Fundamentals of Gas Dynamics

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Week - 08
Lecture – 30
Normal Shocks Relations – 1

We will continue with whatever we are discussed about Normal Shock. So, I will start with this relation.

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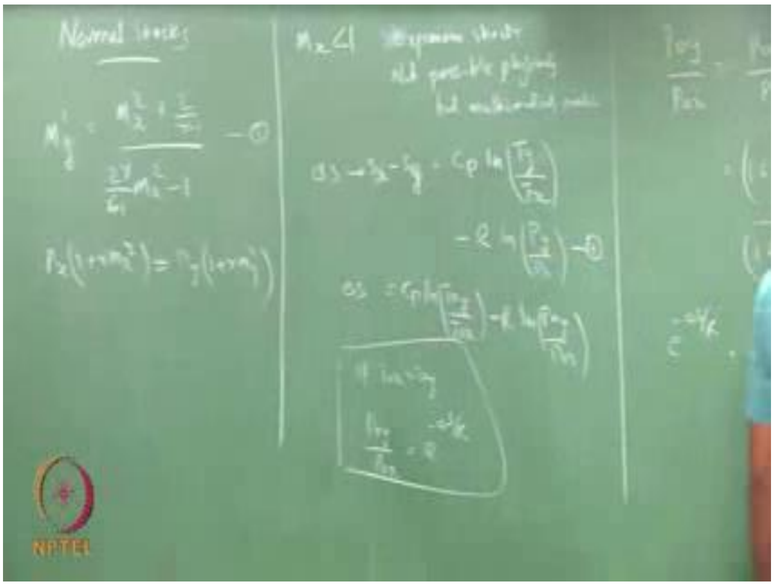

We have derived this equation which is the mach relation of mach number with respect to the mach number before the shock. We have seen this. So, these are the two equations that we had seen last in the previous lecture. We were trying to see why M x less than 1 expansion shock is not possible physically even though the solution excess in this particular equation. So this is not physically possible.

So, one way of showing this is not physically possible is by calculating the entropy change. If I go back to my Gibbs equation which is delta s which is s 1, s 2 minus s 1 or s x minus s y is nothing but c p l n by T y by T x minus R l n P y by P x, which is also equal to your c p l n T 0 y by T 0 x minus R l n P 0 y by P 0 x because that is entropy change that is connecting with two states. Or you can say entropy change between any two points and then you have this point. Which we had seen to be if T 0 x equals T 0 y we have seen P 0 y by P 0 x is e power minus delta s by R. Which we can we can write in this in terms of mach number.

Student: (Refer Time: 04:25) since P 0 y is greater than P 0 x (Refer Time: 04:30).

So, before this we will have to show P 0 x relation between P 0 x will b (Refer Time: 04:41).

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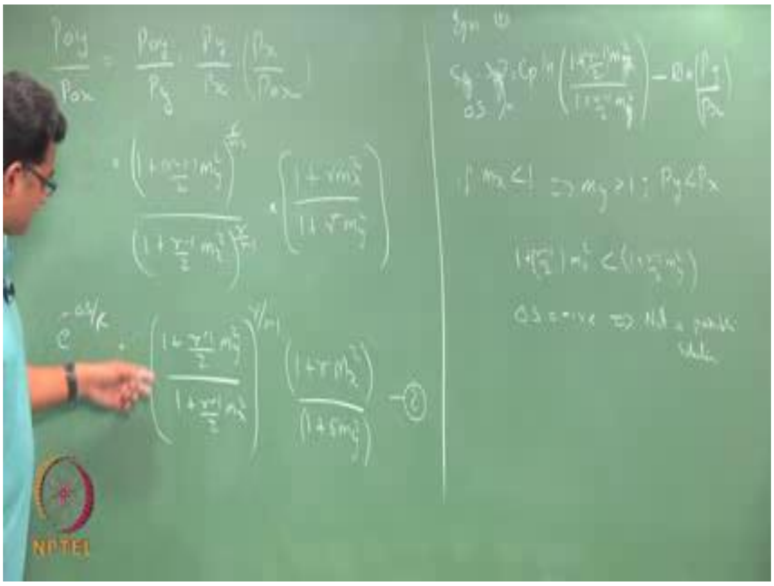

