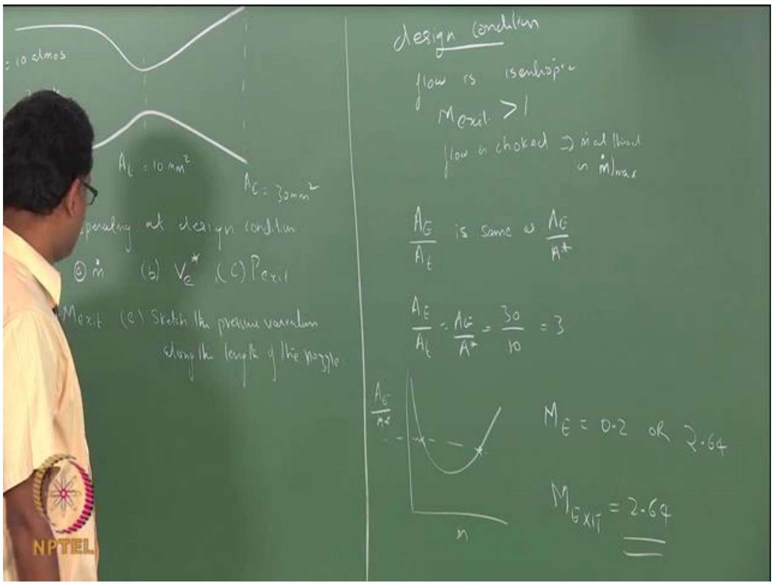
Fundamentals of Gas Dynamics

Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week – 07
Lecture – 26
Discussion on C-D Nozzles – 1

In this lecture we will do some numerical problems to understand C-D Nozzles further. So, whatever concept we have been discussing in the last few classes we will try to see it from numerical point of view. So, keep your gas tables near you. So, the first question I have here is.

(Refer Slide Time: 00:39)



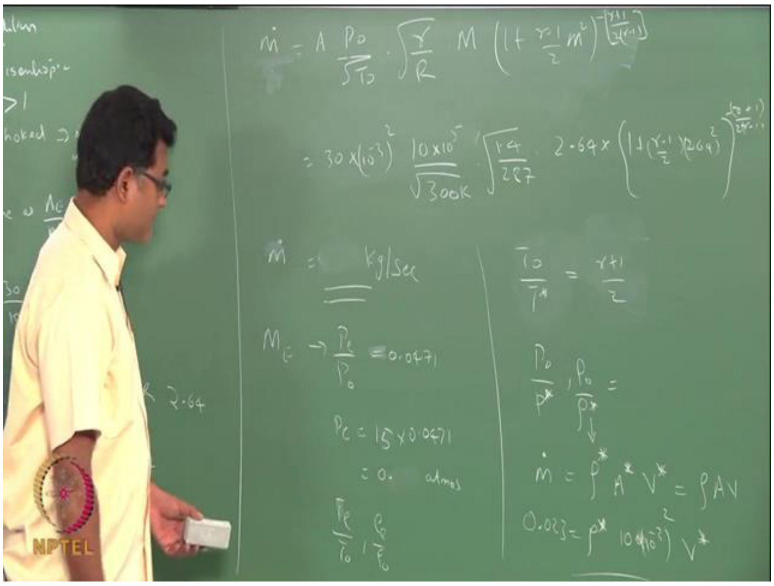
So, I have a converging diverging nozzle, the throat area is 10 millimeters, the exit area is 30 millimeters, the stagnation pressure is 10 atmospheres and T 0 is 300 Kelvin.

Nozzle is operating at design condition find m dot; find V star at the exit, c - P exit and M exit, length of the nozzle. So, I will read the question, I have a nozzle here whose throat area and exit dia is given, so it is throat area. So, this is millimeters of square, 10 millimeter square and 30 millimeter square. So, nozzle is operating at the design condition for this particular stagnation pressure and temperature. Find m dot V star at the exit and P exit, static pressure at the exit, mach number at the exit and sketch how the pressure variation along the length of the nozzle.

First information is operating a design condition. So, when you read this the first thing that comes to your mind is flow is isentropic and your M exit is supersonic flow, flow is choked implies m dot at your throat is here m dot max, these three information should come along with this particular statement which is the design condition. So, if your flow is choked then it means my A E by A throat is same as A E by A star.

