Fundamentals of Gas Dynamics
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Week – 07
Lecture – 25
More on C-D Nozzles

We will continue discussing the Convergent-Divergent Nozzle.

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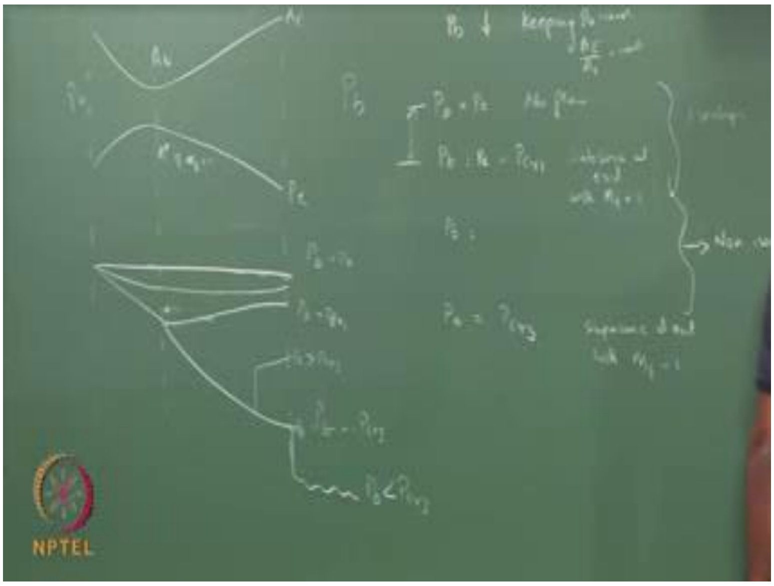

What we had see in the other day is, we have a convergent-divergent nozzle the area ratio A exit by A throat will decide everything that is happening in the divergent section once the flow is chopped. This would also be your A star if Mach number at throat equals 1.

And what we had seen the other day is if my back pressure that is exits P 0, if my back pressure is same as my stagnation pressure there is no flow, so I would have a straight line here back pressure is same as my P 0, if I reduce my back pressure a little bit there would be a some change in the flow pattern, so there will be a subsonic flow. If you reduce it still further at some value of back pressure you would get your throat value; Mach number at throat will be equal to 1. And this we know has two solutions again depending on your back pressure.

So what I have done here is, I have reduced my back pressure keeping P 0 constant and my A e by A throat also constant, so we are doing this experiment on a given converging-diverging nozzle. When this is equal to 0 no flow, when P b is reduced to some value where my throat Mach number is 1, so if I reduce P b to some value such that your P b equals P exit which is a I call it as P critical 1. Once you I reach P critical 1 I will have a subsonic flow at exit with Mach number at throat equals 1. In between these two values I have infinite cases which are all isentropic solutions; between this I have infinite isentropic solution. Meaning, I can have any value between P b and P critical 1 and all those solution would be isentropic solution, but the Mach number at throat will not be equal to 1.

Now if I reduce this further I will have some value, I do not know what this situation is I will try to explain that, but I continue decreasing my P b it would reach a condition P critical 3 which would give me a supersonic flow or supersonic Mach number at the exit with obviously my Mach at the throat is 1. If I keep reducing by back pressure till it reaches P critical I have again infinite number of solution, but they are all non isentropic. So any solution between any pressure that pressure between P critical 1 and P critical 3 are non isentropic, so I use critical 3. So, there is somewhere in between there is P critical 2 which I will explain it in a while.

Now if I reduce my back pressure further down. I would draw that here, so I would reduce P critical, so my back pressure here is less than P critical 3. I have a situation here my P exit is greater than P back pressure. So, every condition in between P critical 1 and my P critical 3 I have here back pressure greater than P critical 3. This scenario may be something which we will deal it a later. I can have a shock inside the diverging section which would give me subsonic flow at the exit, so all these values would have subsonic flow at the exit.

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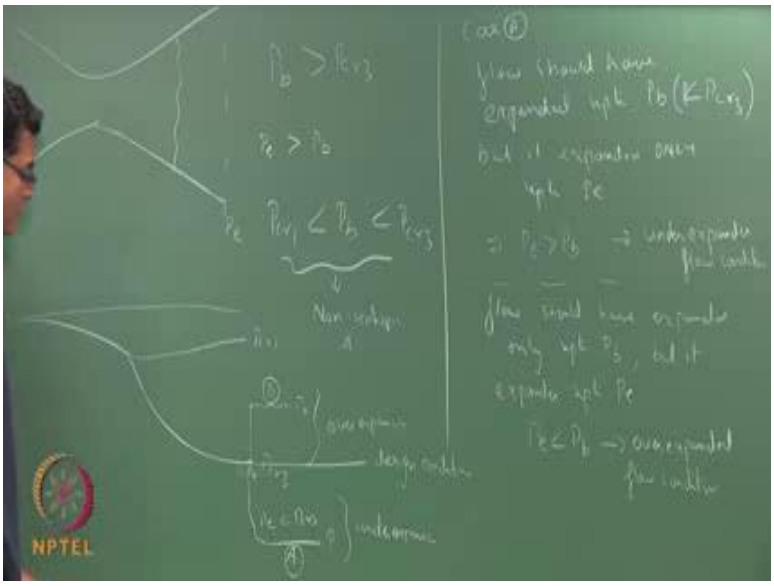

So, what happens here is for some pressure P b which is a now greater than P critical 3, so your P exit is now greater than your P b. I would have a shock here. So, my P exit is here. This shock which we will explain what is a shock in a later class, so it just means that this is a non isentropic process. So any value between P b line between P critical 1 and P critical 3 will produce a non isentropic process which for the time being it is a simply a shock. We will explain what is a shock and how do you find the properties across the shock in a later class.

