Fundamentals of Gas Dynamics

Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week – 09
Lecture – 33
Discussion on Normal Shocks – 1

We will continue our discussion on a Normal Shock. We will do some numerical problems and try to understand the normal shock further. So, the first question I have is.

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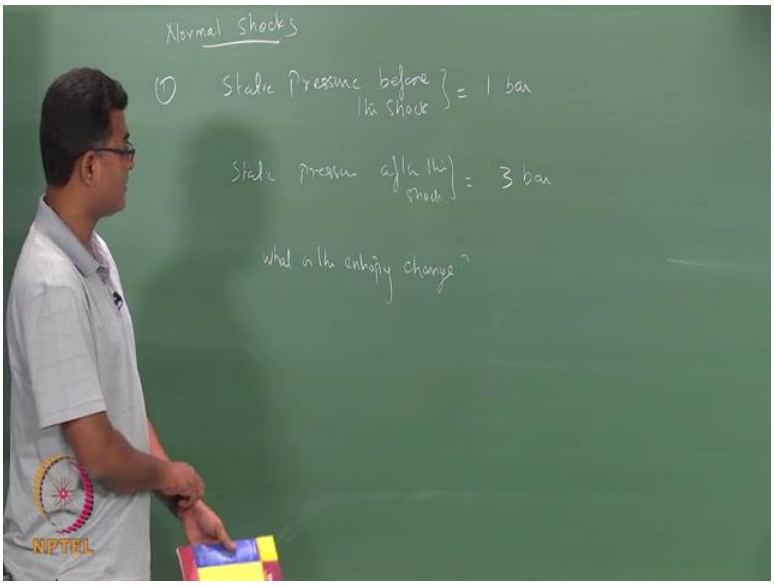

First question I have is, you are given static pressure before the normal shock and static pressure after the shock. Let us put this as 3 bar and 1 bar. So, what is the entropy change due to this particular process, where you have a shock and I know the static pressure before the shock and static pressure after the shock; so what would be the entropy change? I can do this several ways; first I will try to derive my entropy change in terms of pressure and try to find the numerical value of entropy and we will also do the same thing in a probably different way which we will see.

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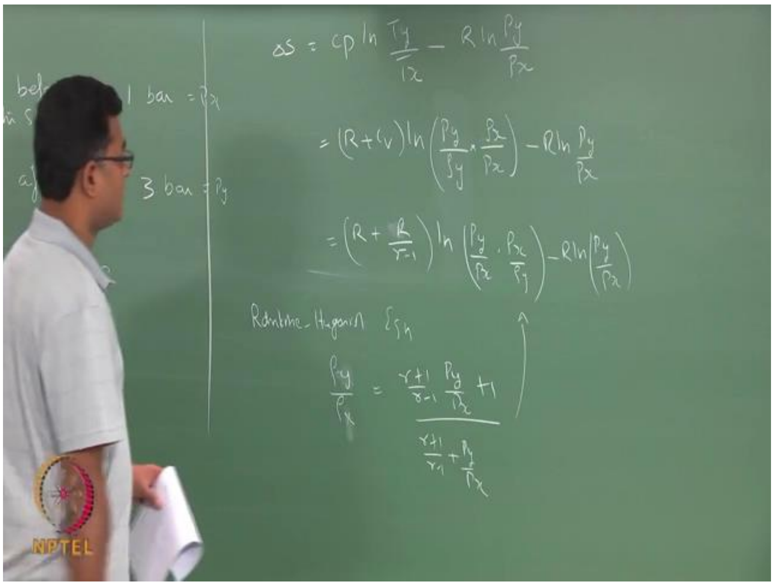

The first is we will start with the Gibbs relation delta S equals c p l n T y minus T x minus R l n P y by P x. I am given only my P y and this is P x and this is P y, so I know only P y and P x so what I will do is I will try to eliminate T in terms of P. So, I will write the delta S in term of pressure ratio alone and try to see if we can infer something from that. So, what I am going to do is I will replace c p with R with as R plus c v l n T y by T x, and T y by T x I can replace it using the state equation as a T y is P y by rho y by a T x is T x by rho x minus R l n P y by P x which I can write c v as R gamma R by gamma minus 1 l n P y by P x into rho x by rho y minus R l n P y by P x.

