**Fundamentals of Gas Dynamics**
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**Week - 08**
**Lecture - 29**
**Introduction to normal shocks**

This week we are going to study something on normal shocks. So, normal shocks are non-isentropic process that is happening in the flow.

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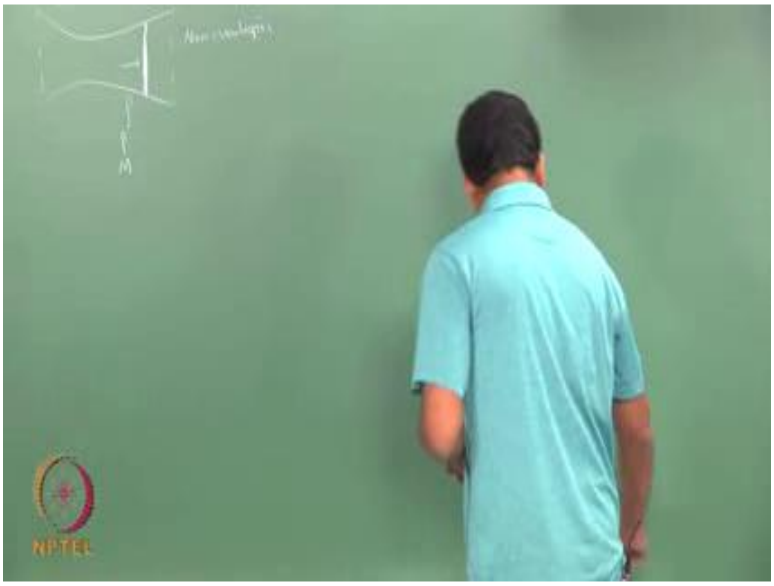

So, if I have a nozzle like what we had discussed in the last few classes for some pressure issue, you are going to have non-isentropic solutions. What essentially it means is that you are going to have some kind of shock that is hitting in the diverging section of the nozzle. What do I mean by that? I will show you a video.

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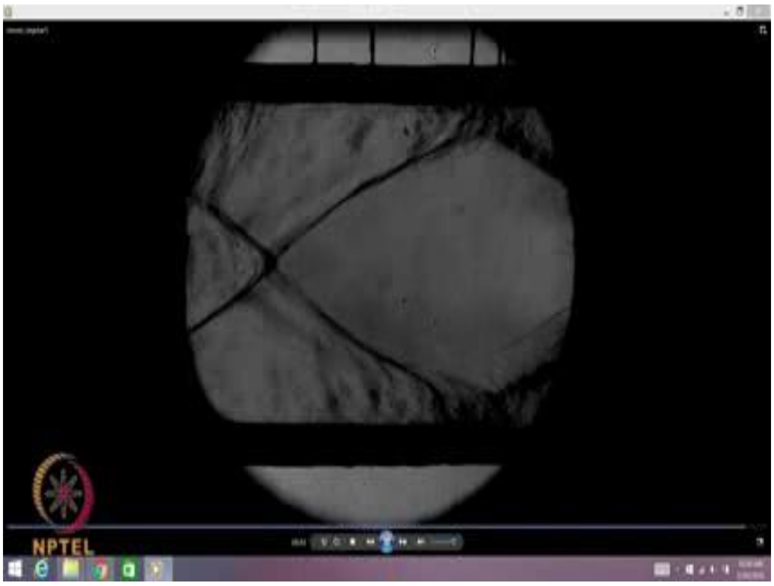


So, in this video what you see here as this dark lines are the shocks. So, you see a slanting shock here and then, you have the other shocks there. So, I will replay that again. So, this is as a normal shock, this is oblige shock and all those things we are going to discuss in this course. So, as these dark bands are larger density gradient values, so what you see here is something called Schlieren photograph in which the gradients of density is being captured.

This property, this change in your flow pattern at this particular junction or a thin layer, thin region where you have discontinuity in density and pressure and other quantities or typically called as shock, this is a non-isentropic process which essentially would give me flow pattern in this particular case.