So, we will have P 0 y by P 0 x as e power minus delta. Now we will see how this can be represented in terms of mach number. So, your P 0 y by P 0 x I can write it as P 0 y by P y into P y by P x into P x by P 0 x. So this is isentropic process, this is the stagnation state associated with y so that would be 1 plus gamma minus 1 by 2 M y square to the power gamma by gamma minus 1. This is also the isentropic process that I would write it here 1 plus gamma minus 1 by 2 M x square to the power gamma by gamma minus 1 into P x P y we know as 1 plus gamma by gamma M x square divided by 1 plus gamma M y square.

Now this is my entropy change, e power minus delta s by R as this 1 plus gamma minus 1 by 2 M y square divided by 1 plus gamma minus 1 by 2 M x square to the power gamma by gamma minus 1 1 plus gamma M x square divided by 1 plus gamma M y square. So you can show M x less than 1 not possible by 2 ways; one taking this equation. Let us take this equation or take this equation and show. So, let us take equation 1 which is s x minus s y c p l n T y by T x is we had shown the other day to be T x by T y is just 1 plus gamma minus 1 by M y square divided by 1 plus gamma minus 1 by 2 M x square, M y T y by T x so this would be x by y minus R l n P y by P x.

Now, if M x is less than 1. Let us see what happen. This is s y minus s x or simply delta x that is evaluate delta x and see the sign of delta x. So if M x is less than 1 and implying we have seen M y should will be greater than 1 and your P y will be less than your P x. These two information you would get it from this relation m x. So if M x is less than 1 you would get M y to be greater than 1, and hence my P y would be less than P x, so these two information we will use it here.

So, if my M x is less than 1 my M y is greater than 1 and P x is less than 1 which implies if I evaluate this, so my this term is less than this term, so this should be less than 1. So, this l n of something less than 1 and here P y is less than your P x, but this comes with a negative sign. This if again a smaller quantity which means your delta s would be negative, which implies not a possible solution. We can also show that here, so if my M x is larger gamma is always greater than 1. This term M x is always going to be greater than 1, M x is greater less than 1 M y greater than 1 and hence if you look at this terms this term would be larger and this term would be larger and since this is gamma minus gamma, gamma is greater than 1 gamma minus 1 is always less than 1.

So, this quantity is always greater than 1, so this would be larger quantity and then this is a smaller quantity this is a smaller quantity so you need to find the magnitude between these two M x square this is going to be larger. So, you are going to have a small positive quantity here, which means your delta s would be negative. Implying your delta s is negative, for M x less than 1.

For M x greater than 1 this equation still holds and you can show that this is going to be a positive quantity. I will rub this.

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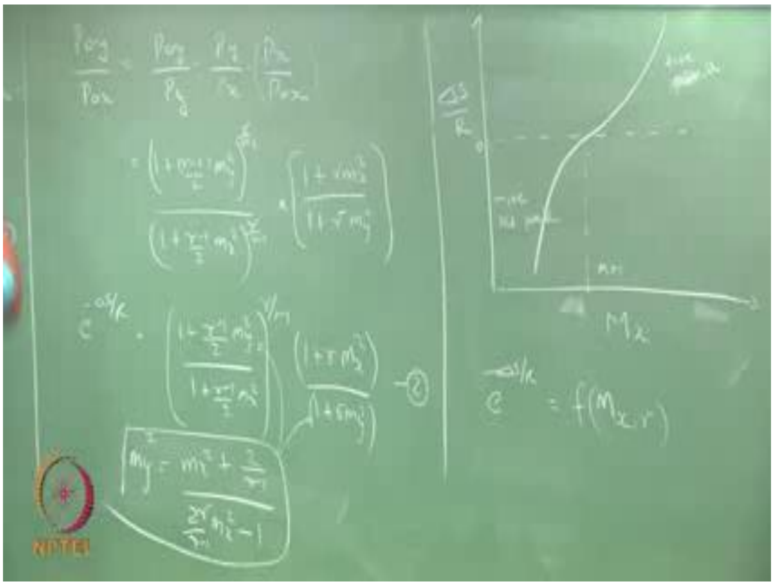