If I take that information, is nothing but 30 by 10 that is 3, for this A E by A star and mach number I have 2 solutions. Now, also another exit mach number is 1. So, my M exit associated with this particular ratio is either the subsonic solution or the supersonic solution. So, I take this table look for gamma equals 1.4, A by A star is 3. So, I will look for A by A star is 3 which is going to be around 0.2 or 2.64. Since we already know this is a design condition my M exit is 2.64 and not the subsonic one. So, it is my M exit is non, I can substitute my mass flux a into P 0 by root T 0 root of gamma by r into M into 1 plus gamma minus 1 by 2 M square minus gamma plus 1 divided by 2 into gamma minus 1.

(Refer Slide Time: 07:47)



So, my area the exit is 30 into 10 power minus 3 the whole square, P 0 is 10 at most 10 into 10 power 5 divided by root of 300 Kelvin into root of 1.4 divided by 287 into M exit is something which we already know now - 2.64 into 1 plus gamma minus 1 by 2, 2.64 square to the power minus 1 by gamma by 2 gamma minus 1, that would be your m dot.

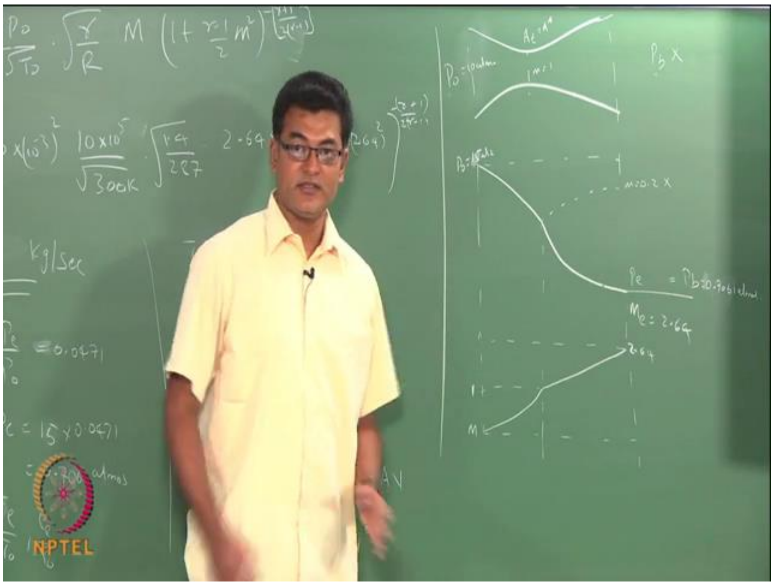
So, once you have your M exit this is my m dot in kg per second. So, once I have my M exit I can get P by P 0 or P exit by P 0 which I have to look at the gas tables to find 2.64 for 2.64 my P by P 0 is 0.07, P by P 0 is 0.0471 from which I can get P exit as 10 into 0.0471 which is 0.471 at most. Likewise I can also get my T exit by T 0 or rho exit by rho 0 if I know the values are also rho, rho 0 I can get it from pv equals rt or p equals rt and then I can get the exit density of the fluid, exit temperature of the fluid.

Now, since gamma is known I also know this value which is a constant from which I can get P naught by P star or rho naught by rho star. So, if I know these values I know already m dot, from m dot I can write rho star into A star into A star into V star at the exit, at the throat equals my rho AV. So, the throat area you know, throat area is 10 millimeter square which is 10 minus 3 to the power square, rho star I would get from here which is again a constant from m dot I already have computed what this value.

Student: 0.023.

0.023. So, I substitute that value here that is my m dot, rho star from this relation for gamma 1.4 from which I can get my V star which is my v at the throat. So, the question here if V at the throat, V star at the throat, since it is already choked condition the V here would be your V star because m equals 1 at the throat. So, your V at the throat is your V star from which you can get V star. So, we have computed m dot V star P exit and M exit. Now, sketch your pressure variation.

(Refer Slide Time: 13:21)



So, I have my nozzle here P 0 is 10 at most I do not know my P b, but I know my A throat is now my A star and mach number here is 1. So, P 0 is 10 at most, this might have reduced some value and it (Refer Time: 14:10) taken the isentropic route to get M exit to be equal to 2.64. So, this solution m equals 0.2 we have not taken because it is already not there, it is working at the design condition. So, this is my design condition. So, my P exit is Pb. So, even though we do not have that information about the Pb with the back pressure, the information that the nozzle is walking in the designed condition means my exit pressure is same as my back pressure. So, which is all equal to what? Which is equal to 0.471 at most, likewise you can draw the mach number variation also.

So, at somewhere along the throat my mach number is 1 and then it reaches 2.64 at the exit, here it is 1, this is my mach number variation, fine. So, the same nozzle throat area is same exit area is same except that the stagnation pressure is now increased to 15 atmosphere and it is working in the design condition. So, it means that there is no change in here, so this area ratio is same, the exit mach number is same we have not taken the subsonic solution everything is decided by the area ratio as we have already discussed before. Everything here is decided by this particular area ratio and this mach number is a fixed, designed mach number is fixed.