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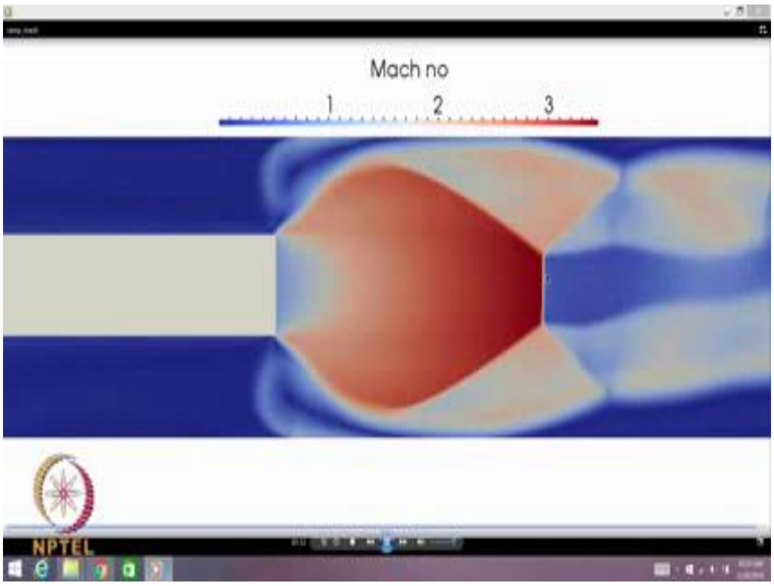


So, I have a jet that is coming out and this would create a difference in Mach number because my density is now different. So, if you see here, these are larger Mach number regions and the blue regions are your low Mach number regions and in between you see a thin line that is where the jump in Mach number happens and if you see a bit more, these larger Mach numbers once it crosses this thin region, suddenly it turns into a subsonic flow or a blue region. So, the red region is now changed to a blue region once the flow crosses this particular line. So, we are going to see in some special cases. Some within the assumption of 1D steady flow whatever I have shown you is an unsteady flow, but the analysis that we are going to do is something on the steady flow.

So, what do I mean by that? I have a shock that is standing in time and at some location and it is not changing with respect to time. So, we are going to study some location where I have a shock here and this Mach number would be large Mach number comes and something happens here and then, the Mach number reduces. So, your density changes, pressure changes and there are few other quantities that changes and we will see how to analyze this within this particular assumption. We will come back to this once we are discussing supersonic flows. So, whatever we have shown you in the movie, we are going to discuss in detail the oblige shocks and the normal shocks. So, the topic which I am going to discuss now is standing normal shocks.

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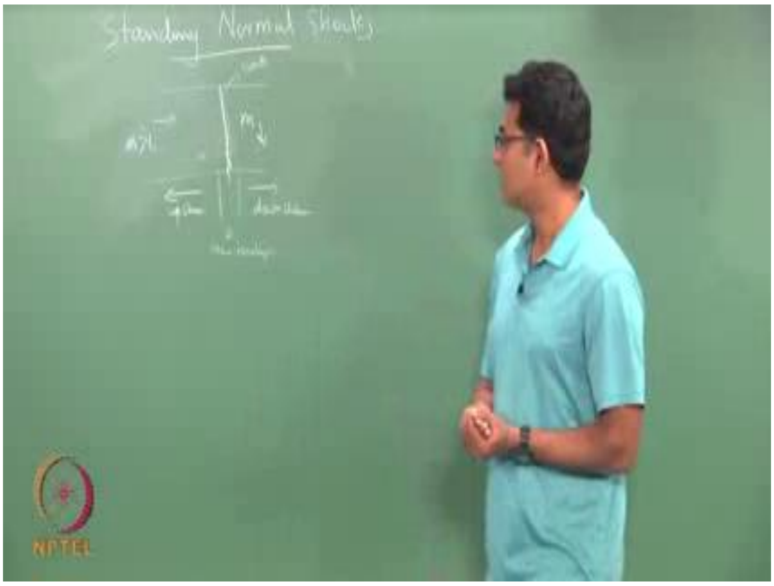

So, this unlike what we have seen in the movie, the shock is standing there still and then, we are trying to analyze it. So, what are shocks before we start? So, if I have a fluid particle that is traveling at some velocity, some large velocity at some location, something happens which forces the fluid to decelerate or forces the fluid to stop. Now, if this is m greater than 1, this information cannot be going back or going upstream.

