Fundamentals of Gas Dynamics
Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week – 09
Lecture – 35
Normal shocks in C-D nozzles

This class we are going to discuss the Shocks - that is happening in a C-D nozzle in the diverging section of a C-D nozzle. So, if you look at the C-D nozzles that we have discussed.

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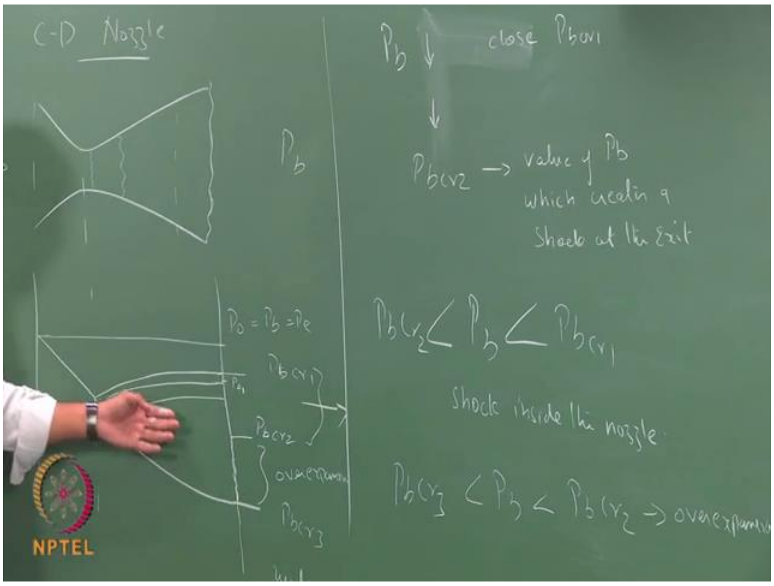

So, we have a C-D nozzle and then depending on the pressure that pressure here we have scenarios like, I have the throat region here when P 0 is same as my P b that is same as my P exit there is a nerve flow. And when I slowly decrease my back pressure you would get A throat condition which is choked throat, a choked condition at the throat and then in the diverging section you would get either a subsonic or supersonic isentropic solution depending on the value of your back pressure.

Now, in between, this we called as a P b critical 1 and this as P b critical 3. If you have a P b value somewhere between these two, you can expect a non isentropic solution which involves shock. So, there is some value of P b which will create a shock inside the nozzle for some value of P b more than P critical 3 you will have a shock somewhere outside the nozzle. So, we are going to consider that particular case where you have shocks inside the diverging section of the C-D nozzle.

So, if I decrease my P b to value smaller than P critical 1, if you decrease it if your P b, let us take this as some value. So, I am decreasing my P b now this is smaller than P b critical 1, but closer to P b critical 1 - meaning the flow in the throat is choked, but it somewhere here, the exit pressure is somewhere here closer to P critical 1 the flow is still chocked. So, you will have mach 1 here and it is an (Refer Time: 03:07) non isentropic solution. So, there is a shock somewhere sitting at the diverging section.

In this condition you would have a shock that is closer to your throat. So, I have a shock that is closer to the throat which will create a shock here which would so for some region it would be supersonic and then suddenly it faces the shock and the pressure increases to some value here. So, this is say let us take it as P b 1. If I have a pressure that is still further down and then I would have a shock that is seating further downstream we would go here and if I keep on doing that I would reach a pressure where I would have a shock seating at the exit. So, there will be some case some value of P b which would create a shock at the exit and that back pressure is your P critical 2. So, you would reach P critical 2 value of P b which creates a shock at the exit, exactly at the exit. So, this is my so called P b critical 2.

If your pressure back pressure is between P b critical 1 and P b critical 2 I have shock inside my nozzle, I have shock inside nozzle. We are going to discuss this scenario today. So, if your P b is between P b critical 2 and P b critical 3 then we have the over expanded nozzle over expanded case which we will discuss it later.

Earlier we had said the over expansion is for P b somewhere between these 2 values, now we have narrowing it to this particular case where you have an over expansion, so this is my over expansion and if I produce further I will have under expansion which is again something that is happening outside the nozzle we will discuss it later for the time being you are going to discuss this case. This non isentropic case where my P b value is between P r critical 1 and P r critical 2, this is what we are going to discuss today. So, I would revisit the movie which we had shown in few classes back.