Now, when it comes to the mass flow rate that would be different because now P 0 is different, this value would be something else which means my P exit ratio is same, P 0 is different, so my P exit is also different. Instead of 10 it is now 15, 15 into 0.047.

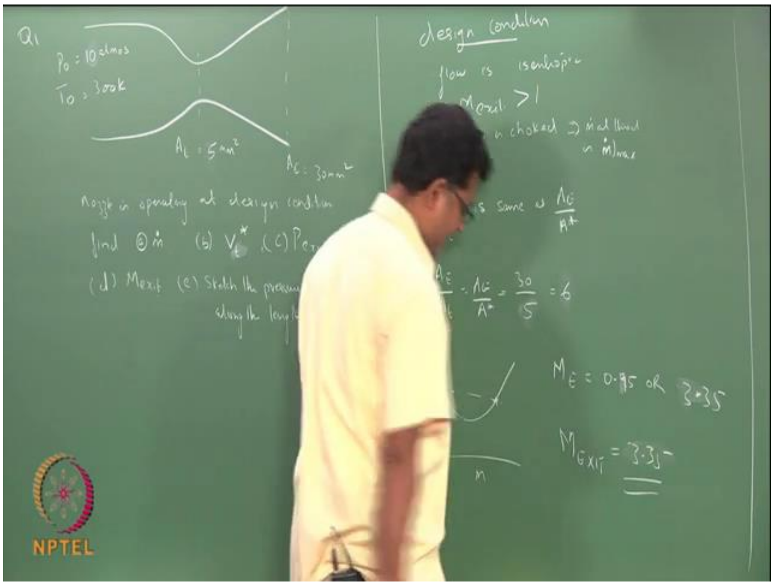
Student: 0.706.

0.706. So, my exit is different, my m dot is different, P exit is different again it is in the same design condition except that P 0 has changed. And here again the T 0 is same, so this ratio would be same, accordingly your P star would change since the P 0 has now a different value, your rho star would also be a different value. So, here again your m dot is different, your V star would also be different.

And here same plot except that this value is now 15 at most, that pressure would be something else which is 0.706, exit mach number has remained the same there is no change there and the mach number plot is the same, the stagnation value it is just subtracted a bit. So, the throat mach number is still m equals 1, the pressure ratio is adjusted such a way that the exit condition is (Refer Time: 18:46).

So, as I discussed before this is a steady state solution when I say the stagnation pressure has been increased to 15 at most it is a different case, it is different steady. So, the stagnation reservoir pressure that is supplying air to the nozzle is now 15 at most that is all. It is a steady state solution; remember that, it is a steady state solution. There is no unsteady part that has been discussed here. Now, we will go to the next question.

(Refer Slide Time: 19:47)



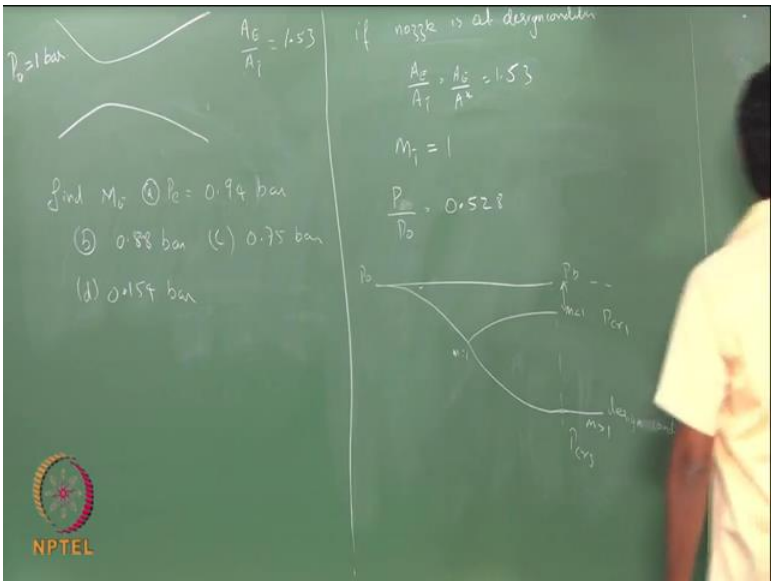
So, the next question is - let us go back to the first question where my P 0 is 10 at most. Now I change my A throat to 5 millimeter square, again now it is in the designed condition find m dot V star, P exit, M exit and sketch the pressure variation. So, if the nozzle is now different, the pressure is same this stagnation pressure is same, stagnation temperature is the same as before 10 atmosphere, and 300 Kelvin except that the throat area is now reduced to 5 meter square.