So, if this is my upstream direction and this is my downstream direction with respect to the point we are talking, so at this point something happens, so that the fluid stop suddenly. So, the fluid particle that comes here does not have this information till it reaches that information and that is what we have learned from the propagation of supersonic waves. So, the fluid here comes and stops or decelerates suddenly to a large value, decelerates to a large velocity gradient.

So, the fluid behind does not know this information comes and hit the same fluid and you will have an accumulation of fluid particles at this particular region, at this particular point to coming and hitting and that thin region is called your shock. So, the sound waves are infinite decimally small pressure waves, shock is a finite value pressure wave. So, the large pressure gradient that is moving is here shocks are called shock.

The analogy that is given in a few books which I have read is that if you have few skiing, if you are skiing, a group is skiing down the hill, at some point something happens and the first skier stops suddenly and then, the person skier that is behind him does not know what has happened comes and hit and the following person comes and hit and there is a sudden havoc. You know change in velocity of the group of the skiers that is professionally something like this, it comes and it stops or decelerates to a very small value. So, the Mach number here would change dramatically at this thin region. So, as soon as it comes and crosses this, the Mach number change because Mach number changes the other property also changes.

Now, we will see what the relation between the Mach number before the wave and before this shock and after the shock and the relation that is connecting these two, ok. So, this particular region is non-isentropic process. So, in this region it is non-isentropic because it involves sudden changes in properties, it involves entropy change and it involves friction and other things. So, it is a non-isentropic process. So, what we will analyze is the following.

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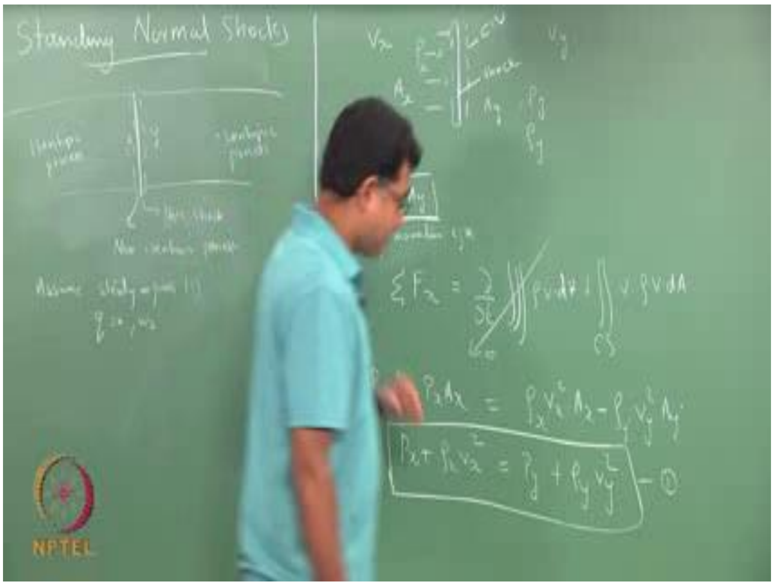


We have a thin shock. So, typically the thickness is of the order of our mean free path. So, I have a shock here, very thin up to this, it is an isentropic process, after this is again isentropic process, but within this thin region you have non-isentropic process. We will also assume steady and quasi 1D as before we will also assume our heat transfer is 0 and shaft work is 0. So, it is an adiabatic process, irreversible adiabatic process steady quasi 1D.

So, if I take the shock alone and the control volume here, so this is my control volume and this thick line is my normal shock. So, there is I call this region just properties before the shock as x and after the shock as y. So, if I say m x, it is the shock Mach number before the shock m y is the Mach number after the shock or you still can use one and two, but for convenience, we will use x and y and see how it changes across the shock. So, if this is my control volume, if p is acting on the control volume in an area a, so this is p x a x after the shock is a y and p y you have rho x and rho y approaching with a velocity v x and v y.