S by R versus m let us take the 0 line somewhere here and mach number somewhere here, this is my 1. If I plot this function where I substitute M y in terms of my m x, so I can substitute this equation 1 here. So, what I do is I substitute M x square M y square as M x square plus 2 by gamma minus 1 by 2 gamma by gamma minus 2 M x square minus 1. So, this I substitute here and here I would get e power minus delta s by R as a function of M x and gamma. If I substitute my M y here I would get this quantity e power minus delta s by R as a function of M x and gamma.

Now I plot m x, this delta s by R I would get a curve of this form, so at M x equals 1 I will not have any change in my entropy for that particular jump which we had seen. For m 1 greater than one I have entropy change positive, M x less than 1 I would have negative entropy. This is my negative entropy change not possible for some fluids; it can still possible for some special case. Possible in in general fluids like air and other things, but if you use some other fluid for example case where you have an increasing pressure when you expand.

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So, when you expand. Suppose you have a fluid P V diagram, typical process we have is this is the expansion. So, I have a pressure that is reducing by density decreases or the specific volume increase that is our typical case of air or CO2 or whatever. Now, if you have a fluid which has a slope somewhere here, so when I increase my volume I have an increasing in pressure. If I have a fluid like this I can typically have an expansions or shock. But again there the pressure is increasing. So, you have an increase in pressure even though the volume is coming down.

So that kind of fluid can still have this kind of thing. But for the general purpose and for general fluids that we encounter let us assume that if M x is less than 1 there is no shock possible. Only if M x greater than 1 shock possible you show that you think entropy change starting from Gibbs relation and get it this particular variations, this is for a given gamma. So, let us take gamma equal 1.4 or so. For other gamma you will have a slightly different curve.

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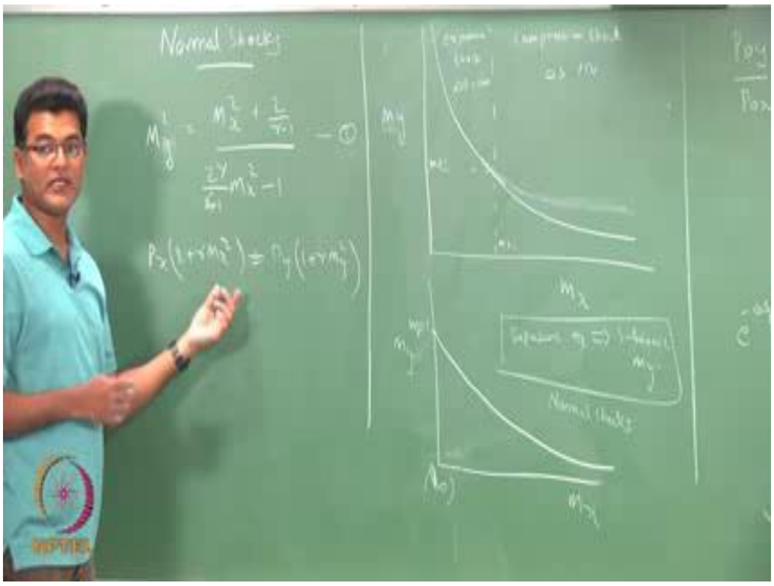

Now, if we have decided that this is true. The subsonic shocks are not possible, now we can go head and draw the variation of M x and M y m x. Let us take something like this, so this is my so called Expansion Shock, this is my Compression Shock. This would be slightly more stepper curve. This is delta s positive, there is delta s negative.

