Fundamentals of Gas Dynamics
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Week – 07
Lecture – 28
Examples and applications of flow through C-D nozzles

We will end the discussion on Converging-Diverging Nozzle with a small numerical simulation that we have done. So, the example that I am going to show is what we had already discussed.

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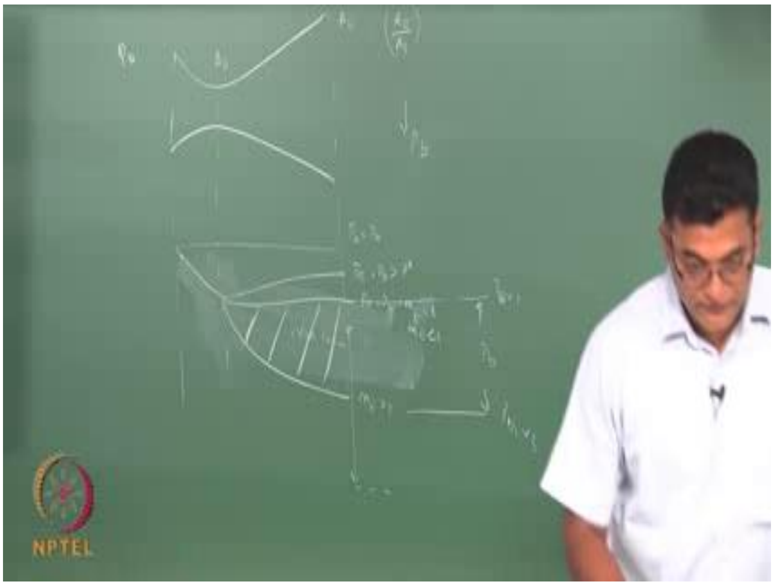

So, it is a converging nozzle, I have kept my p 0 constant for some area ratio which is A exit by A throat. So, for this particular value of nozzle that we have considered for this particular stagnation pressure I keep on decreasing my p b, and what we have obtained is this when p 0 equals p b, there is no flow, otherwise you would get a subsonic flow at some condition and supersonic flow when you have A. Again then you have p exit equals p b which is greater than p star, p at the throat conditions then you have some value, again it is not like this it would equals and throat equals 1, which also gives me a supersonic value, here m exit is subsonic in between I have a non-isotropic solution. So, there is a p b value between p critical, p b critical 1 and p b critical 3 where I get a non-isotropic solution.

So, in the numerical simulation what I am going to show you right now is between some values here to some value lower than p b critical 3. I kept my p 0 constant for a given nozzle I start from somewhere here, some non-isotropic solution and then I go down here and what I am going to show you is how the flow develops into this. This is also a precursor to what we are going to discuss in shocks. So, I am going to show you the occasional shocks moving from 1 position to some position and at some value of p b, I have the shocks sitting at the exit which we will define it later to be our p b critical 2.

So, that scenario will also be shown, but remember the simulation is a viscous simulation. So, there will be a slight change in the values as well as the shock structure that we are going to prove. So, what we are analysis now is strictly 1 d flow, but in the analysis it is 2 d flow that its actually 3 d flow, its 3 d accelerometery flow. So, we had taken a section and then we are going to do this.

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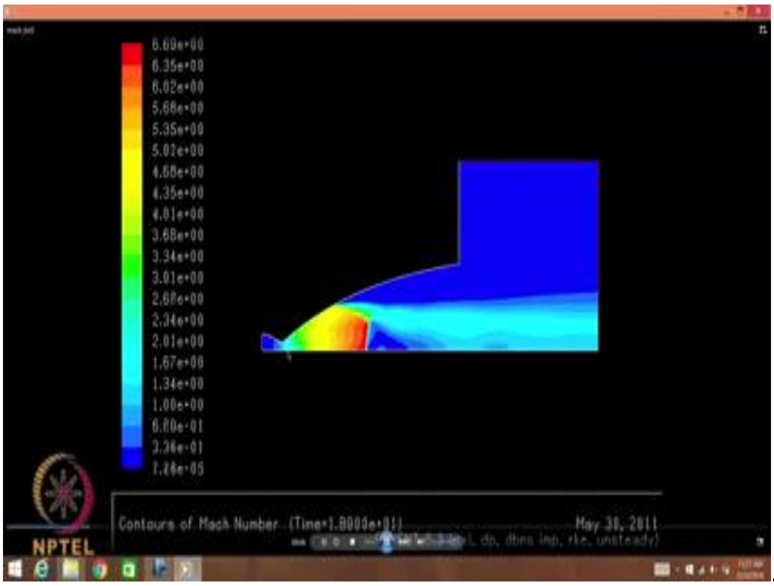