Now, since it is at the designed condition all this inferences that we made earlier is true flow is isentropic, M exit is greater than 1, flow is choked, so m dot at the throat is the maximum m dot that you can get through the nozzle. A E by A throat is same as my A E by A star, but now the A star value is different, from 10 it is now reduced to 5. So, this is 30 by 5. So, the value is 6. So, accordingly the mach number at the exit is now different if you look at the gas tables A E by A star value of around 6 is 0.095 and around 3.35. So, actually you are doing the (Refer Time: 22:20) between 3.35 and 3.40.

But for the explanation purpose let us take 3.35 and since this is at the same condition we would take the supersonic value which is 3.35. So, your area ratio is now different M exit is different. So, the moment you have changed the area ratio, you have change the design mach number at the exit accordingly everything else will change. So, even though your P

0 and T 0 is same, now the exit mach number is different, so everything has changed. And you can continue doing whatever we have done for the earlier problem likewise.

(Refer Slide Time: 23:50)



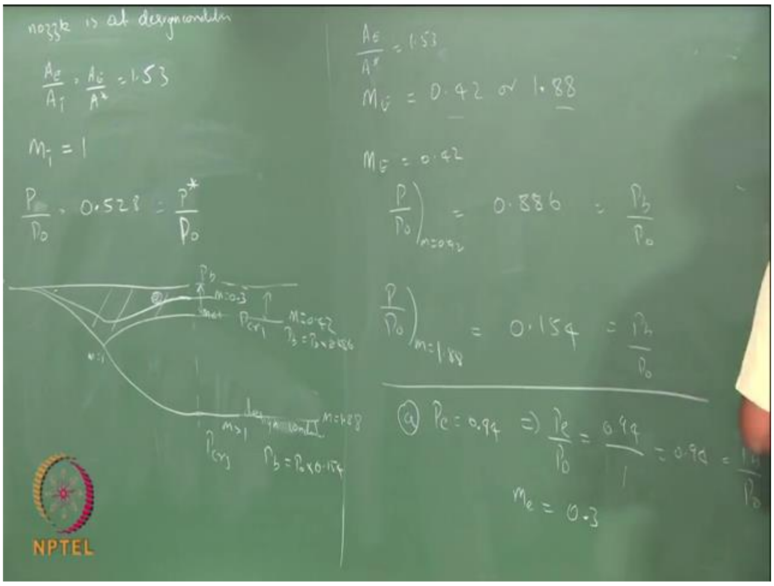
Now, we will move on to the next question. Nozzle A exit by A throat is given as 1.53 P 0 is 1 bar, find M exit for P exit to be 0.94 bar, 0.88 bar, c - 0.75 bar and d - 0.154 bar.

So, I have the nozzle with this particular area ratio, I have a reservoir pressure of 1 bar now if the P exit is changed to these values what is my mach number. So, if it is a design condition we do not know whether it is a design condition or not, but let us take if the nozzle is at the same condition A E by A throat is your A E by A star which is 1.53, which means your mach number at the throat is 1. So, if A E by A star is 1.353 and this is my; if I did nozzle is at designed condition the throat mach number is 1, if that is the case then my P by P 0 associated with this particular mach number is - P by P 0 associated with mach number 1 for gamma equals 1.4 is 0.528.

So, if the pressure ratio is such that this particular pressure ratio is reached you would see a design condition. So, if I plot the pressure values P 01 atmosphere goes to some value and then it takes to a supersonic which is your designed condition, but this can also take other isentropic solution which is my m less than 1, this is m greater than 1, here m equals 1 and if I change my Pb in between. If the Pb is such that my P exit equals Pb is somewhere between these 2 we have also called this is as P critical 3 and this is P critical 1. So, if the pressure is between these 2 values I am going to get a subsonic mach numbers, isentropic solution of subsonic mach numbers

The question here is the P exit is progressively decreasing. So, I have case 1 here may be here, may be here, may be here. So, I need to find what exactly that is and accordingly find my mach number. So, that is all I need to do here. But identifying this curve is the important part in solving this problem. All I need to check is if the pressure ratio is above or below or this condition. So, this would be my so called P star by P 0. So, my A e by this is 1.53.