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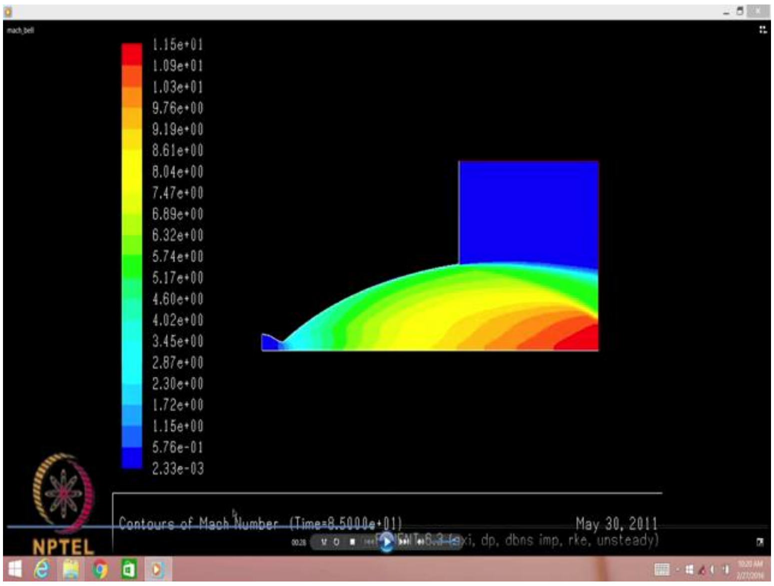

So, this is the case where you have the P b is getting reduced and your P b is between P critical 1 and P critical 2. So, there is a shock inside converge diverging section and as soon as the shock reaches the exit we call that as our P b critical 2, somewhere here.

For example in this contour the shock has just reached the exit. So, in this case the value of P b which creates this kind of shock that is seating just at the exit is your P critical 2 and any critical pressure lower than this, but more than P critical 3 is called our over expansion. Now, you see an over expanded jet outside the nozzle. So, if I play it further from here on it is over expanded, it is all over expanded now, it is again over expanded.

Now the contour the exit is smoothly out of the nozzle, you have a nice mach number contours which are continuous. So, you have this P critical 3 now, any pressure between P critical 3 and P critical 2 is over expansion, but today's class we are going to discuss the back pressures between P critical 1 and P critical 2.

I repeat we are going to consider this case where you have a shock seating inside the diverging section. So, we are trying to see again if this is a normal shock. What you see here is a normal shock with boundary layer and other interaction because this is a viscous flow simulation and it is 3D, 3D simulation; what we are analyzing is 1D simulation steady 1D simulation. So, there will be a standing shock that is here. So, instead of this picture what we see will be a normal shock seating at this location for a given P b which is been (Refer Time: 09:03).

So, now we will come back to this. We are going to analyze this case today.

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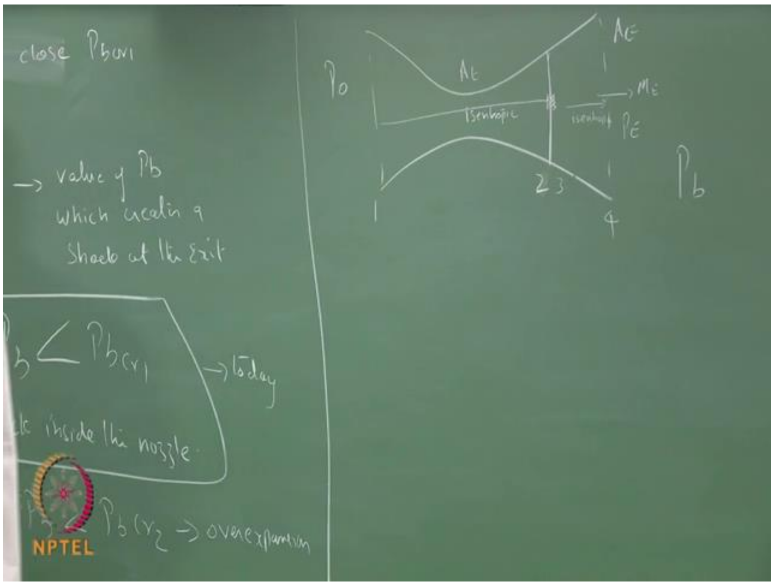

So, if I have nozzle and I have a shock here at some location, let us call this as location 2 this is entrance is location 1. Location 2 I have a shock after the location I called that as 3 and exit is 4 (Refer Time: 09:50) and that pressure is P b. For some value of P b for a given area ratio you have this particular case where there is a shock that is seating inside. Now, what would be the mach number here, what would be the exit pressure here is something that we going to see how do we analyze this.