So, I will show that first. The flow is developing from the left to right and the p b is now decreasing. So, I will pause for a minute, the p b here in this chamber is decreasing, but the stagnation pressure at the inlet of the nozzle is kept constant. So, this is only half of the nozzle and this nozzle is a converging diverging section. The converging inlet area is very small then it comes to the throat and then it increases to a large value and which exit into a chamber which is where we decrease our back pressure.

So, in this flow the back pressure is decreasing in time, but in our analysis the back pressure is not a time varying thing. We have change back pressure and we have found a steady state solution. So, those are the differences between the simulation and what we analyzed again this is a viscous flow. So, you would see there will be a boundary layout growth and other thing along the walls given that condition will try to see what happens when your p b is somewhere in between p b critical 1 and p b critical 3.

Now, I will play this, the location of the shock is this that is moving and when it comes to this condition the shock has moved out and this condition if you look at here the flow stream line neatly go out of the nozzle without any discontinuities. So, the mach number, the flow here there is no shock, this is actually a mach number condo. So, there are no shocks here, mach number increases steadily from the throat to the exit this is the so called p b 3. So, I am getting a nozzle flow with the mach number greater than 1 at the exit and if you look at the graph which we have shown it neatly goes out.

So, your exit, back exit pressure is also your back pressure and the mach number goes neatly out. So, there are no discontinuities. Now, if I decrease my back pressure further something else happen. So, your flow would go here which is what we called as our under expanded nozzle. So, from our expanded what we have come here is an under expanded nozzle. Now, I will stop here and rewind and show this again.

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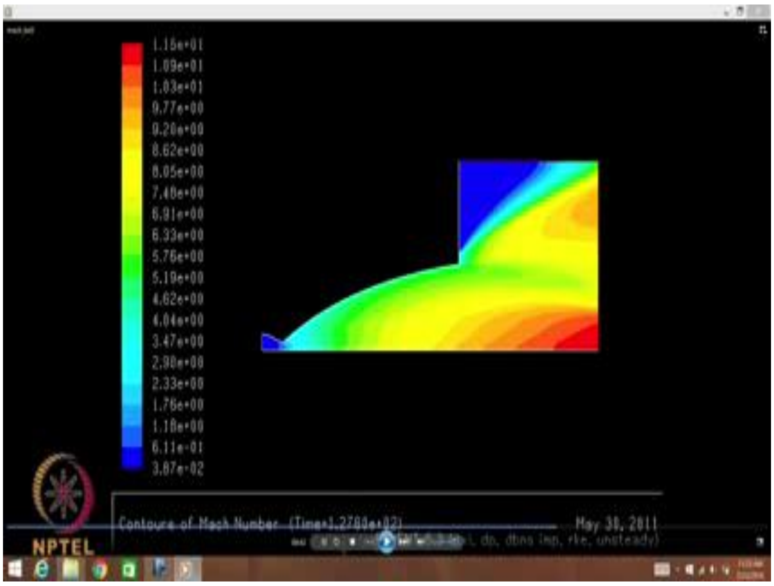

So, from the beginning I have a situation where the pressure exit p b is between critical pressure 1 and critical pressure 3. Now, the nozzle is over expanded as we had described in the previous lectures. So, there is some non-isotropic solution that is happening here these are the shocks which we will see it at a later point of time. So, the mach number is very high here and it encountered as something called a shock and then suddenly you have a subsonic mach number here which is your blue region. So, the red region is where you have the larger mach number.

Now, when I decrease my p b this shock moves towards the exit. So, I play this, the shock is slowly moving again. This is p b changing in time which is different from what we have analyzed I am showing it I viscous numerical solution. So, the shock is slowly moving out, this is still within inside p b 2 and p b 1 and p b 3, now it has reached it is going to reach the p b three values. So, this has now reached the p b value now it has gone up. So, it is reduced further the p b is still reducing further. So, there is some other solution that is happening outside the nozzle, if you look at inside the nozzle it is not changing at all.

