Fundamentals of Gas Dynamics
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Week – 08
Lecture – 31
Normal Shocks Relations – 2

So, we will continue with the normal shock relation. So, I will write down the equations which we have derived the other day. The first equation in a normal shock is, of course T 0 one, T 0 x equals T 0 y from h 0 x equals h 0 y.

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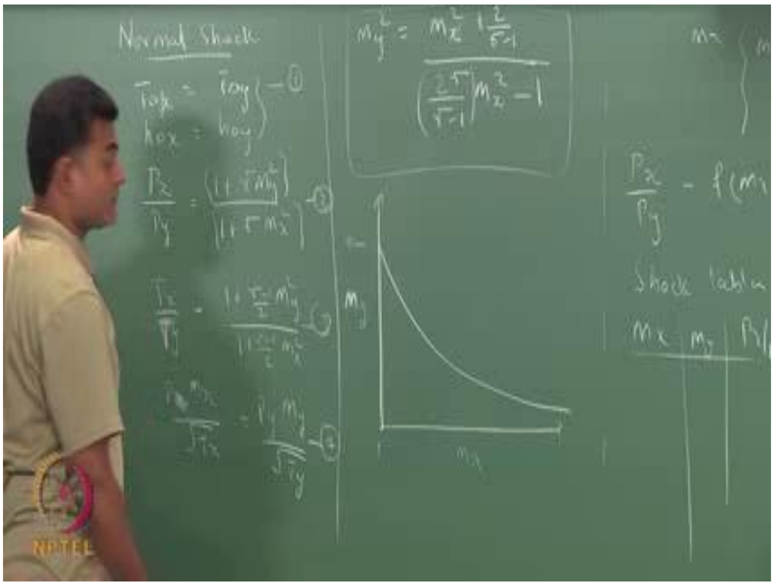

And, from continuity we have derived P x by P y equals 1 plus gamma m y square divided by 1 plus gamma m x square; where x and y are states before the shock and after the shock. We have also seen T x by T y to be 1 plus gamma minus 1 by 2 m y square divided by 1 plus gamma minus 1 by 2 m x square. Our P 2 from continuity equations, P x m x root t x equals P y m y root of T y.

We have also derived the equation for Mach number after the shock, in terms of Mach number before the shock. So, your m y square is m x square plus 2 by gamma minus 1 whole divided by 2 gamma by gamma minus 1 m x square whole minus 1.

Now, we will derive some equations further to this. So, I have this equation which we have also seen to be of this form; this is 1, this is m y, this is m x, 1. Now, what we will do is; so, now we know our m y in terms of m x.

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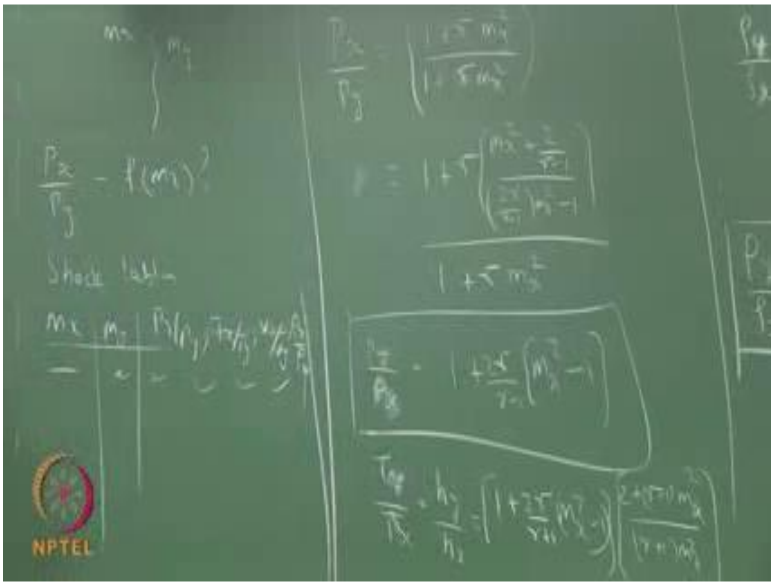

So, if I know my Mach number before the shock, I know my Mach number after the shock. So, likewise if we can relate these ratios in terms of m x? Ok. Or, T x by T y or rho x by rho y. If that is possible, then we can have something called shock table, where I would write my m x, then I can get my m y, then I can get these ratios which we have been talking about.