Hence we are not going to discuss this particular case were M x less than 1, so subsonic expansion shocks we are not going to discuss we are going to consider only this compression shock. So, if I redraw that curve if I start my coordinates from 1 I have m x, m y; so this is 1 this is 0. I start my x axis from 1, so my m equals 1 here and m equals 1 here, so I would have a curve that is going like this. So, for every supersonic m I would have a subsonic m y. That is a strictly for normal shocks.

So, again you can look at for the gas tables you can relate for every supersonic mach number you can have a subsonic mach number after the shock. So, there is depends only on your m x. The moment you have a relation between the mach number that exits before the shock and after the shock I can replace all these m y's in terms of m x. So, my P x by P y can be in terms of m y, so I would rub this.

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My M y is now a function of M x and gamma. We have in the earlier process we have derived this function of M x and M y and gamma T x by T y and rho x by rho y everything in terms of M x and m y, which now we can use this to get P x by P y in terms of M x and gamma and likewise T x by T y and rho x and rho y everything in terms of my mach number before the shock and get the so called table which we had seen before. In the gas table for every M x you will have M y and you will have a P x by P y, T x by T y, rho x by rho y and so on. So we can use these tables to do our problems, but before that let us also consider the star mach number or the m star values before and after your shock.

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So, I can relate my m square in terms of star values as this something which we had already seen in the previous class; how to relate my m in terms of m stars, for every m there is a m star that is associated with it. And we have also seen what is the m star when m tends to infinity, all such things that we had seen plot looks something like this, this is my m tends to infinity, this is my asymptotic value of m star which is root of gamma plus 1 by gamma minus 1. So all those things we had seen. Now we are going to use this for M x and M y and see if my m star x has anything to do with M y star can you relate my M x star with M y star using this relation across a shock.

So, what I do is I take this equation which we had written several times today itself which is 2 gamma by gamma minus 1 M x square minus 1. Now I am going to substitute this particular equation to this particular equation, so my value of M x in terms of M x star. So, I substitute instead of M y I substitute this, this would be 2 by gamma plus 1 M y star square divided by 1 minus gamma minus 1 gamma plus 1 M y star square equals; so I would write it here so that I can write it comfortably.

Instead of M y I would write this, so this would be 2 by gamma plus 1 M y star square divided by 1 minus gamma minus 1 by gamma plus 1 M y star square equals instead of M x I again write this 2 by gamma plus 1 M x star square divided by 1 minus gamma minus 1 by gamma plus 1 M x square plus 1 by gamma minus 1 whole divided by 2 gamma by gamma minus 1 instead of M x again I write this 2 by gamma plus 1 into M x star square divided by 1 minus gamma minus 1 gamma plus 1 M x star square whole minus 1.

So, the denominator in this and denominator in this are the same, so I can cancel out that. I can reduce this to 2 gamma minus 1 by gamma plus 1 M x star square plus 2 into 1 plus gamma minus 1 by gamma plus 1 M x star square divided by this quantity 2 gamma into 2 by gamma plus 1 M x star square minus gamma minus 1 into 1 minus gamma minus 1 by gamma plus 1 M x star square.

So, here gamma plus 1 and gamma plus 1 get canceled, so this would be 2 M y star square divided by 1 minus gamma minus 1 into M y star square. So here again gamma minus 1 and gamma minus 1 get canceled, so this would be 2 gamma minus 1 M x square plus 2 into gamma plus 1 gamma minus 1, gamma gamma cancels and 1 and 1 and remain as 2. This is bracket here, so this will be plus 1 that plus 1 2 M x star square divided by; so this without the denominator. Again the same quantity it would be 2 into M x star square.

So, there is something wrong somewhere. Final equation is this star is 1. If I do the algebra little more carefully I should get this relation which is prandtls relation.