So, if you look at inside the nozzle what you see here is an isotropic flow and everything all problems or all non-isotropic flow is happening events are happening outside the nozzle. So, the nozzle ends here, inside the nozzle if you look at it there are no changes there are no non-isotropic events that is happening, but when it comes to a pressure ratio before your critical pressure ratio 3, you have all kinds of non-isotropic events that is happening inside the nozzle. So, I will play this again, you have degrees in your p b. So, your non-isotropic events are moving out of the nozzle. Now, it is out of the nozzle, inside the nozzle there is not much change.

Now, you have reached around p b through the critical 3 and now further in p b 3, you would see events that is happening here which is now an under expanded nozzle. So, the nozzle goes from an over expansion in this case to an under expansion when you reduce your p b. So, this is now an under expansion. So, there is something called a shock that is happening here. Now, we will come back to this. So, what I have shown now is from value of p b 3.

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Somewhere here, I have reduced my p b for the down some where here. So, the nozzle has gone from an under expanded this separation is which I had shown you in the simulation is due to your viscous simulation. So, what we have analyzed is an invest simulation the viscosity is not there, that is why it is an isotropic flow. Now, within this region there the chances of separations are there and the simulation is viscous flow simulation its non isotropic this event is non-isotropic.

So, there is a separation and then you have a jet that is going like this as you decrease your p b further it would go something like this. So, from an over expansion the flow has gone to an under expansion. So, the moment you see a rocket nozzle with this kind of jet it is a under expanded jet and the moment you see a nozzle with a this kind of flow this is my nozzle exit and this is my jet the moment you see a smaller jet like this is an over expanded nozzle.

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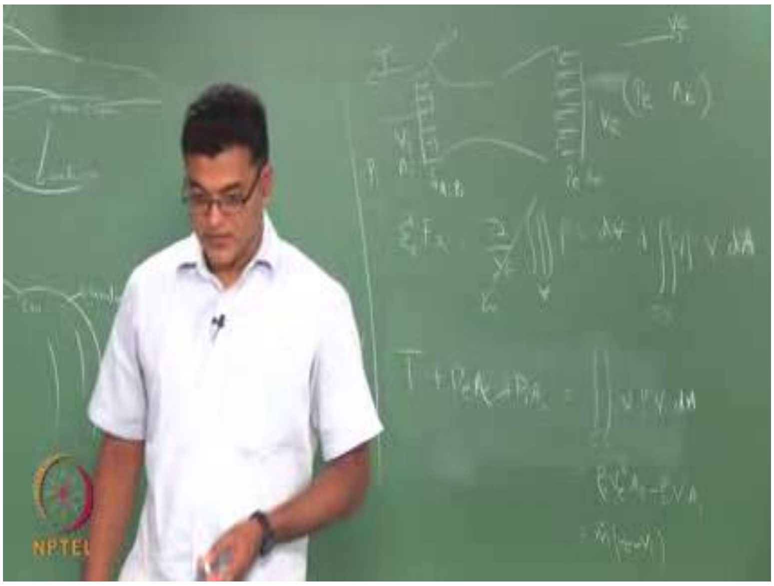

We can also calculate the thrust that is developed by the nozzle. So, if I have the flow here flow going from here to here, if I write the, take the control volume as this I can apply the momentum equation f of x equals dou, dou is the t of triple integral rho v d v which is my volume. So, this is my volume control, volume integrated for this control, volume plus all the control surface rho v v bar d area for a steady state which is what we are been doing all this classes this term would be 0 sigma x, f of x would be the thrust that is developed if this is the nozzle that is going the thrust is this way. So, this my v exit this is my v inlet and I have a area inlet pressure inlet, I have a pressure exit and a area exit. So, the first that is due to your pressure is this which is my p i into a i that is generated due to this is my p exit into A exit which is in this direction.

