Fundamentals of Gas Dynamics
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Week – 10
Lecture – 36
Moving Normal Shocks

So, this week or this lecture we will discuss how to find a property across a moving shock. So, when a moving shock is something that, what we have going to consider is, a shock that is moving in a stagnant fluid. So, we will not consider a moving shock in a moving fluid. So, the example that we are going to consider for this course is just, a shock that is moving in a stagnant fluid, like a shock that is moving when there is bomb explosion.

So, we have explosion, and there is a shock, and this shock moves across the space, and as it moves it changes the properties behind the shock. So, still then there is no change, as soon as the shock crosses a point, the properties behind the shock changes. So, that is what we will consider, and we will consider only a normal shock for the time being.

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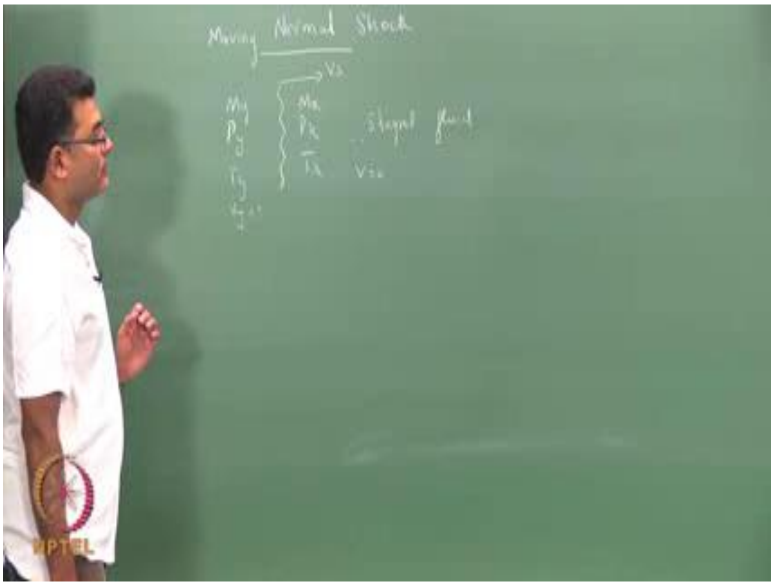

So, I have a shock which is moving with some velocity. So, this is stagnant fluid or a gas, and as it moves, it changes the properties here. So, here it is my p y m y and t y. So, before the shock, its my m x p x t x. So, the fluid here has 0 velocity before. So, v equal 0 in this case, but as the shock comes and hits the point the velocity changes. So, I have a v y. Now we will do the proper notations now.

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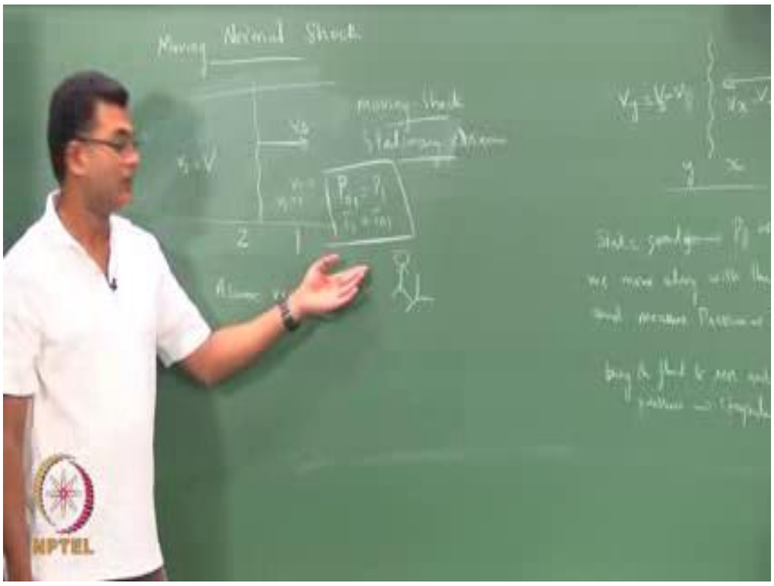

So, I have a shock, and I am an observer standing outside this. So, there is a frame of reference associated with the observer, like what we have done when we evaluated the velocity of sound. So, I have a pressure wave that is moving. Now here pressure wave is not infinitesimal small, but a finite value pressure wave that is moving, which is a shock, unlike the sound velocity which is an infinitesimal small pressure wave. So, I call this as. I will use the notations. So, this is a moving shock, or stationary observer. So, the fluid, this is my direction of shock moving.