Now, I will replace this density ratio using Rankin Hugoniot equation which we have derived the other day. Rankin Hugoniot equation which tells me the ratio rho x by rho y in terms of; so rho y by rho x is gamma plus 1 by gamma minus 1 into P y by P x plus 1 divided by gamma plus 1 by gamma minus 1 plus P y by P x. I substitute this in here.

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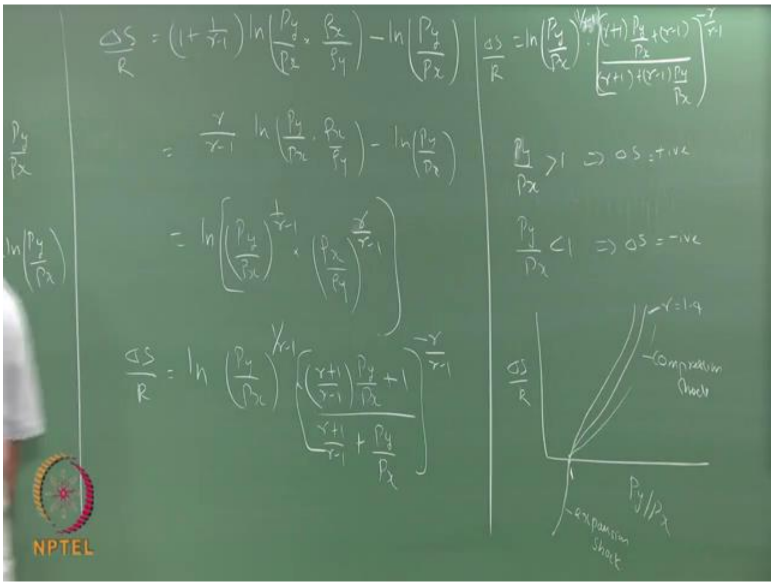

So, I will get my delta S as delta S by R as 1 plus 1 by gamma minus 1 into l n P y by P x into; let us for the time being let us write this as this minus l n P y by P x. This is nothing but gamma minus 1 by gamma l n P y by P x into rho x by rho y minus l n P y by P x, which I can replace this as l n P y by P x to the power 1 by gamma minus 1. So, I have taken this exponent inside and subtracted one from that into rho x by rho y to the power plus gamma by gamma minus 1, you are following. So, what I have done is I have taken this exponent inside so this would be P y by P x to the power gamma by gamma minus 1 and then that is this and that is that.

Now here I will apply the Rankin Hugoniot equation take this equation and substitute it there so I would get a lengthy equation which will look something like this l n plus 1 divided by minus 1 which is nothing but l n P y by P x to the power 1 gamma minus 1 into gamma plus 1 P y by P x plus gamma minus 1 divided by gamma plus 1 plus gamma minus 1 into P y by P x to the power minus gamma by gamma minus 1.

So, this is a sum equation where; so if my P y by P x is greater than 1 since gamma is always greater than 1 this quantity would be greater than 1 and this quantity would be greater than 1 I would get delta S positive, if P y by P x less than 1 I would get delta S negative. If I plot delta S by R versus P y by P x if it is 1 it is 0 so this is 1 and then it has some curve like this for gamma equals something, for other gammas you would have similar curves all coming in doing this. If P y by P x is less than 1 you will get this thing.

So, this is your compression shock this would be your expansion shock which is not physically possible. In the problem we have P y by P x is 3, so I substitute 3 here.