(Refer Slide Time: 29:02)

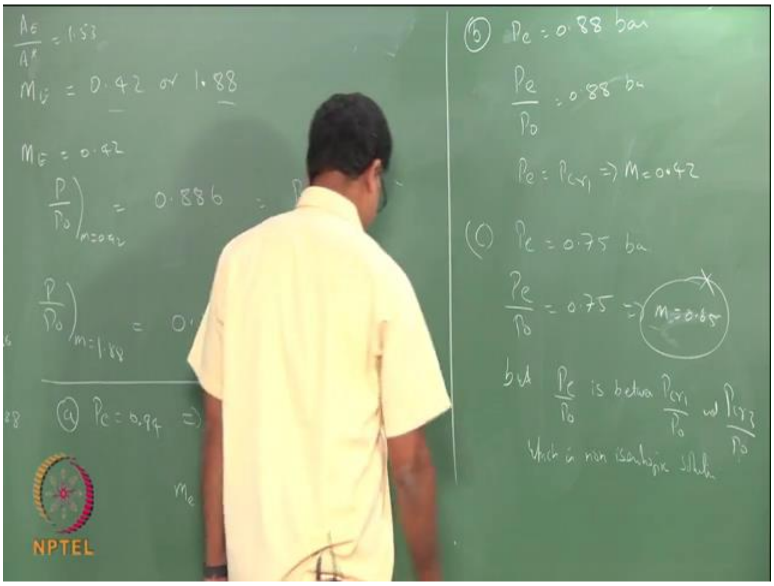


So, the M exit will have 2 values, M exit will have 2 values subsonic value and supersonic value. So, A E by A star 1.53, 1.53 gamma equals 1.4, 1.53 is m equals 0.42 or 1.53. So, I look at this table m equals 1.53, m equals 1.88 which means this is m equals 0.42, this is a isentropic solutions, this is my 1.88. So, this area ratio decides these 2 mach numbers. So, that is decided by this.

Now M equals 0.42, my P by P 0 at m equals 0.42 is I look at the tables' m equals 0.42, I would get P by P 0 as 0.886 or m - P by P 0 at m equals 1.88, m equals 1.88; P by P 0 is 0.154, what does this mean? So, get if my Pb which is also my Pb by P 0, it is also my Pb by P 0. If my Pb by P 0 is this pressure I am going to get this mach number which is 1.88. So, if my Pb is equal to P 0 in to 0.154 I am going to get this mach number or if my Pb is P 0 into 0.886 I am going to get the subsonic mach number both are isentropic solutions. So, any pressures in between will give me a subsonic between this value and decrypt the critical pressure 1 will give me subsonic solution. So, all I need to check is whether these pressures are above this pressure ratio or below this particular pressure ratio.

Let us check the first value - A is P exit equals 0.94, my Pe by P 0 is 0.94 divided by 1 which is 0.94 which is greater than this, which means it is going to lie here in this regime, some curve here is going to be your k is A. And the mach number associated with this P exit by Pe 0.94. So, I take the tables look for 0.94, 0.94 is 0.3. So, my exit mach number for this particular back pressure which is also my Pb by P 0. So, in this region my Pb is also equals my P exit which is 0.94 for that particular pressure ratio I would get my exit mach number to be 0.3.

(Refer Slide Time: 34:22)



Now I take the next pressure, b - P exit is 0.88 bar or my P exit by P 0 is 0.88 bar which is close to our isentropic solution of the subsonic case which is our P is P critical 1, will give me exit mach number or 0.42 which we have already found. So, if my exit mach number to get an exit mach number of 0.42, which is an isentropic solution of the nozzle, which we have given the pressure ratio; should be 0.88 which we have already seen.

Now, take the third case which is 0.75 bar again do the same process 0.75. So, I look at the tables 0.75 is m equals 0.65. Now the problem here is this pressure ratio is between these 2 values, hence we know that this is a non isentropic solution, we cannot use this relation to get the mach number because; this is the isentropic solution, but Pe by P 0 is between per 1 by P 0 and P critical 3 by P 0 which is non isentropic solution.

How can you prove this? You can from for this particular pressure ratio for this particular mach number you look for your area ratio, show that your throat area is larger than your star value A star. So, A star is larger than your throat area. So, hence he this cannot be possible solution. This pressure ratio, easier thing is you we have just demonstrated that particular pressure ratio is between P c r 1 and P c r 3 which is a non isentropic solution. Hence your pressure ratio will not give you this mach number, this mach number evaluated from your isentropic solution is wrong. So, this cannot happen. You do not know what is the mach number there you will have to do, after studying shocks we will have to evaluate what is the mach number. We will do that particular case later.

Now, let us look at the fourth case. Fourth case is the pressure is 0.154 which is what we have seen to be our isentropic solution here. So, your mach number at the exit is going to be 1.88. So, that is case that is drawn here. So, this is my a, this is my b and there is some other solution here which is my c and this is my d. So, all this points a b c and d - shown in this graph a b c and d.