Now, the thrust that is developed is opposite to the velocity that is at the exit which is this direction. So, I can add up the f of x which is t plus p e into A exit plus minus p i into A i which is nothing, but this quantity which is nothing, but rho into rho exit v exit square into A exit minus rho inlet v inlet square into A i which is nothing, but m dot into v e minus v i, you can use this to find the thrust developed in a nozzle. So, if I know these quantities then I can find the thrust. Now, we will address something that.

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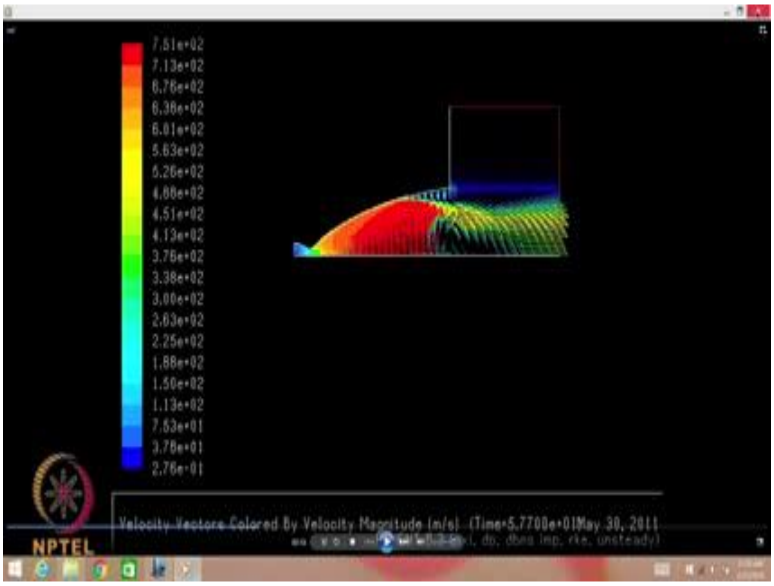

You would see in a nozzle, for example, we are just now said there is a shock. So, if you look at the velocity plot of the same thing that we have a shown, if you look at these are the velocity vectors and again the experiment is the same thing, we are decreasing our back pressure, the back pressure is reduced further and further. Now, if you see that we had seen some shock that is moving further some non-isotropic solution, I will repeat that. Now here is some location of the shock. The velocity vectors are large in magnitude before the shock and then suddenly it goes down. So, this is 1 of the main characteristics of a normal shock.

So, the mach number would be large before the shock and then suddenly it decreases and which is opposite to something like an expansion fan. So, what are the shocks is what we are trying to look at it. So, you look here and I will pause, these red regions are the region where I have large magnitude in velocity and then after that it suddenly there is a dramatic reduction in my velocity.

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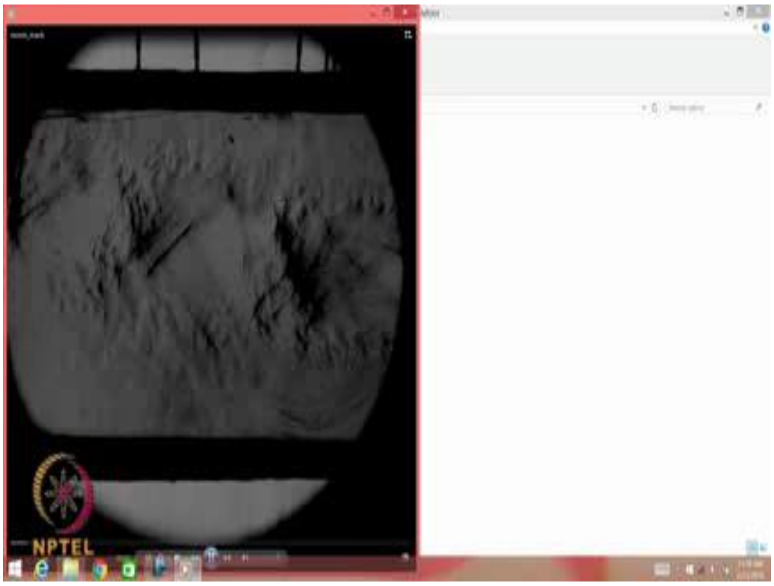

So, how does it look like when you do a density condo? So, if I take this particular case, these dark regions what you see here are some of the large density gradient flows. So, this is something called a Schlieren's photograph where you try to photograph the density variations. So, density gradients have been captured in visualization. So, this dark regions are essentially these lines are essentially shocks. Now, you see the flow is from right to left and these are these are shocks that is generated. So, what are these shocks? There is a large velocity that is coming here and then it faces due to some reasons the flows suddenly needs to slow down, there is some non isotropic process there.