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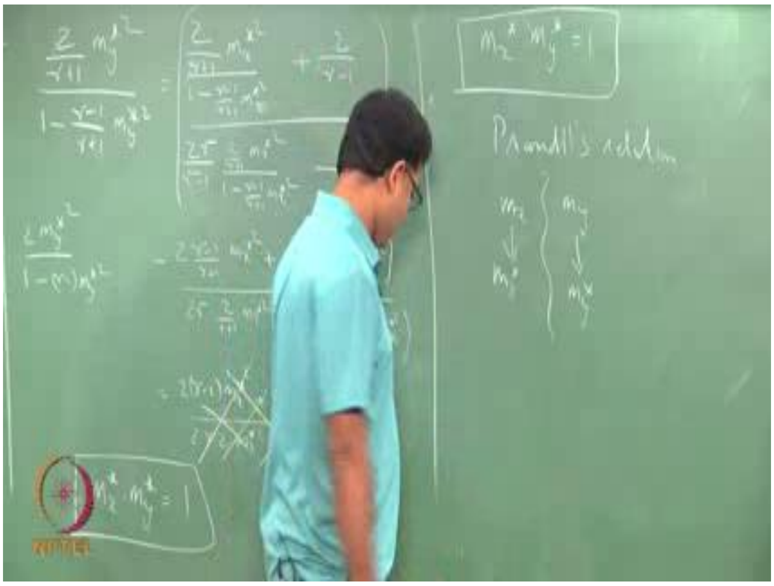

So, this just talks about you have a shock I have an M x and I have an M y associated with this I have an M x star I have an M y star, the product of this is this which also leads to.

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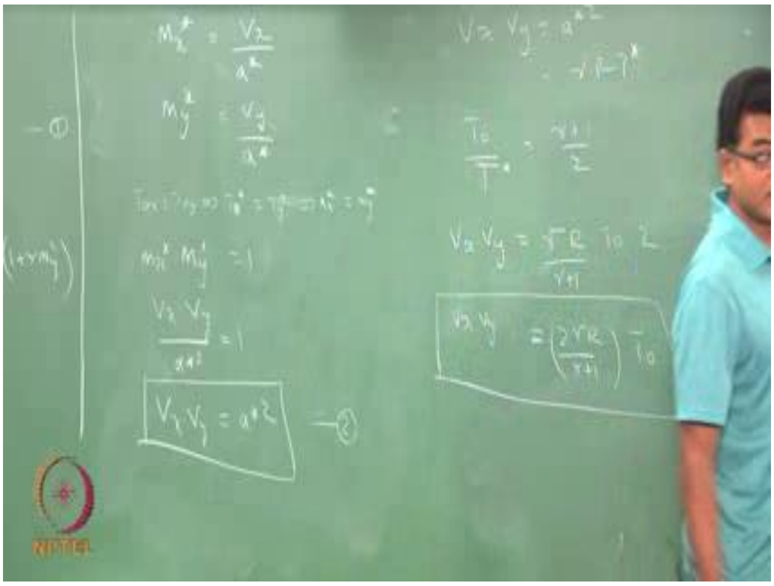

So, my M x star is nothing but v x by a star, and M y star is nothing but v y a star because your T x star and T y star is the same quantity that we had seen in the last class, because your stagnation temperature does not change. T 0 x equals T 0 y implies your T x star equals T y star implies a x star equals a y star.

Now, the product of this is what we had seen just now, which means M y star equals 1. So, v x into v y by a x square is 1 or if I multiply the velocity before and after the shock I should get this. So, now I have v x equals v y equals a x star square which is nothing but gamma R T star. Now you also know T stagnation by T star equals gamma plus 1 by 2. So, I can rewrite this equation v x into v y as gamma R T 0 into 2 divided by T star divided by gamma minus gamma plus 1. I have replaced T star with this value 2 into T 0 by gamma plus 1.

So, this is v x into v y and some constant 2 gamma R by gamma plus 1 into T 0. The product of this velocity is now just a function of your stagnation temperature, or if you see this as your total entropy change then your v square which is also the kinetic energy; now this is something that is kinetic energy lost here during the irreversible process of shock. So, the product of those velocities depends only on your stagnation temperature and the fluid of course gamma and R.

These are the other important relations which that is coming associated with your prandtls equation; m star, M x star plus my star equals 1.