quantity what you have written here the total property N that is moved out of this control volume is this, likewise the fluid property that is moving in to the control volume is this what you have written here. So, this is this quantity, essentially the integral C S eta rho V dot D A delta t is the flux of property N through the control volume, through the control surface or if it as a region 3 and this is eta rho V dot D A delta t, it a flux of N through region 3 as this is the region 1 eta rho V dot D A into delta t is the flux of N through 1 in delta t time, or the net efflux is the flux out minus flux in which is integral 3 eta rho V dot D A D t delta t minus or simply integral around the control volume rho V dot D A in delta t time.

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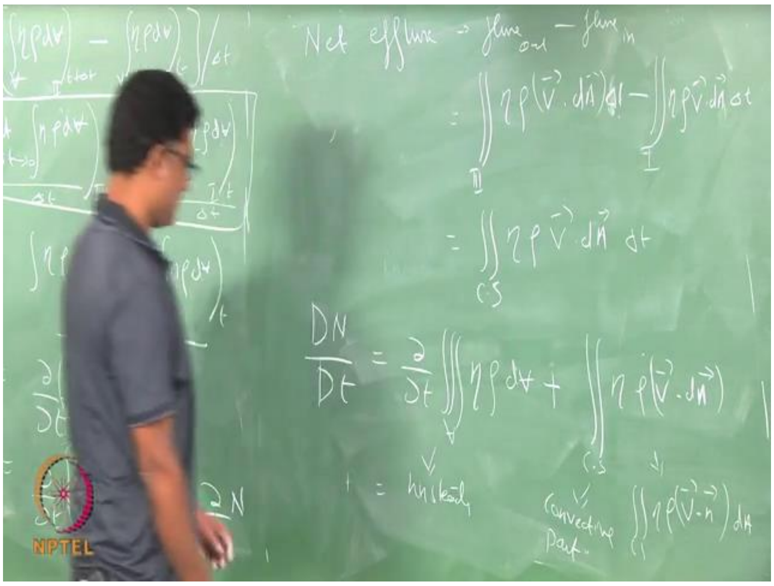

So, this second and third term in the right hand side of the original equation is now equated to the net efflux that is happening. So, your D A by D N by D t the material derivative of property N is dou by dou t, your triple integral which is volume integral eta rho D V which is this particular term, which we are written originally plus the net efflux term which is this the property N that is getting out minus property N that is getting in which is triple integral C S eta rho V dot D A, or simply we can replace this as eta rho V dot N dot into D A. So, this is your unsteady part of your property change and this is your convective part and this equation as typically known as the Reynolds transport equation because you are trying to find the transport of the property N.

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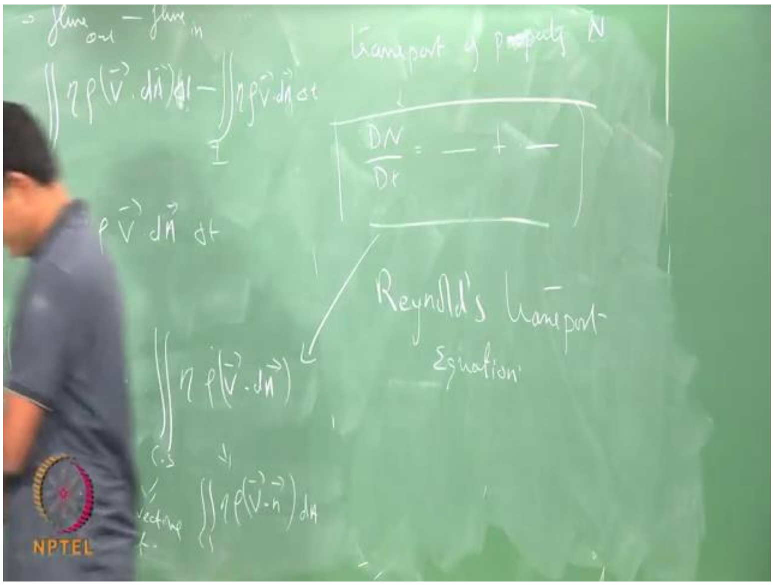

Property N is given by this equation D N by D t equals these 2 terms and this equation is Reynolds transport equation.