So, the fluid that is coming from here, I will pause again. So, the fluid coming from the right does not know that something is happening here. So, it just comes and slow downs and hit this particular 1 then the fluid after that will also do the same thing. So, there is a accumulation of these fluid coming and hitting at that location and suddenly the density would be quite large there and that large density, large pressure location is what we called as shocks.

So, it is actually a finite pressure wave that is moving the small pressure wave, we call it as sound wave, but this is a large pressure gradient that is moving in the flow or standing in a flow or I would being faced when the flow is coming and the fluid is facing this large pressure gradient. So, it does not know what to do it just flows down and the mach number dramatically comes down and we will see on what condition this will happen?

It is going to happen only in a supersonic flow, it is not going to happens in a subsonic flow we will see why it is, but essentially what I want to show you in this particular plot is that there is a density jump in this particular case, where there is a shock there are other regions of black region, black color which we will for the time being we will forget. We will concentrate on this particular line which is a normal shock, which we will call it as normal shock is what we are going to analyze at present.

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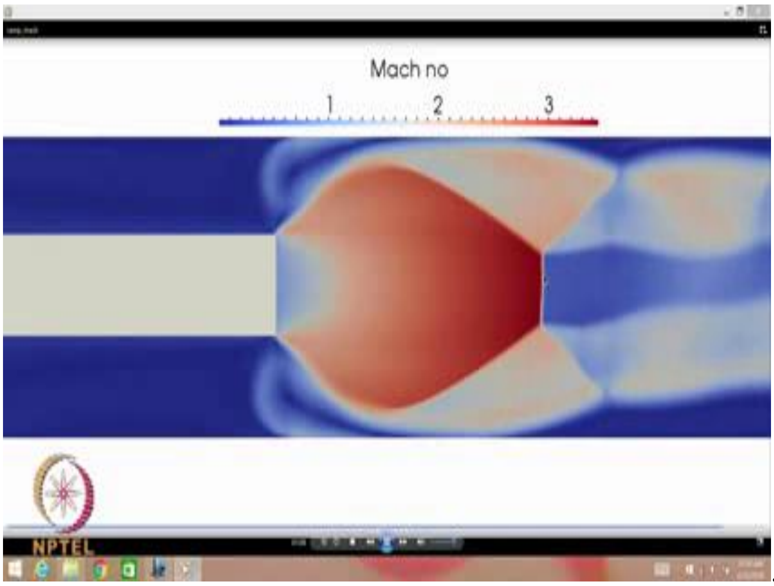

I will also show you a numerical simulation what was shown there is a experiment we will show you a numerical simulation in which we obtain a similar scenario, for example, this is a jet that is coming from left to right and this jet is going into a chamber which is kept at some pressure. So, let us forget about this unsteadiness, will show you when it reaches a steady flow there is a large mach number that is generated here and then suddenly it faces a shock after which the mach number reduces to a sub sonic value.

So, the red is the region where you have large mach number and the blue is where you have a small mach number or subsonic mach number and ones it faces this particular region or small location.

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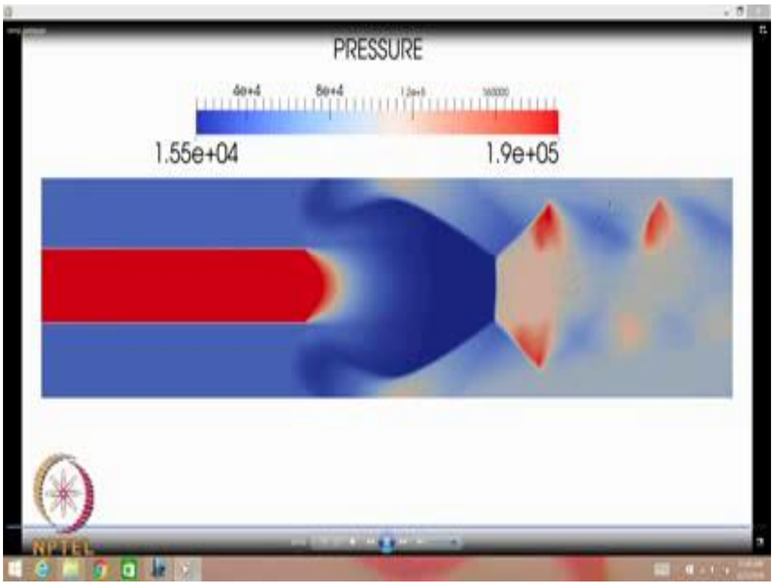