Any locations, before the shock is 1, any location after the shock is 2. And as soon as the shock hits the point, the velocity here would change from 0 to some value. So, v 1 is 0 here, v 2 is v. So, this is an unsteady problem. So, I have a shock moving and stationary observer. So, I see the shock that is moving across. Now v s is constant, or I have a uniform velocity. In that case like when we derived our velocity a sound, I can have an observer that moves along with the shock and try to evaluate, convert this unsteady problem to a steady problem. So, I would use at moving observer, or stationary shock.

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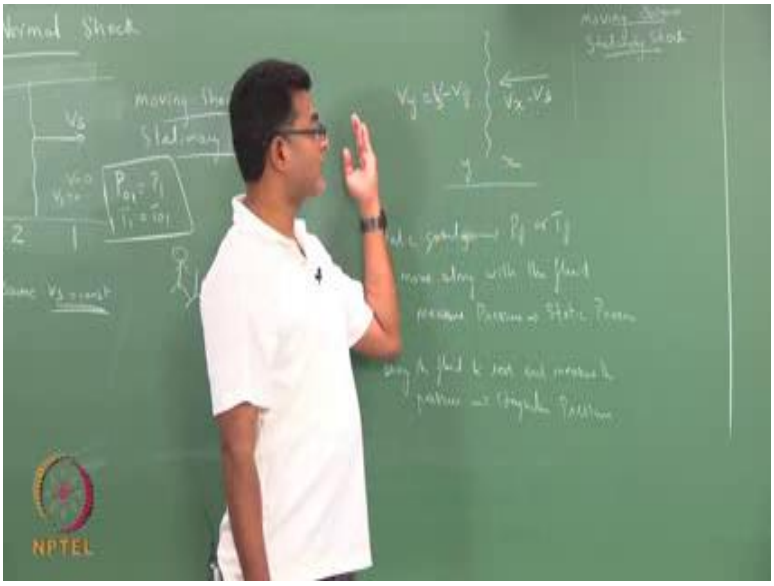

So, if I am moving along with the shock I would feel the shock to be stationary. If the stock shock is stationary, I can use the ratios that we have derived in the previous classes to evaluate the properties. So, let us see what all the properties that would change when we do this. So, here I would have x and y x is before the shock y is after the shock or front of the shock behind the shock as we call that in the stationary shock. So, the shock if you are an observer standing in the shock, I would see the fluid that is approaching me, with a velocity v s, and I would see the fluid that behind me residing at v minus v s, or v s minus v.

So, I am moving along with the shock. So, for example, in this case I a standing here and watching the shock. So, that would have velocity v s which is moving fast. So, I am moving along with the shock, I would see the fluid that is approaching me with v s and residing me with v v minus v s; that is what we have. So, my v x is v s v y has this. Previously we had v 1 is 0 and v 2 as v here my v x is v s and v y is. In fact, that is the only notation that we leave, as long as we are we do not confuse these 2 notations we are into the problem.

So, we have assumed constant v s. Now what is a static quantity, how do we measure static quantity; for example, p 1 or t p or t, if we move along with the fluid, and measure my pressure, then I am actually measuring what my static pressure. If I bring that first, and measure my pressure, I get stagnation pressure. So, if I have a fluid, if I moving along with the particle and measure, my pressure or temperature, I would get my static pressure or static temperature. If I isentropically bring it down to 0 velocity, and measure my temperature or pressure it is called a quantity that we measuring as this stagnation pressure and temperature. This is something which we had defined it during our definition of stagnation state.

So, now, we have situation here where we are moving along with the shock. So, if I measure the pressure that moving, if I move along with a shock I would be measuring my static pressure. So, I measure the temperature when I move along with, it will be static temperature. So, if I look at case 1, where I have a stationary observer and I have shock that is moving, the pressure here before the shock approaches with a stagnant fluid. So, the pressure here is my stagnation pressure, which is also my static pressure. So, my p 1 is also equal to my p 01.