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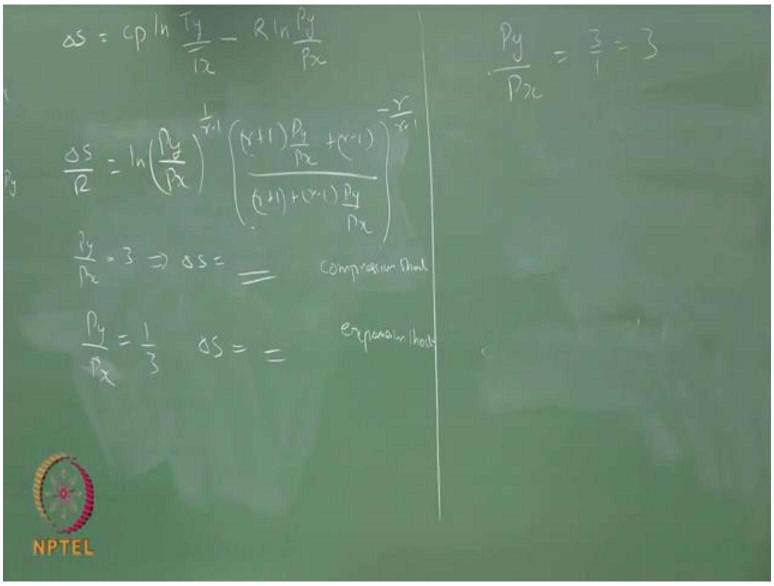

So, all I do is I will retain this equation because I need that equation to do the same problem in a different way. So, if I substitute 3 in that equation I would get delta S by R equals; I will write that equation again for the understanding purpose, gamma plus 1 P y by P x plus gamma minus 1 divided by gamma plus 1 plus gamma minus 1 into P y by P x to the power minus gamma by gamma minus 1. Now, P y by P x is 3, so I substitute 3 I would get my delta S.

Since I know the value of R and gamma I can substitute this and get the value of delta S. If by chance P y by P x is the other way around. So, if the question is this is after the shock and this is before the shock I would get this as 1 by 2, in such a scenario you can compute this and see for yourself what would be the value of delta S which would be negative and hence this is not going to be a possible solution, this is the expansion shock this will be the compression shock.

Now this is for understanding purpose; what exactly you have the entropy change given a pressure ratio that is conceptually this is what it happens. But to do this particular problem you do not need to remember all this equation.

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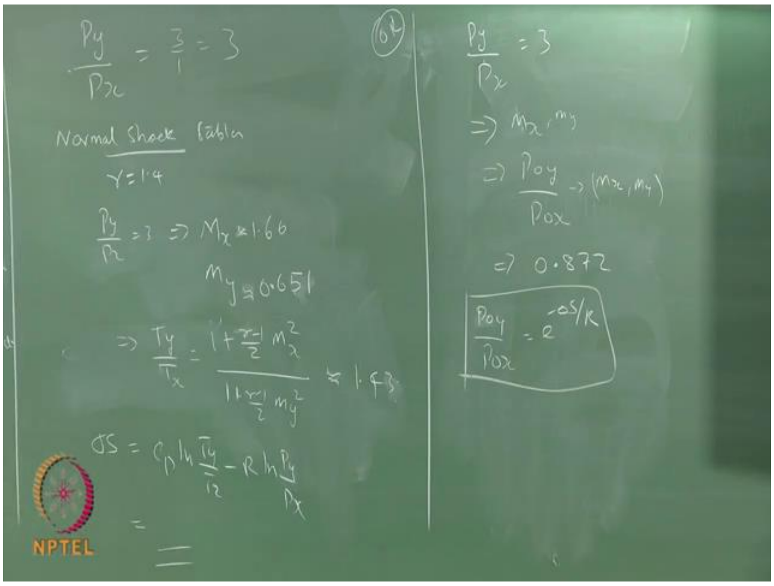

What you can do here is, now P y by P x is given; P y by P x is 3 by 1 this is 3 that is given. So, what we do I take the normal shock tables for gamma equals 1.4 I take P y by P x 3; so I go to the table take P y by P x which is 3 which will give me a value of M x this is 1.66, M y is 0.646 this will also give me T y by T x. If you know this you can anyway get T y by T x, how do I get it? I can use 1 plus gamma minus 1 by 2 M x square divided by 1 plus gamma minus 1 by 2 M y square. Use this equation we can get this ratio, but this ratio is also given in the tables which I can get it as 1.43; this is approximately this. Typically you need to interpolate and get the exact value, but for demonstration purpose 0.651.