So, as we have done before up to this point its isentropic flow then something happens which is a non isentropic process then again I have an isentropic flow. So, I will use isentropic tables to find the value up to the shock then, we use the normal shock table to find the properties across the shock then, we again use isentropic tables to find the shock after that. So, bus way to do this kind of analysis is to do a numerical problem and try to understand what exactly is happening. So, will just do a numerical problem and discuss that.

So, we use a combination of isentropic table and normal shock table question one.

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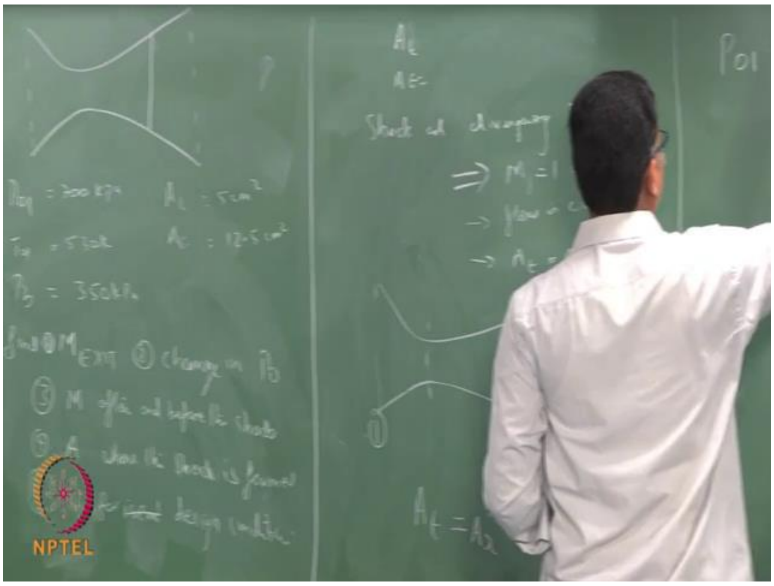

So, I have a shock somewhere at the diverging section. The stagnation pressure of the inlet is 700 kilopascal and the stagnation temperature is 530 Kelvin, area of the throat is 5 centimeter square, area of the exit is 12.5 centimeter square, back pressure is 350 kilopascal. First we will try to find M at the exit, but this question 1, question 2 change in P 0, M after and before the shock area where the shock is formed and what is your P b for isentropic or what is the P b for design condition.

So, your throat is given, A is given - now there is a shock that is formed at the shock at the diverging section implies M equals 1 at the throat or flow is choked at the throat or your A throat is also your A star that A star before the shock. So, if I use this as, inlet is 1, exit is 2 I have a shock somewhere here I call that as x and y, if this statement is true I could say A throat is my A x star. Now A x star we also know that P 01 into A 1 star equals P 02 into A 2 star for a non isentropic process which is what we have right now.

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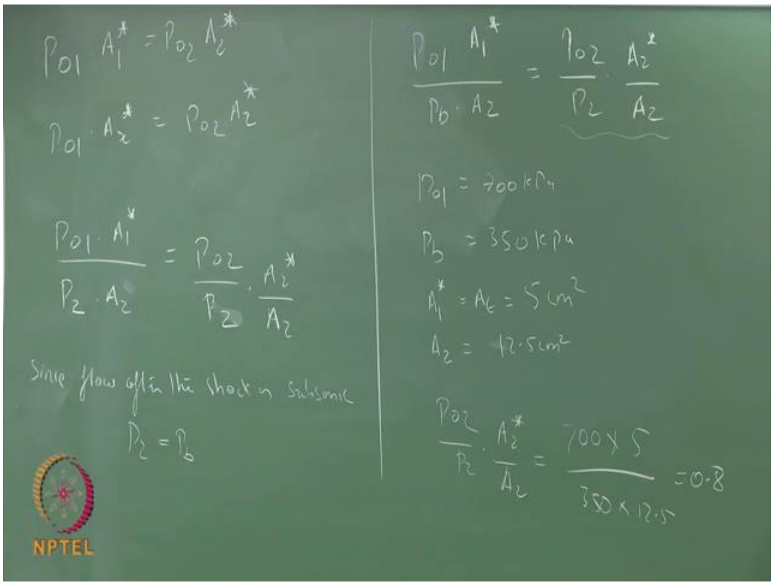

But A 1 star is now our A x star. So, my P 01 A x star is my P 02 into A 2 star. So, what is happening here I have an isentropic flow here just before the shock and then I have a non isentropic flow after that I again have isentropic flow. So, if I want to find this I divide by P 02 into A 2 star or let us take it as 2, then 2 here P 02 into A 2 star by A. Instead of P 02, I divided with P 2 because P b is given; P b is given, so I divided with P 2. Since flow after the shock is subsonic and the isentropic my P b 2 is equal to P b which is also my exit 2 is my exit, 2 is my exit. So, P b 2 is my P b.