And hence my t 1 that is measured is also equal to my t 01. So, that is the first information we have. Now we go to the other coordinate system case 2 where there a standing shock now fluid approaching with v s and fluid proceeding with v s minus v. Since it is a standing shock we know that the stagnation temperature is the same. So, I will write here as 2 cases; 1 is stationary observer, and the moving observer, or you can write this as a moving shock and stationary shock.

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So, in stationary observer my p 01 is same as my p 1, and my t 01 is same as my t 1 and moving observer we have a standing shock, which means my t 0 x is same as my t 0 y. and now this static quantities which we are measured from this moving stationary coordinates which is also same as my moving coordinates, because I moving along with my fluid, the p x that is measured here, same as my p 1.

So, I would write this quantity. So, the p x is what the static pressure, which is measured when I move along with the fluid; that is same as my p 1. Likewise my t x which is a temperature that is measured when I move along with the shock is same as my t 1. So, my t 1 is same as my t x. Now, I use continuity equations, momentum equation, and other set of equations to derive the following relations.

So, as I wrote here, I would complete this; v x is my v s in the moving coordinates, v y is my v s minus v, v 1 is 0 v 2 is v. As long as we are comfortable with this notation we can proceed further. And this static quantities aftershock is also. So, you can write p 2 is my p y and p y is my p 2. Likewise t 2 also, because it is a static quantity, which is not changing, because of the coordinates whether it is moving or this thing; in any case the static quantities measured when you move along with the fluid. So, whether the observer changes its position or not, it is not going to change your definition of your static quantities.

So, static quantities are going to be the same. So, now, we apply the momentum equations and other things. So, the notation I would again rephrase it here. So, stationary coordinates or observer, I have used x and y, and moving coordinates I have used 1 and 2. This notation we would keep it. This notation is very important, unlike in other classes where we could mix up with our notation and still get convening argument. Here if the mess up with argue notation we are going to mess up with the arguments. So, now, I have m x; m x is v s by a x.

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Stationary coordinate is the 1 and 2 moving coordinates is x and y. So, we keep this notation, which defines new quantities, mach number x y before, and mach number across the shock in a moving coordinate system and mach number across the shock in a stationary coordinate system. So, we define v x by v x by a x v x is v s. So, I would write v s by a x, and m y is v y by a y which is v s minus v by a y. m 1 is v 1 by a 1 which is 0, m 2 is v by a 2 which is v 2 v by v 2 by a 2 which is v by a 2.

So, my a x is root gamma r t x, which is same as root gamma t 1 which same as my a 1 a y is root gamma t y which is same as root gamma t 2, which is my a 2. So, my a x is same as my a 2, a y is same as my a 2, because of this quantity. So, a y is same as a 2, a 2 same as a y and a 1 is same as my x . So, I can write my m y in terms of m 1. So, I will be assuming keeping this notation interact. I am going to rub this particular portion.

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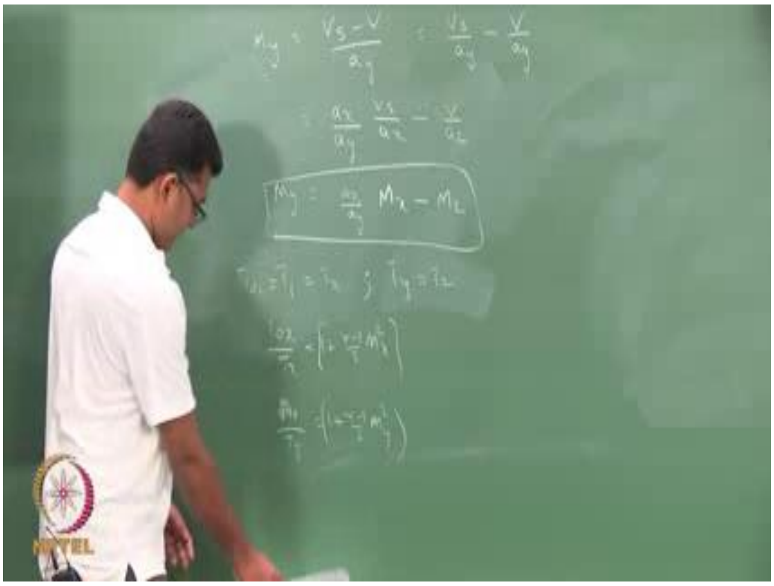