In such cases when you are P exits. So, if I look at here for the scenario pressure that goes and then you have the P critical 1 then you have P critical 3. Now here the back pressure is somewhere between these two values. So, your back pressure is here, so I my P b is somewhere here. My flow should have expanded up to this, but it expanded more than that. If I write it in words for example, here your P exit has less than P critical 3 let us write about this case. Let us take case a, the flow should have expanded up to P b which is now much less than P critical 3. The flow should have expanded up to your P b, but it expanded only up to P exit. Meaning, your P exit is greater than your P b.

So, your flow should have expanded up to this particular pressure, but it expanded only up to this such cases called your under expanded nozzle or flow condition. So the flow should have expanded up to P b, but it expanded only up to P e which is your P exit that is your under expansion. Likewise, for my case b which is a your P exit the above this, so the flow should have expanded only up to P b. So, the flow should have expanded only up to this point, but now it has expanded up to P e. So, in this regime you have your over expanded flow and this is your under expanded flow.

Remember the way we call it this is under expanded because the flow should have expanded up to P b, but it expanded only up to P e. The other way the flow should have expanded up to P b, but now it has expanded to P e. So this is over expansion and this is under expansion this nomenclature has always created problem in the students identifying a, which case is what, but remember these statements. So, if you remember these statements you would it would be very clear. So, if the back pressure is much lower than this then your flow is under expanded if it is slightly higher than it is this.

So, if you consider a rocket nozzle before that your flow this is under expansion and this is over expansion the region where P e is exactly equal to P b is our design condition. So, this particular nozzle with this particular area ratio is designed to work at this particular back pressure, any back pressure other than this is either an under expansion or over expansion. The purpose of CD nozzle is to have supersonic flow, so if you want a supersonic flow the throat Mach number should be 1 the flow should be chocked. Once you are reached that it depends on the back pressure what is the Mach number at the exit. If the back pressure is anything other than your P exit it is either under expansion or over expansion.

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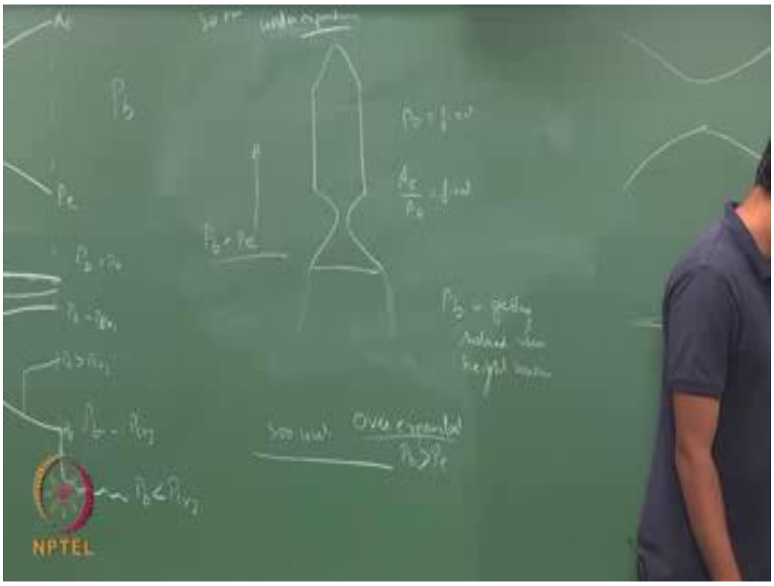

So, if I have a rocket nozzle. I have designed this nozzle which means my A exit by A throat is fixed I am not changing the area ratio and I am also assuming my P 0 is fixed or constant, but this is moving up the atmosphere which means your P b is getting reduced when height increases. This is precisely what we have done here. We have the constant P 0 and constant area ratio and as it moves up in the atmosphere your P b get reduced. Now assuming that the choke conditions happens at the throat the Mach number is 1 at the throat, I have solution here now your P b will decide whether the flow is under expansion or over expansion.

So, this nozzle will work for 1 back pressure in his trajectory. The nozzle is going from sea level to some height say let us take a just 30 kilometer will worry about just this height. So, somewhere in this height your P b is equal to P exit that is the only design condition that is available for this nozzle. All other condition it would be either under expansion or over expansion. So, near the sea level back pressure is quite large, so it is over expanded here. Your back pressure is much larger than your P exit that is what it is here; your back pressure is much larger than your P exit.

As it moves up in the air it goes from over expansion to your under expansion, everything depends on your P b so at some particular height you will have the design condition. We will talk about what will happen to the flow of structure everything after discussing the shock and supersonic jets. For the time being let us just stick to this. This is the difference when you are changing your P b when I keep my P 0 and area ratio constant.