So, I can rewrite this equation as P 01 into A 1 star divided by P b into A 2 equals P 02 by P 2 into A 2 star by A 2. P 01, we know P 01 is 700 kilopascal P b is 350 kilopascal, A 1 star is my A throat which is 5 centimeter square, A 2 is my A exit is 12.5 centimeter square. So, this ratio P 02 by P 2 into A 2 star by A 2 is this 700 into 5 divided by 350 into 12.5 which is 0.8. P 02 is also your P 0 y because that is your stagnation pressure after your shock and P 2 is your P b, and your A y star is your A 2 star because again that is an isentropic flow after your shock, this is after my shock.

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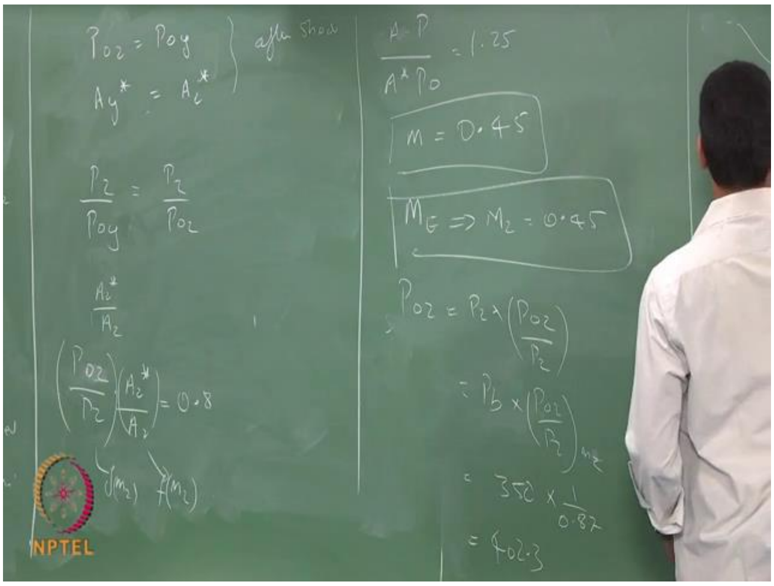

So, P 2 by P 0 y is my P 2 by P 02 and you have A 2 star by A 2 now this product P 02 or P 2 by P 2 by what is written there is P 02 by P 2 into A 2 star by A 2 equals 0.8. Now if you look at the tables. So, this is a function of M 2 this is a function of M 2. So, if I know the product I should get my M 2. So, if you look at the tables isentropic tables for gamma equals one point four you have this ratio given. So, this ratio is in the tables it is the reciprocal of that, in the tables you will see A into P by A star into P 0 which is our reciprocal of this should be 1.25, how much it is 0.8 – it is 1.25.

So, you look at 1.25, if you look at the tables for this particular value your mach number you would get it as 0.45, 0.45. So, this is the mach number that is happening at section 2 which means this is my M exit. So, I have my M exit here. Now if I have my M exit, I can find P 02 which is P 2 into P 02 by P 2, for this particular mach number we know the ratio from the tables again. So, your P 2 is now P b into P 02 by P 2 at M equals 2 at M 2. So, P b is 350 kilopascal into ratio P 02 by P 02 by P 2 for M equals 0.45 P 02 by P 2 would be 1 by 0.87. This would give me 402.3. So now, what have we done here?

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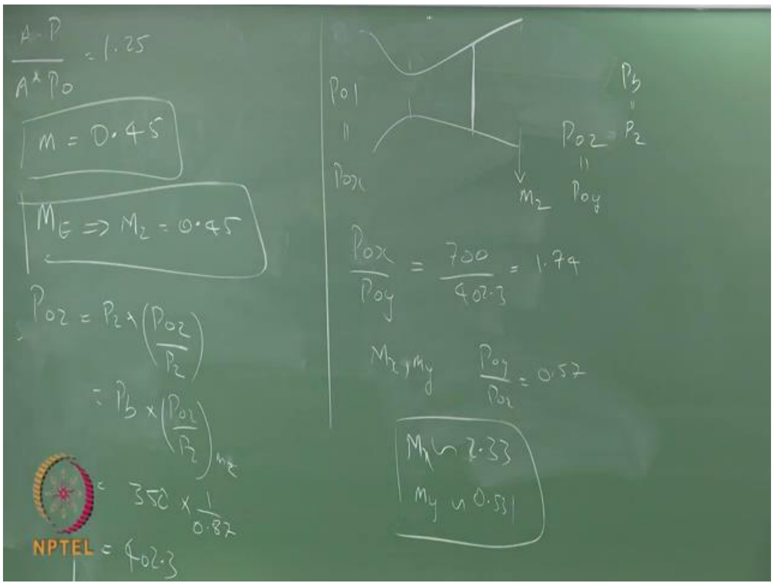