If we get this value I can substitute this in the Gibbs equation to get the entropy change; c p value I know, R value I know, this ratio is now 1.33, this ratio is 3. I do not need to remember this complicated equation, but the concept is that you are not going to have an event if P y by P x is less than 1 to which we have showed it to be the case. So, P y by P x is always greater than 1. There is a pressure jump across the shock.

You can also do it if you write this in terms of density and get the density jump which is not given in the table, but you can use Rankin Hugoniot equation and get those. This is one method, this is another method so I know this value, if I know this value I can get my M x and M y as I have told you here I can also get P 0 y by P 0 x. How do we get it? We have derived it in the previous class this in terms of mach number and before and after the shock and hence you can get this. This ratio is also given in the table, this ratio is around 0.872 for the problem that we are considering. The moment you have this ratio I can always get my delta x which is related in this form. So since I know this ratio I can get delta S.

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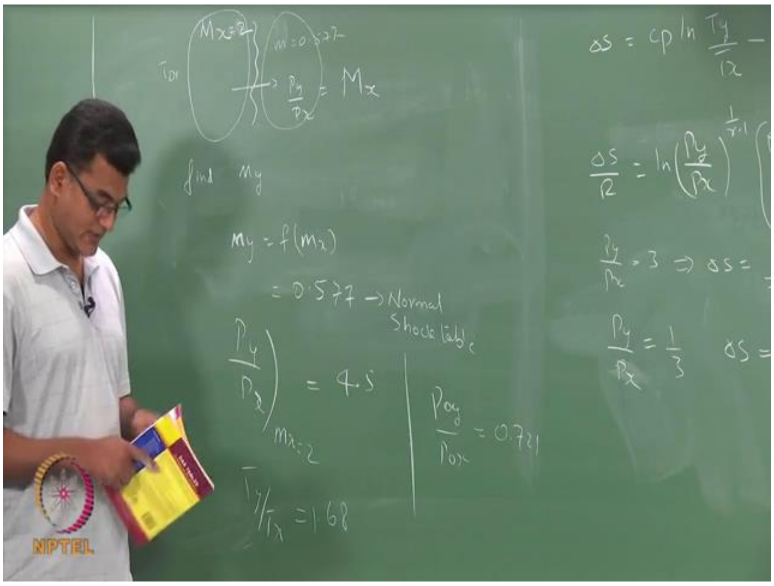

So, you can do this in many different ways. We will do another simple problem to get used to the normal shock table. The simplest problem would be, I have a shock at M x equal 2 find M y, so that can be the most simplest of your problem. So, you know M y in terms of M x. If you would remember that equation you can use it otherwise use the tables.

So I take the tables, so M x equals 2 I get M y as; the moment I have this value I know all the isentropic relation here, I know all the isentropic relation here for m equals 0.577. So, if I know any of these values I can compute the corresponding static quantities everything using isentropic relation, this also using isentropic relation. But in the shock tables you also know the pressure jumps P y by P x again in terms of my (Refer Time: 21:52) mach number, so my P y by P x for M x equals 2 is given as 4.5.

Similarly, T y by T x is 1.68, P 0 y by P 0 x is 0.721. If you look here, these are all greater than 1 except the stagnation pressure so only the stagnation pressure decreases across the shock while all other quantities increases across the shock which we just there tabulated in your tables.