So, if I assume a x equals a y, apply my momentum equation here, we have not used momentum equations in the previous few classes. Now, we are going to use that here. So, this is going to be sigma x or sigma x equals assuming it to be 1 t, you have dou by dou t of control volume rho v d v plus control volume c s v rho v d a assuming steady this term would go 0, f of x is the pressure that is acting on this area. So, it will be p y a y minus p x a x equals, this would be rho v x square a x minus rho y v y square a y. So, a x equals a x equals a y, I can cancel out these terms. So, my p x plus rho x v x square same as my p y plus rho y v y square. So, that is our equation 1. So, x is the pressure before the shock, y is the pressure after the shock density and velocity.

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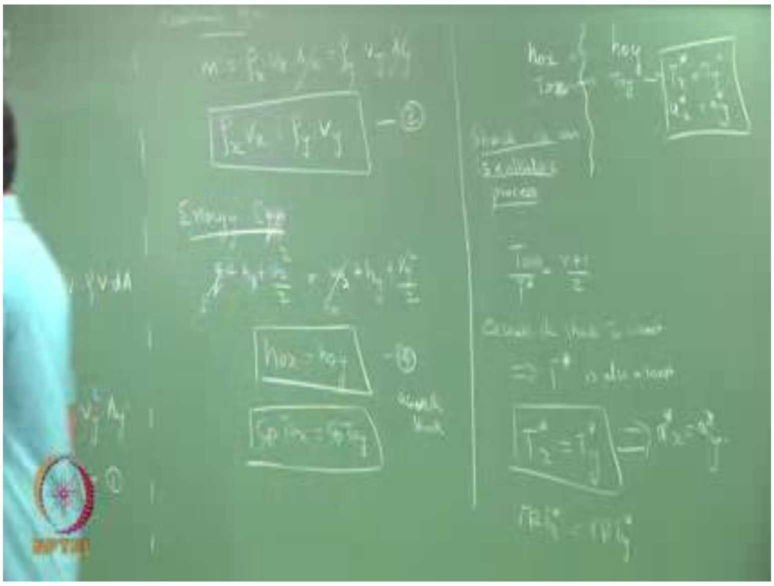

Now, we will use the continuity, apply continuity equation which is m dot equals rho x v x a x equals rho y v y a y. So, a x and a y are equal. So, I cancel out and my rho x v x equals rho y v y. So, that is equation 2. Now, I apply energy equation which is q plus h one h x plus v x square by 2 equals w s plus h y plus v y square by 2. So, we have assumed q equals 0 and w is equal 0. So, we end up with h 0 x equal h 0 y. So, these are across the shocks for perfect gas. I can write c p t 0 x equal c p t 0 y assuming c p to be constant; you will get t 0 x equal t 0 y. So, what I mean by that is, if I have shock here, the enthalpy here is same as my enthalpy after the shock.

So, even though the process is non-isentropic which does not impose this condition to be wrong, we have assumed q equals 0, w is equal 0 is enough to have my stagnation temperatures and stagnation enthalpy to be same. So, the process here is isenthalpic process is shock is essentially is an isenthalpic process. So, we just assumed our heat transfer and shaft work to be 0. That is the only assumption we have taken and we have arrived at this particular relation.

Now, we also know that your t 0 x or t 0 by t star is gamma plus 1 by 2. Now, across the shock point t 0 is constant which implies my t star is also a constant. It means my t star x is same as my t star y. We are going further if this is true my gamma r t star x is same as my gamma r t star y which implies my a star x is same as my a star y. Across the shock now I have my t star x equals t star y and a star x also equals to star y.

So, all this from the assumption that q equals 0, w is equal 0. Now, we will take this equation 2 and try to substitute that in terms of pressure and see what we get. So, like what we are done before, we will write these equations here, so that we can discuss it at a later point of time.