So, I have this shock here now we have found our M 2, we have also found our P 2, we already know our P 1, we know our P b which is also equal to our P 2. So, this is the subs this is the isentropic flow, this P 02 is going to be same as P 0 y which is equal to your P 0 y this is same as our P 0 x. Now, I know P 0 x by P 0 y which is which is 700 divided by P 02 is 402.3, that is my ratio.

Now, if I get that ratio I can find M x and M y. So, that ratio is how much 1.74 and that brings. So, I take the tables - normal shock tables I look for this particular pressure ratio P 0, the inverse P 0 y by P 0 x which is 1 by. So, I look at 0.57 P 0 y by P 0 x 0.57 – 0.57 is around 2.3. So, it is around 2.3, yes?

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2.33, approximately 2.33 which also tells me the M y to be 0 0.53 something, that is where I have my shock. So, now, if I know this shock location I also need to find the area at which the shock is being form. Now I all the quantities that is required to find the area I know my M x is 2.33, M y is 0.531, P 0 x, P 0 y is 402.3.

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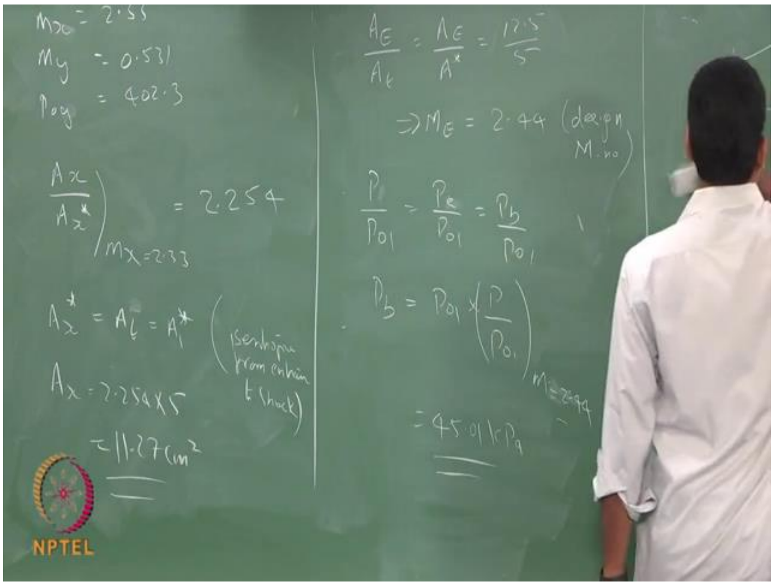

So, we know now we can find our area A x by let us take A x star for M x. So, what is the A x is the question. So, M x is 2.33. So, this again from the isentropic tables 2.33 would fetch me area ratio a by A star of 2.254, 2.254. But A x star is our A throat star - A throat which is our A 1 star because that is an isentropic flow before from (Refer Time: 29:34) to shock which is given as 5.

So, my A x is 2.254 into 5, 11.27 centimeter square, at this location I will have my shock. Now, what else? Now P b at the design condition, the P at design condition that is very easy problem which we have been working on, you can look at the area ratio. Just look at the area ratio, A E exit by A throat. Find that the appropriate mach number which will give you an isentropic flow and that mach number will give you the pressure ratio. So, A exit by A throat equals A E by A star equals 12.5 by 5 this will give me a mach number at the exit, a supersonic solution. So, this will have 2 solutions, you take the supersonic solution which is 2.44. So, this is my design mach number.

The movement I have this, I can find my pressure ratio P by P 0 which is my P exit by P 0 which is my P b by P 0. P 0 is already given, since it is an isentropic flow P 0 at the entrance is going to be the stagnation pressure throughout the nozzle. So, I will have this P 01 which would give me P b value, what? 45.01 kilopascal, we can also find the entropy change when you had the shock.