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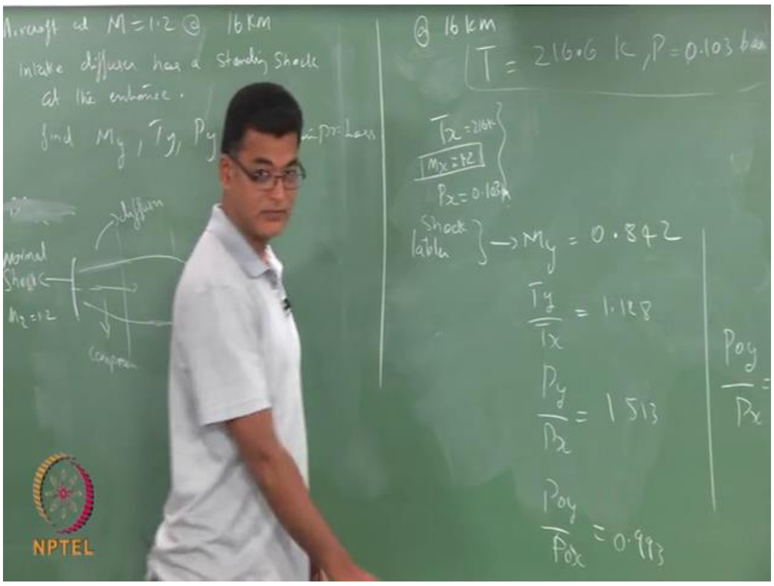

So, when I aircraft at mach 1.2 at 16 kilometers so the engine we intake diffuser has a standing shock at the entrance. Find after shock what is your mach number? What is your temperature, what is your pressure, and your stagnation loss. So, I have an aircraft that is moving at 1.2 at 16 kilometers that is the only information I have, and there is a standing shock in front of the diffuser. So, before the engine you have a shock that is coming, so this would typically be a bog shock but for the time being we will assume this as a normal shock.

So, I have a normal shock standing in front of the diffuser which goes into the engine, so the purpose of the diffuser is to compress the engine as far as possible and collect over maximum air that can go into the engine. So half of the compression typically happens here, since we already know that there is going to be a pressure jump across the shock you can achieve a compression inside at the inlet of the diffuser. And the flow will be subsonic and to get increase in further pressure you need a diverging section here, so the air goes in here to the combustion chamber and then to the exit.

So, the mach number here is given M x equals 1.2 and this happens at 16 kilometers. Question is what is the mach number after the shock, temperature after the shock, pressure after the shock, and what is the stagnation pressure loss. First thing that at 16 kilometers your temperature is 216.6 Kelvin, so this is your temperature before the shock T x is 216 kelvin and mach number is 1.2 before the. And your pressure at P is 0.103 bar at 16 Kelvin. So, this also is given in the tables. Table 2 in this addition 6 will have a according to the height you will have the temperatures given here, so you have P y P x and T y T x given as this. So, your P x is 0.103 bar.

Since, you know your mach number before the shock from the tables you can find M y which is I take the tables normal shock table I look for m 1.2. M y is 0.842 and T y by T x is 1.128, P y by P x is 1.513, and P 0 y by P 0 x is 0.993, and we have also know P 0 y by P x 2.407. With these values you have to find.