So, what do I do? I substitute m y in these equations. Say, for example, I substitute m y in P x minus P x by P y equation. So, I would eliminate my m y with this. So, I take equation. Let us say equation 1, this is equation two, equation three and equation four. So, I take equation two, which I will rewrite here. It is P x by P y equals 1 plus gamma m y square divided by 1 plus gamma m x square. I substitute for m y, in terms of m x.

So, I am not going to derive the simplified form of this equation. But, I will tell you the procedure to get this. This is nothing but mere algebra; divided by 1 plus gamma into m x square or divided by 1 plus gamma m x square. So, I have replaced m y in terms of m x. That is all I have done.

So, the final equation would look something like 1 plus 2 gamma by gamma minus 1 into m x square minus one. So, I have the pressure ratios before and after the shock, in terms of my Mach number before the shock.

Likewise, I can get T. So, this is. So, this is T y, P y by P x. There is a correction here. This is P y by P x is this. And T y by T x, which is also my h y by h x equals, some very lengthy equation, into 2 plus gamma minus 1 m 1 square divided by gamma plus 1 into m x square.

So, I have my pressure ratio before and after. The static pressure ratios before and after the shock. Static temperature pressure ratio; the temperature ratios in terms of Mach number, I can also get my density from. That is a very simple relation, which I can get it as this.

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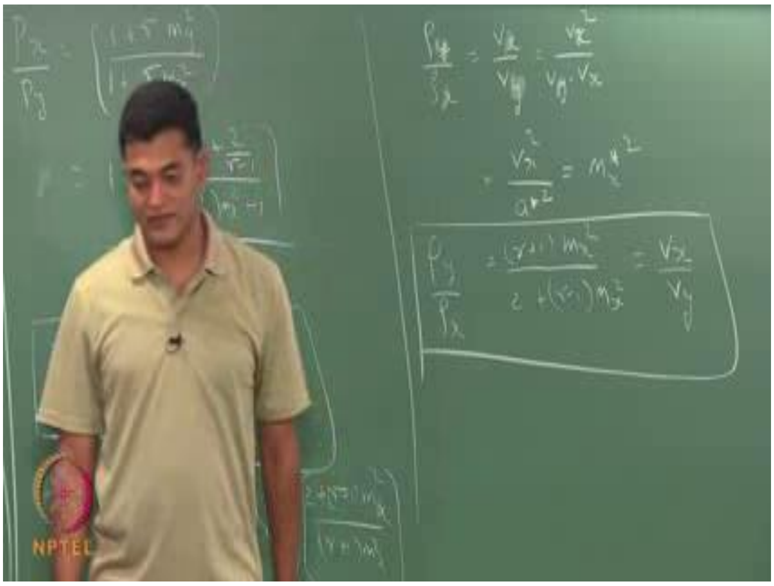

We can write this as V 1; we can write this as V 1 square by V 1. I will write it as y and x; is the notation that we were using. So, I will continue with that. So, this would be V x square by V y into V x, which V x dot V y we have derived to be.

Student: (Refer Time: 08:13).

Ok. There is a bracket here. There is m x square minus one, there is a bracket there, yes, thanks. So, the denominator V x dot V y, we know it as a star square; which is now nothing but m x star square. Now, m x star; we know already in terms of m x, which is nothing but gamma plus 1 divided by gamma plus 1 m x square divided by 2 plus gamma minus 1 m x square. Correct? Yes. So, this is rho y by rho x, which is also equal to V x by V y. So your star, m star, before the shock is your ratio of your densities. So, now I have written these.