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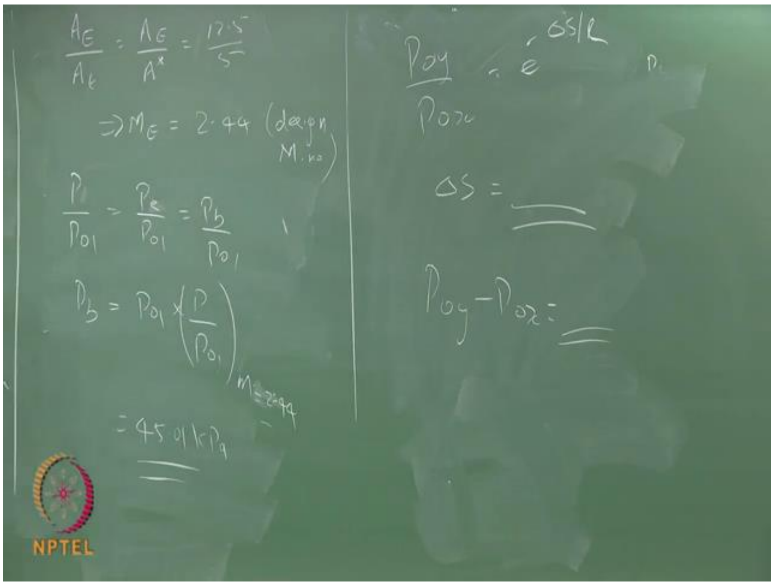

Since you know P x and P y your P 0 y and P 0 x is e power minus delta S by R, you can also find your delta A. We can also find your stagnation loss this way; now, will do your next problem.

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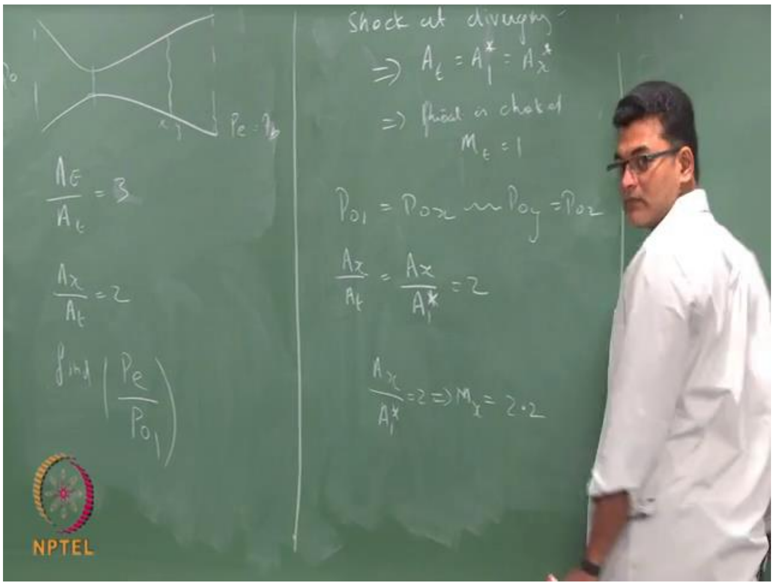

I have a nozzle where my A exit by A throat is 2 and there is this 3, and there is a shock sitting at some location where my area ratio is 2. Find P e by P 01? So, I have a P 0 I have a P exit now after the shock it is going to be subsonic. So, this can also be your P b. So, the pressure ratio P exit to P 0 that is what we need to find all that is given is this area ratio, nothing else. From this area ratio you have to find your pressure ratio P e by P exit. Again we start from the information we have, there is a shock diverging section implies A throat is your A star 1 which is your A star throat is choked M throat is 1. So, whatever pressure you have P 01 is same as your P 0 x then it encounters a shock your (Refer Time: 36:06) some of your stagnation pressure which would be retained till the exit.

Now you are given A x by A t which is also A x by A star. So, this is A star 1 because that is something that is happening before the shock, we call this as this is 2. So, the mach number associated with A x equals 2 plus I look at the shock tables the isentropic tables. Find A x by A star 2 will give me a mach number of mach number of 2.2, mach number of around 2.2. Now the shock is happening at M x equals 2.2.

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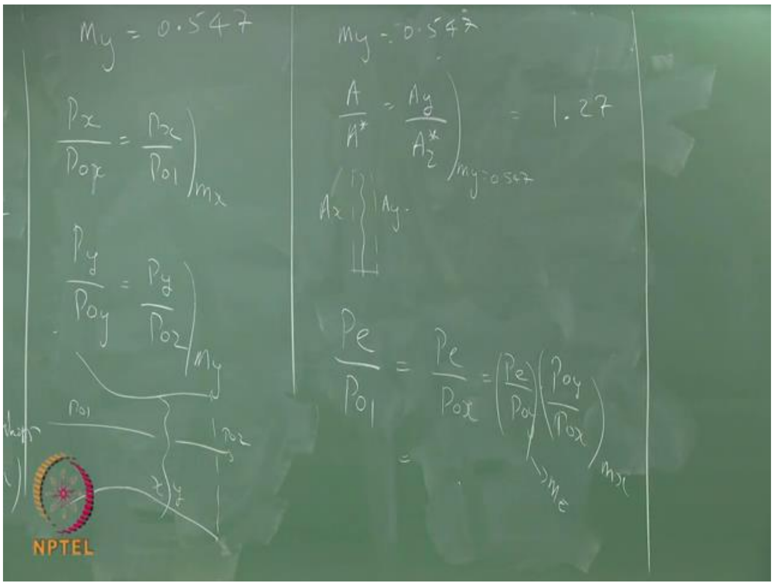