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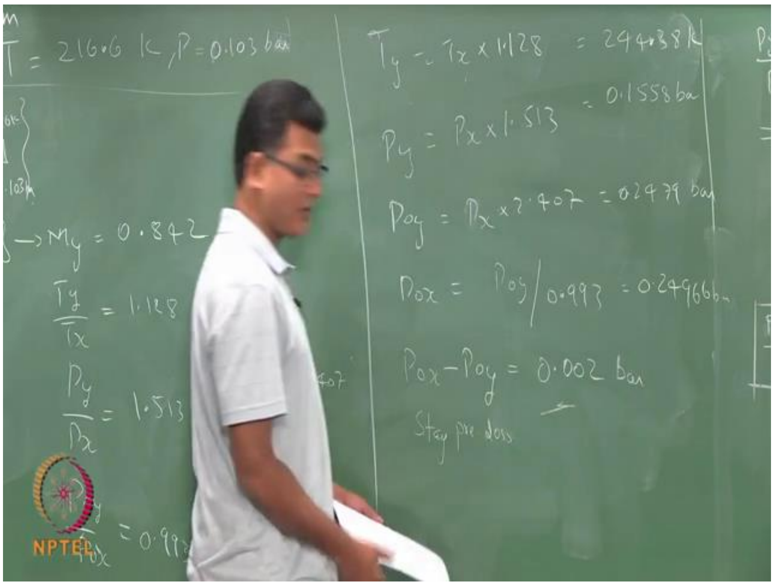

Thus, this is a very simple substitution you know T x so you can get T y, so T y is nothing but T x into 1.128, P y is nothing but P x into 1.513 there is a point here and P 0 y is P x into 2.407. The moment you get P 0 y I can find P 0 x which is P 0 y divided by 0.993. If you get these values the stagnation pressure loss is nothing but P 0 x minus P 0 y. So, you get typically all the values. It is a one of the simplest problem you can design to get used to the normal shock tables.

So, the values we have here is T y is 244.38, P y is 0.1558 bar, so this is Kelvin. P 0 y is 0.2479, P 0 x is 0.24966; these are all bar, these are all, this is also in bar. So, P 0 x minus P 0 y is your stagnation pressure loss which is 0.002 bar. You can also find the velocity at the entrance. If this is my diffuser entrance what is the velocity which it is entering the diffuser. So, the since we know T y and M y.

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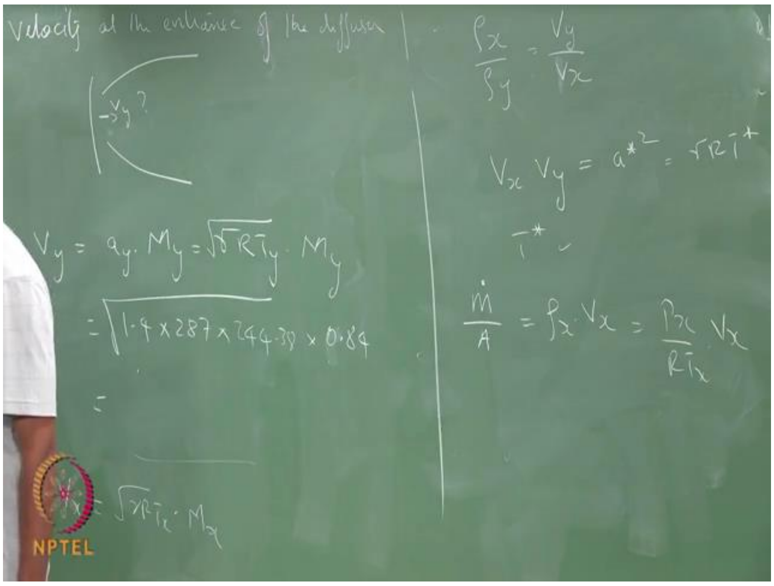

So, this is the diffuser and there is a shock here, what is v y? So, v y is nothing but a y into M y which is gamma R T y root into M y which is 1.4 into 287 into T y yes something which we computed 244.38 root into M y is 0.84. So, you would get the velocity with which the fluid is entering. You can also find V x which is gamma R T x into M x. If you know V x and T x you can find the density ratios rho x by rho y, which is nothing but V y by V x. If we know V y and V x you can also find the A star or T star; y is a star square which is your prandtl equation which is nothing but gamma R T star. You can find T star which is constant across the shock.

You can also find mass flux that is entering the diffuser which is nothing but rho x V x, which is nothing but P x by R T x into V x. We know all the quantities, so you can also find the mass flux that is entering the flow, so all these from the normal shock table and the combination of normal shock table and isentropic flows.