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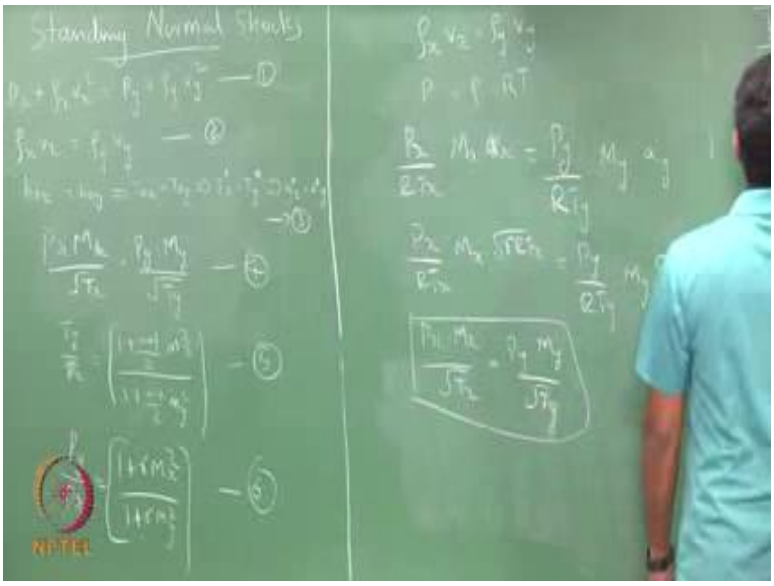


That is equation 1. Equation 2 is this which is continuity equation and then, we have h 0 x equals h 0 y which implies t 0 x equals t 0 y which implies my t star x equals t star y which implies a star x equals a star y. So, we have derived, we have seen these many equations. Now, I am going further in the derivation. So, what I will take is let us number the equation.

So, that is equation 1, this is equation 2 and this set is equation 3. So, what I will do is, I have rho x v x equals rho y v y I would have my p x equals rho x into r t x or I would rather say my rho equals p r t. I would get rho x by rho y as v x by v y. Now, I will substitute this there. So, this would be rho x. Instead of rho x, I would substitute as p x by r t x into m x into a x equals p y by r t y into my into a y. Now, ay also I would substitute in terms of temperature. So, this would be p x by r t x into m x into root of gamma t x equals p y by r t y into m y into root of gamma r t y.

So, I can reduce this to px into m x divided by root t x equals p y into m y by root t y. So, this is from a simple continuity equation and substituting the ideal gas equation. So, that is my equation 4. So, this quantity is going to be constant across the shock. So, that is my equation 4.

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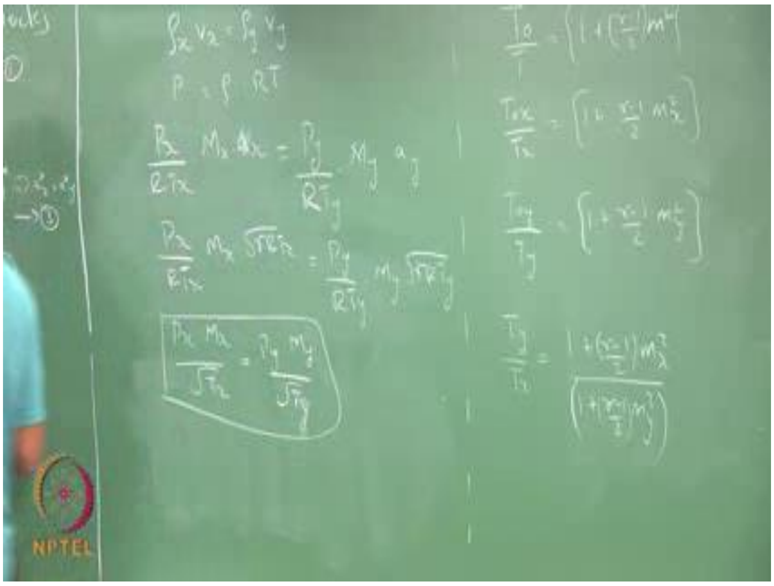

Now, I can relate t by t 0 by t as 1 plus gamma minus 1 by 2 m square which means I can relate my stagnation quantities before the shock and after the shock in terms of Mach number m x square and m y square. Now, we know t x t 0 x and t 0 y are same. So, I can write t y by t x equals 1 plus gamma minus 1 by 2 m x square divided by 1 plus gamma minus 2 m y square. So, this is our t 1 minus t 2. What we have done previously?