So, your M y equals this is from isentropic tables. Now M y is, I look at the shock tables for mach number 2.2, M y is 0.547. So, I have M y that is generated at this, your P x by P 01 or P 0 x is P x by P 01 at M x and your P y by P 0 y is your P y by P 02 or P 0 exit at M y. So, what I have written here is, have a shock then this is x, this is y, if that is your value this is my P 02, this is my P 01 which is consent up to the shock and consent after the shock, that is one information.

Second information, I can get A y by after the shock I have M y equals 0.545 which implies my A by A star associated with this is my A y by A 2 star A y star or A 2 star.

But in the shock derivation we have taken the control volume around the shock and we have assumed A x equals A y. So, this I can get it for M y equals 0.74 which is I look at the isentropic tables I get A by A star of A by A star is 0.547, A by A star is around 1.27.

Now I know my A x. So, I need to find P exit by P 01 which I can rewrite it as P exit by P 0 x which is P exit into P 0 y into P 0 y by P 0 x into P 0 x by. So, I can rewrite the P 0 x in this particular form. So, I know this pressure ratio P exit by P 01 as P exit by P 0 x into P exit, I rewrite in this particular form. This is associated with my M x, this is associated with my M exit, if there is a shock and if there is a pressure that is here. So, I need to find my M exit.

So, if I know my M exit I can find this ratio and that multiplied by this should give me the pressure ratio; now something.

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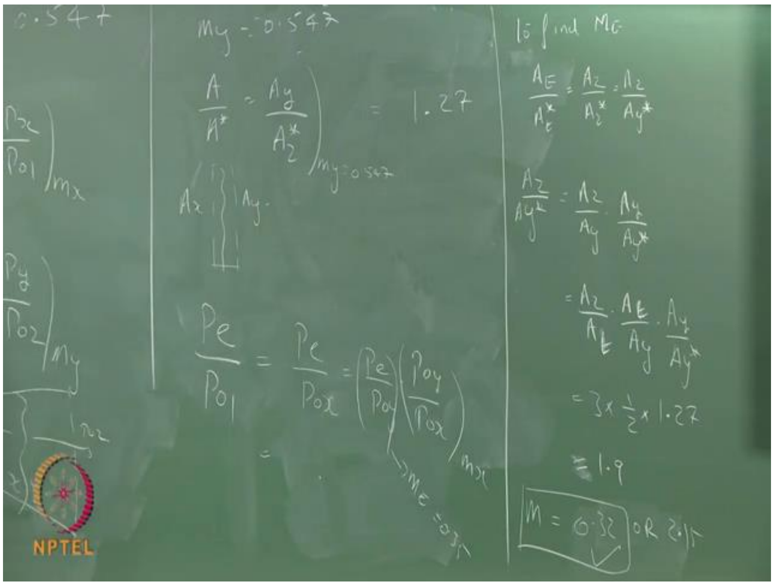

To find M exit I need to find A exit by A exit star which is nothing but, what is we called it as A 2 by A 2 star A 2 start is also my A y star. So, I need to find A 2 by A y star which I can rewrite it as A 2 by A y into A y by A y star, which I can write it as A by A 1 into A 1 by A y into A y by A y star.

So, I need to find A exit by A exit star, so I replace this quantity in terms of quantities I know. So, A 2 by A 1 is already given which is A 2 by A 1 is A exit by A 1. So, I will write this as my throat area. So, I divide it by throat area. So, A 2 by A throat area and A throat area by A y and A y by A y star. So, A 2 by A throat area is our area ratio which is 3 and A throat by A y is the location where your shock is happening which is given as 2, it is 1 by 2 into A y by A y star we already have found based on my M y which is 1.27. So, this is my A exit by A exit star which is 1.9. So, your M associated with this 1.9 is, look at the isentropic table 1.9 is subsonic value, we will write both or 2.15.

Now, we are talking about a section which is after the shock. So, a supersonic cannot happen there it is always a subsonic flow, so this is our M exit. So, your P by P by P 0 corresponding to 0.35 is your value that has to be substituted here.