If you look at the tables which I was talking about, I have written all these ratios; T x by T y, V x by V y, rho x by rho y, all in terms of my Mach number, before the shock. So, if I know the Mach number before the shock, I get all these values. So, I can tabulate that, and that is what you have it in shock table. This depends only on your gamma and probably (Refer Time: 10:25).

So depending on your gamma value, you get all these results. So, look at your gamma in the tables. Look for the shock tables. This is the shock tables that is similar to your isentropic tables, what you have. So, in the book after the isentropic table, you will see something called a shock table. So, look for the appropriate gamma, find your Mach number before the shock. And, you get all these quantities after the shock.

Now, it would be interesting to see how this varies, when m 1 and m x approximates to 0, approximates to infinity. So, where is the duster?

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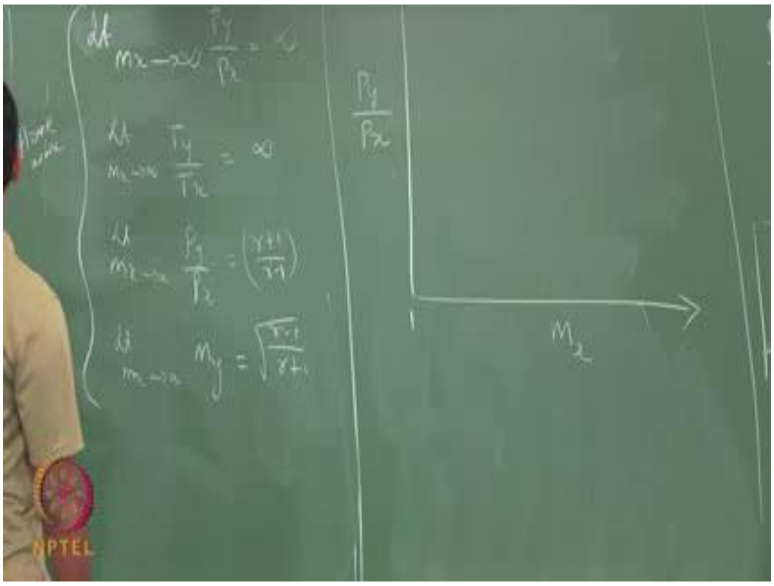

In the limit m x approximating to infinity, what is your P y by P x. So, I would get P y by P x in the limit approximating m to infinity is infinity. Then, limit m x tending to infinity my T y by T x also approximates to infinity, if you look at this equation. I can get infinity there. But, what about the density? Rho y by rho x approximates to; if I differentiate that I would get gamma plus 1 by gamma minus 1.

And, what about m 2 or m y? So, m x and m y are the notation that is given in the shock tables. Typically, is many books will deal with as 1 and two; it is the same thing. So, so if we look at your equation here as m tends to infinity, what is your Mach number at after the shock? So, you can do this. And, you would find this to be gamma minus 1 by gamma plus one. So, you can try these things as your homework. Get these equations, find the limits. As m x tends to infinity, what are these values?

So, these 2 asymptotically reaches to these values. So, if I plot these values, I would, I will rub this.

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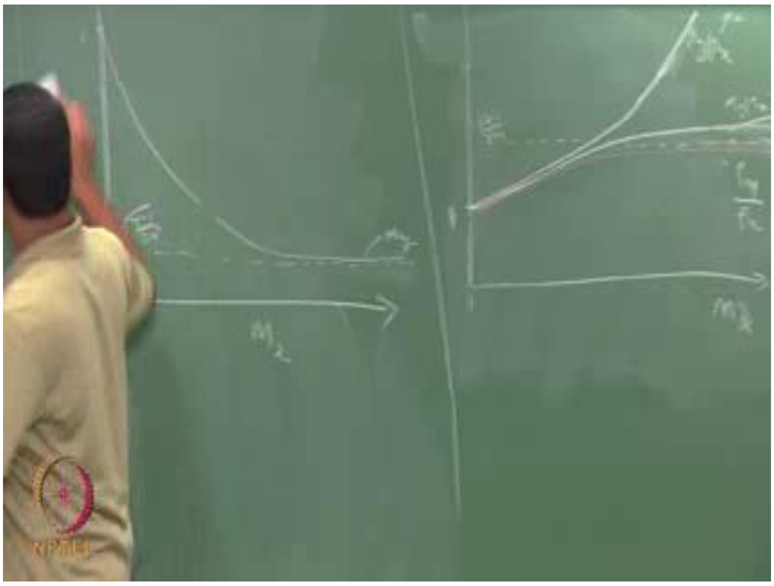