So, my m y is v s minus v by a y, which is v s minus a y minus v by a y, which I multiply by x a x minus v by a y is v by a 2 v by a 2 is my m 2. So, I can write this as a x by a y into v s by a x is m x minus m 2. So, if I replace this with gamma r t I can write it in terms of t. Now, we know t 1 equals t x which is also t 01. So, t 0 x by t x is 1 plus gamma minus 1 by 2 into m x square t 0 y by t y is 1 plus gamma minus 1 by 2 m y square, but t y is also t 2. So, I can replace this with 2.

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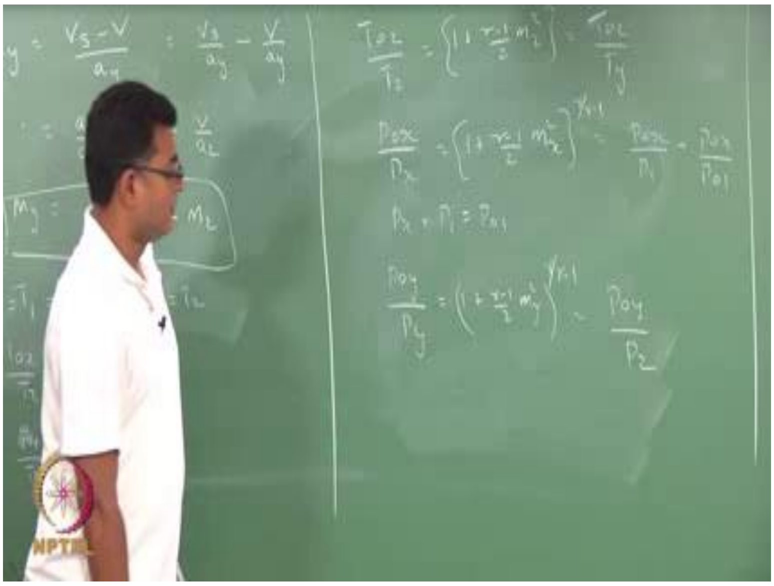

So, I can write, likewise I can write for p also p 0 x by p x is 1 plus gamma minus 1 by 2 m x square into to the power gamma by gamma minus 1. Now we also know p x equals' p 1 equals p 01. So, I can replace this with these quantities, any 1 of these quantities which is equal to p 0 x by p 1 which is equal to p 0 x by p 01. Likewise p 0 y by p y is 1 plus gamma minus 1 by 2 m y square to the power gamma by gamma minus 1, which is equal to p 0 y by p 2, because the static quantities are same. Now I am not going to rub this, but I will rub this portion and write this equation, maybe I would re write that equation here; m y equals a x plus a y into m x minus m 2.

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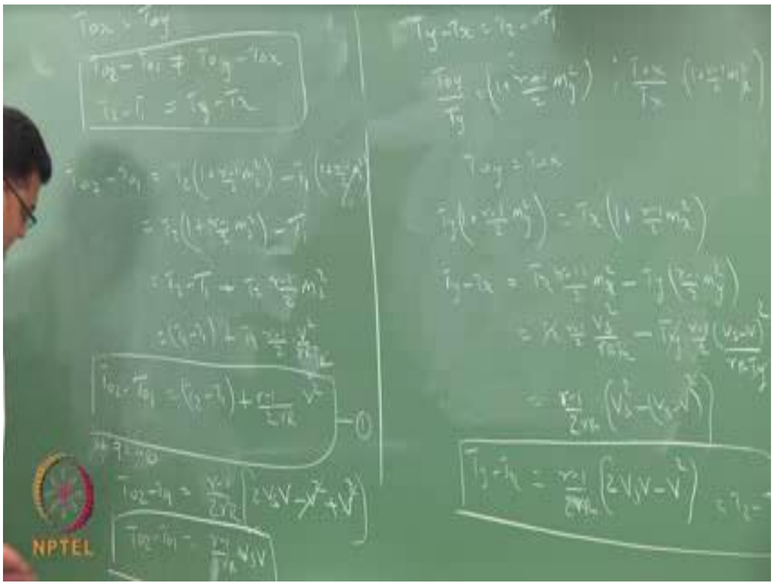