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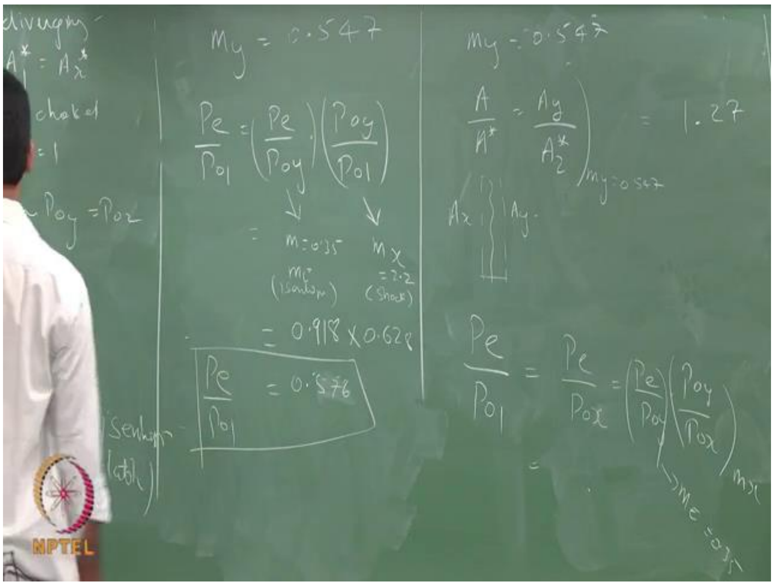

So, your P exit by P 01 is P exit by P 0 y into P 0 y by P 01, this is corresponding to M equals 0.35 which is our M exit, this is your corresponding to your M x which is before the shock. So, these 2 values should give me the pressure ratio.

So, M equals 3.5 my P exit by P e 0 is 0.918 multiplied by, I look at the shock tables M x equals 2.2, 2.2 P 0 y by P 0 x is 0.628 this is equals 2.2. So, this is from shock tables, this is from isentropic tables - this is around 0.576. So, that is my P exit by P 01. So, we come back to the question, if I have that pressure ratio P exit by P 01 is 0 0.578, I will have shock at location where you have this.

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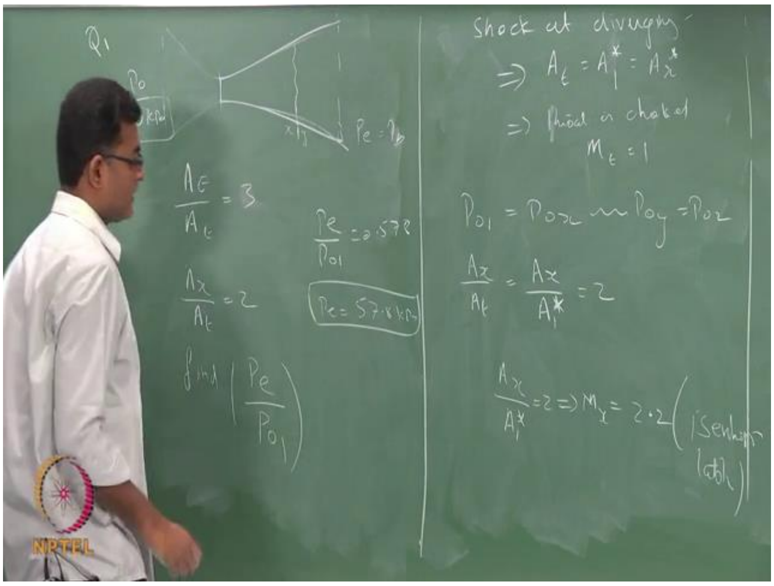

So, if your P 0 is, let us put P exit some value then the ratio would be this. Let us take P 0 say 100 kilopascal. So, your P exit is 57.8 kilopascal, 57.8 kilopascal is a region between P critical 1 and P critical 2, so there is a shock in the diverging section, and that happens at a location where area ratio is 2.

The key is, you consider shock as the non isentropic process everywhere else its isentropic process and what over be the case before and after. If I have long that, you do not need to have the C-D nozzle information where the shocking condition or throat condition is not there, but if it is a nosily you have these condition.

Same is true for a diverging diffuser. So, if I have a diverging portion, if I do not consider this portion and I have a diffuser and there is a shock sitting here you do the same kind of analysis and get the tables to do the problems the values before the shock and after the shock and these normal shock tables for the properties across the shock. That should solve the entire shock problem, now (Refer Time: 51:07) standing normal shock problem.

Thank you.