Mach number, and let us say P, this is what I have written here is P y by P x. So, here Mach number starts from one. So, this is m x. So depending on my m x, I will have different values of my property ratios. Or m x equals one, my pressure ratio is going to be.

Student: (Refer Time: 14:46).

So, this would be root of gamma minus 1 by?

Student: 2 gammas.

by 2 gamma. So, as my m infinity tends to infinity, m y is root gamma minus 1 by 2 gamma. Yes. Yes. So, it would be this. Yes. So, this is the correction there. Root gamma minus 1 by 2 gamma is your asymptotic value of your m y. So, let us draw that first; m y. So, it starts from 1 and then goes like this. The asymptotic value is root gamma minus 1 by 2 gamma. So, that would be my asymptotic value. So, it should not cross. As m x tends to infinity, this value will approach to this value.

Now, when m x equals to one. So, this is my m y. Now, I will draw P y by P x. As m x tends to infinity, this ratio would approach infinity. So, at m x equals one, this ratio is one; as we had already seen. Or, if you can substitute that as in the relation, which you have just now seen, you will see this as 1 m x have the ratio value 1 here and this P y by P x. This approaches infinity as m x tends to infinity. T by T x also approaches infinity, when this approaches infinity, but at a lower slope. Now, we can have; this will not cross.

So, your temperature ratio would go like this; your pressure ratio would go somewhere like this. And, your density approaches some asymptotic value. So, let us. So, let us; it again starts from 1, when m x equals 1. So, this would go to some constant value, which is of this, which is gamma plus 1 by gamma minus 1. So, I have my rho y by rho x approaching an asymptotic value, when m x tends to infinity. So, I would; the density line is shown in the red color. It approaches the asymptotic value, which the edges now written off. So, as m tends to infinity, my density would approach to this particular asymptotic value. Both temperature and pressure ratios approaches infinity as m x tends to infinity.

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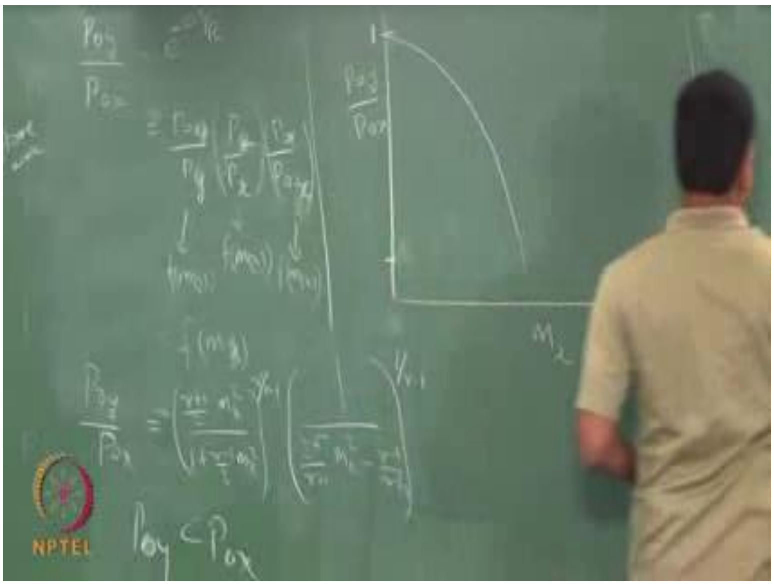

Now, we look at what is the value of P 0 x by P 0 y, which we have written in terms of Mach number the other day; which is, which we obtained it as this. This P 0 in terms of y by P y, this is P y P x P x P y.