We know across the standing shock which is my moving coordinate system my t 0 x equals t 0 y. So, you eventually try to find what is t 01 minus t 0 t 0 2 minus t 01 not equal to t 0 y minus t 0 x where as my t 2 minus t 1 is same as my t x t y minus t x. So, you can show this. So, my t 0 2 minus t 01 is t 1 t 2 into 1 plus gamma minus 1 by 2 m 2 square minus t 1 into 1 plus gamma minus 1 by 2 into m 1 square m 1 is 0.

So, this term is 0. So, I have t 2 minus t 1 I will write the whole thing 1 plus gamma minus 1 by 2 m 2 square minus t 1 I take t 2 minus t 1 plus t 2 into gamma minus 1 by 2; m 2 square, which I replace m 2 in terms of mach number and m 2 in terms of velocity v 2 which is v by a 2 which is ay or a 2 and replace a 2 as gamma r gamma r t 2. So, I strike t 2 minus t 2. So, this would be t 2 minus t 1 plus gamma minus 1 by 2 into r gamma v 2 square. Now if I substitute t 2 minus t 1 in terms of velocity, then I can get this whole equation in terms of velocity which is not equal to this quantity which I will do that here. So, this is my equation 1.

Now, I would try to find t 2 minus t 1. So, my t y minus t x is my t 2 minus t 1. So, I would find this, this is again t y by t y is 1 plus gamma plus 1 minus 1 by 2 in the m y square and t 0 x by t 0 t x equals 1 2 plus gamma minus 1 by 2 m x square since t 0 y equals t 0 x I can write this as t y 1 plus gamma minus 1 by 2 m y square equals t x into 1 plus gamma minus 1 by 2 m x square. So, t y minus t x equals t x into gamma minus 1 by 2 m x square minus t y into gamma minus 1 by 2 m y square which again I replace this in terms of velocity sound and velocity of the fluid.

So, t x into gamma minus 1 by 2 into v x v x is v s square divided by gamma r t x minus t y into gamma minus 1 by 2 m y m y is v y by a v by by a y v y is v s minus v the whole square divided by gamma r t y. So, t x t x cancels t y t y cancels, I end up with gamma minus 1 by 2 r gamma into v s square minus v s minus v is the whole square. So, this is nothing, but gamma minus 1 by 2 gamma r v s and v s are cancels out. So, you end up with the 2 v s v minus v square, which is also equal to my t 2 minus t 1, because the static quantities are same.

So, if I substitute this equation, equation 2 in 1 substituting equation 2 in 1, I would get t 0 2 minus t 01 as gamma minus 1 by 2 gamma r with in brackets 2 v s v minus v square then this plus v square which is nothing, but this quantity 2 minus t 01 is gamma minus 1 by 2 gamma r 2 2 1 can cancels, v s v. So, t 0 2 minus t 01 I will write that here t 0 2 minus t 01 is gamma minus 1 divided by r gamma v s v whereas, t 0 x equals t 0 y. So, that is this difference is different from t 0 y minus t 0 x.

So, your t x is also in terms of t o y minus t x is gamma minus 1 by gamma r to 2 v s v minus v square which is also equal to t 0 t 2 minus t 1 which again is not equal to t 0 y minus t 0 x. So, this is equal to t 2 minus t 1. So, difference in stagnation temperature is not the same, but difference in static temperature is the same. So, if you know these properties you can find the other properties; that is happening in a shock that is moving.

In the next class we will do a numerical problem and try to understand these 2 coordinate system, and evaluate all the properties in moving coordinate and in the stationary coordinates this notation, we would keep the same this is a very important, because that is what it deals all this. So, stationary coordinates 1 into moving coordinates x and y for this particular course.

Thank you.