Where the thickness is of the order of the free molecular, free path and I will show that again. So, the mach number here, these are some transins which we will not discuss it right now. So, these are large mach number regions then suddenly faces the shock and then goes to a subsonic value. If I look at the pressure plot you would see this is a static pressure plot. So, you would see a similar behavior here. So, after the shock then suddenly there is a raise in pressure, after this is a small pressure and suddenly you faces a shock and then you have a large increase in your pressure value.

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So, if I plot the density gradient that will also show a similar trend like what you see in your Schlieren photograph these are density gradients. So, suddenly there is a large gradient that is occurring near the shock. So, this particular line is what we call it a shock and that is as I told you is a very thin region of the order of molecular free path. So, our aim is to study this shocks which are standing at 1 location. So, we will consider two kinds of shocks; 1 is two kinds of normal shock, 1 is when the shock is standing and stationary at 1 location. So, if I have a flow.

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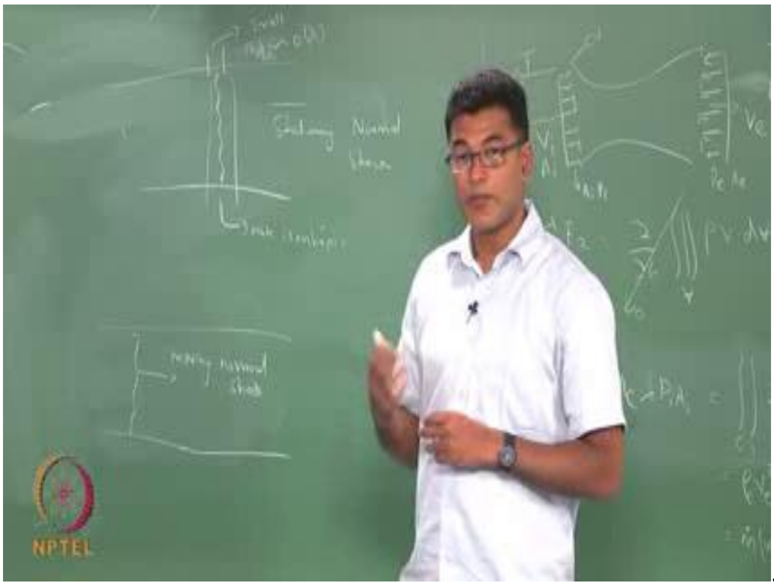

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If I have a shock which is the thickness is very small of the order of thickness of the order of molecular free path. So, this would be your thickness and this is a non-isotropic process. So, this is a stationary normal shock we will also consider when the shock is moving. So, we will in this course consider these two scenarios; 1 I have a shock here the flow is coming and then it encounters a shock. So, the density pressure temperature everything changes it has some discontinuity and then the flow continues.

So, I have a supersonic flow, comes encounters the shock and goes here. I have a stagnant flow the shock is coming in moving alone. These are two different scenarios; we will consider this is like an explosion that is happening. So, if when you have a bomb explosion or 1 of those explosions you have a shock wave that is moving. So, you have shock wave that is, I have a stagnant, there the shock wave is moving and encountering the stagnant fluid and it changes the properties there. In this case, it is like a shock that we will see inside an internal or in a nozzle. So, you have a standing shock and then the flow comes encounters a shock and continues the flow. So, we will consider these two scenarios in our lectures that to follow.

So, with that I will end with that introduction on normal shock. I will end this lecture. So, we will continue, we will take up the isotropic relations combined with non-isotropic relations to try to see what is the properties before the shock and after the shock and we will have to revisit many of the equations that we have done before and we essentially will try to find the relation between the properties before the shock and after the shock.

Thank you.