Student: P x P x.

P x P x, this quantity, now we know in terms of m x; this is any way in m x. This quantity we know in m y, which we can write it in terms of m x. So, all this we can now write it in terms of m x, which is, I will write it down. This again, you can derive it yourself; 1 by gamma by 2 m x square divided by 1 plus gamma minus 1 by 2 m x square whole to the power gamma by gamma minus 1 into 1 by 2 gamma by gamma plus 1 m x square minus gamma minus 1 by gamma plus 1 to the power 1 by gamma minus one.

So, I have my P 0 y by P 0 x in terms of m x. So for a given m x, I have this ratio; which again, we know that P 0 y is going to decrease. So, I would; from the entropy condition, we know that P 0 2 P 0 y should always be less than P 0 x and the ratio would decrease like this. So, I would write P 0 y by P 0 x. If it is; when it is one, you have the trivial solution. And, it goes something like this. And, all these quantities you can get it from the gas tables, where you give, you have a separate section on shock relations. So, the moment you have the flow before, the Mach number before the shock, you can get all the quantities. All the quantities after the shock; all these ratios.

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So, if I look at the T S diagram. So, what I am drawing? I have a flow and then suddenly I see here shock. So, this is x; this is y. So, up to this is isentropic flow. After the shock also is isentropic flow. Shock is a non-isentropic flow. Now, we are drawing; going to draw this particular process. So, this 1 go; the flow goes from 1 to x and then x to y and then from x to y to two. So, I have a flow that is going from state 1 to state x.

Now, this is a expansion process. So, my flow is accelerating or whatever. I have a Mach number at x, which is greater than one; that is only condition I need at present. Now, the moment it encounters a shock, there is an entropy change. So, there is a d x. So, if I take d T y by T x, that is increasing, after the shock it is increasing. So, my T y is greater than T x. So, I have a process that takes me to some location here, which is my y.

Now, from there I still go an isentropic process to two. This stagnation point of x is somewhere here; stagnation point of y is somewhere here. Both are same; T 0 x equals T 0 y equals T 0 1 equals T 0 two. So, the stagnation point of all these, stagnation temperatures of all these points are same.

So, the process from 1 to 2 with a shock in between is this; 1 to x, then x to y and then take another isentropic process to 2 with T 0 1 same. So, T 0 x equals T 0 y equals T 0 1 equals T 0 two, but the T x because of the shock T y has gone up and then comes down.

Now, draw the same thing in a p v diagram. So, I will draw this side; close by. So, I have a pressure. So, this is pressure P 1. So, P 1 isentropically goes to P x. Then, there is a non-isentropic process in which P x by P y, P y by P x increases. So P y, P y by P x increases or P y is greater than my P x. So, there is a sudden jump in pressure, the moment you encounter a shock. So, I have an isentropic process here, and then there is a jump in pressure. So, there would be some jump in my pressure. So, this goes to next line. So, this is my x; this is my y. It may not be like this. So, this is my nu, this is my V. So, my density would also increase. So, it has to go somewhere here such that my, no, it is somewhere here. So, the density has increased; so, the volume has decreased. So, this would be left of my point here. So, this would be P y.

So, there is a decrease. So, my rho x is smaller than my rho y. So, this is my P y. Then, again isentropic process which will take you to location two. So, when you do at the T S diagram, the y is at right of your x condition, your x state; right of your original state before the shock. In the P V diagram because the density is increasing, it will be left of your original state or before this, before the shock.

So, it goes from 1 to x isentropically. Then, a non-isentropic process happens, where you increase your density. So, it goes to left and then from there to state two. So, I will remove this unwanted. So, the result d S that is happening, which is this d S. This d S or delta S is given by the relation P 0 y by P 0 x equals e power minus delta S by R. So, your delta S is given from this relation, where you could give find your delta S and appropriately point to your x location.