So, there is nothing new here for what we had discussed, but just that this is also true across the shock. So, the property t y and t x can be related to the Mach number that is happening before the shock and after the shock. So, we will for completeness, we will write that too. Now, what about p 1 by p 2? So, I will rub this. So, I take the first equation p x plus rho x v x square equals p y plus rho y v y square.

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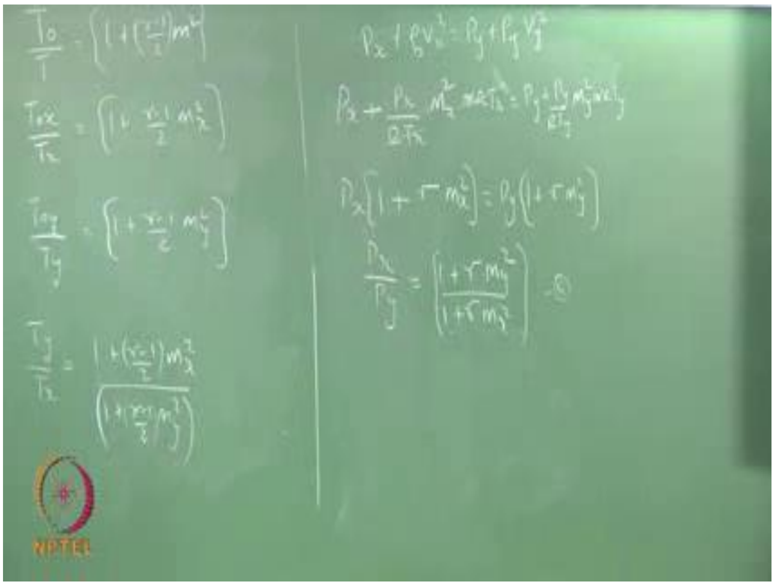

So, yeah I will take p x rho x substitute the ideal gas equation by r t x into m x into m x square into a x square which is gamma r t x square t x equals p y plus p y by r t y m y square into gamma r t y. So, this is p y into 1 plus gamma m x square equals p y into 1 plus gamma m y square. So, I can write p x by p y as this quantity 1 plus gamma r m y square divided by 1 plus gamma m x square. These are simple algebra, nothing more here.

Just you realize the pressure difference, pressure changes according to your Mach number changes. So, this is my equation 6. So, I gather that and write it here. So, I will write p y by p x 1 plus gamma m x square divided by 1 plus gamma m y square. So, this is my equation 6. So, these are the property changes across the shock. Now, what I am going to do is, I am trying to see if there is a relation between Mach number x and y. So, I relate if I know my Mach number x m x, can I find my Mach number m y that is after the shock. So, I am going to use this equation which is our equation 4. P x m x by root t x equals p y m y by root t y which I can rewrite as p x by p y into m p x by p y into root of t y by root of t x equals my m y by m x.

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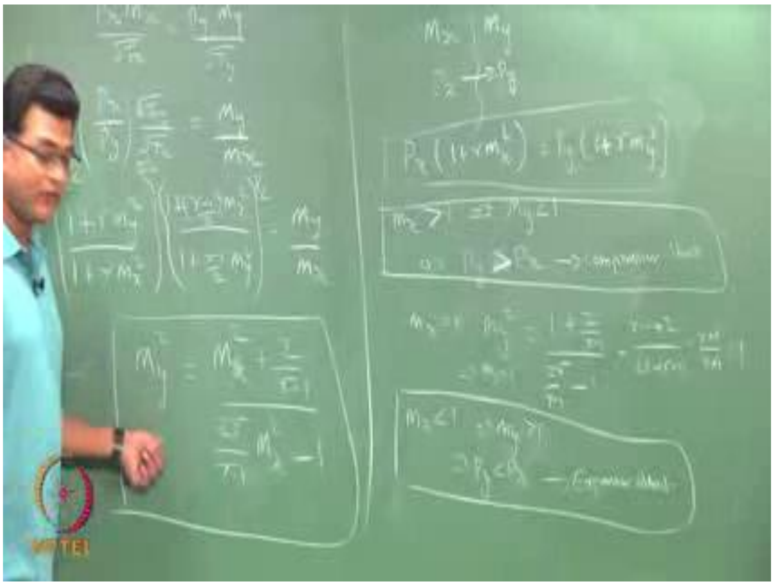

So, I have written this from our continuity equation. I have just taken p x by p y. So, I have taken p x by p y and root t x by root t y by t x. Now, this I know in terms of Mach number which is 1 plus gamma m y square divided by 1 plus gamma m x square t y by t x. Also I know in terms of Mach number 1 plus gamma minus 1 m x square divided by 1 plus gamma minus 1 by 2 m y square to the power 1 by 2 equals m y by m x. So, this is from equation 6. This is from equation 5. So, I know this relation. This is the pressure before the shock and this is pressure after the shock. Likewise, temperature after the shock and temperature before the shock, I know these ratios in terms of Mach number before the shock and Mach number after the shock.

Now, I am eliminating these quantities and trying to get a relation between m x and m y, if I do this, there is a bit of algebra here. I would get the following equation which I would write it here as my m y square equals m 1 square plus 2 by gamma minus 1 divided by 2 gamma by gamma minus 1 m x square minus 1. So, this is my final equation. So, if you do from algebra here, you will get a quadratic solution and this is the solution for m x square. So, if I know my shock number, shock Mach number at location before the shock or just before the shock, when I know my Mach number after the shock that is what it means.

Now, let us look at the other equation and try to see what is happening here. So, I have a shock m x and m y. I also know that my p x into 1 plus gamma m x square equals p y into 1 plus gamma m y square. So, this does not distinguish anything between subsonic or supersonic Mach number. So, let us first consider m x say greater than 1. So, if m x is greater than 1, this ratio implies. So, if m x is greater than 1, you are going to get m y less than 1 here. If you look at this equation implies your m y less than 1 or p y would be larger than your p x p y would be greater than your p x which means this is a compression process.

So, what happens here is you have a Mach number supersonic. Mach number comes and there is a shock and then, you hit a condition where your m y is less than 1 and suddenly your pressure increases to py. So, the flow comes and then sees something all your fluids are to one thin region and then you see the Mach number is decreasing and the other side and your pressure is increasing which is essentially your compression process.

Now, if my m x is less than 1, if I substitute here m x is less than 1, if m x is equal to 1 what happens? So, let us discuss that first if m x equals 1, if this is 1 plus 2 by gamma minus 1, this is 1. So, it is going to be same as let us do that minus 1. So, this would be gamma minus 1 gamma minus 1 goes. So, this would be gamma minus 1 plus 2 divided by 2 gamma minus gamma minus 1. So, this is gamma plus 1 divided by or this would be bracket here. So, this would be gamma plus 1 which is 1. So, your m y is 1.

So, this solution is also present in this particular equation, but this is also the trivial solution which means you have a Mach number and nothing is changing if m y is equals m x equals 1. Then, your pressure ratio here p x by p y is also 1 and hence, your temperature if you look at the equation 5, this will also be 1. So, it is something like you know a flow with nothing is happening. It is a trivial solution. So, that is also a solution to whatever we are discussing here.

Now, our problem is the other one. So, this is also equal to m x less than 1. So, what happens is m if m x less than 1, if you substitute a value m x less than 1 here, gamma is always greater than 1. So, m x less than 1 would give me m y greater than 1. Again you can substitute any random value and see m y is always going to be greater than 1. So, if m y is greater than 1, if you look at this equation relating the pressure, your p y is less than your p x. So, if I have subsonic flow and if there is something that is happening that to take me to m y greater than 1, my pressure is going to decrease which means this is an expansion shock. So, this is a compression shock and this is an expansion shock. So, this is an expansion shock. Now, this is not possible. We will derive and show that this is not possible.

In the next class or the next lecture, we will show that the entropy change during this process is going to be negative and we will also see what is the relation between that when it comes to this, how does m y vary with respect to m x and hence, why this is not possible and only this is possible. We will show that in the next lecture for the time being. So, this is my expansion compression shock and this is my expansion shock. Both of this is possible solution to this equation which relates my m x and m y. So, for any given m x, I have a given y and mathematically that is possible. Basically this is not possible. Again we will show it using entropy change and it gives equation. That is for this class.

Thank you.