Fundamentals of Gas Dynamics
Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week - 07
Lecture – 24
Converging-Diverging (C-D) Nozzles

So, we will start discussing something called converging-diverging nozzles. In the converging nozzle, we have seen that it is impossible to get a supersonic flow at the exit because there is no diverging part.

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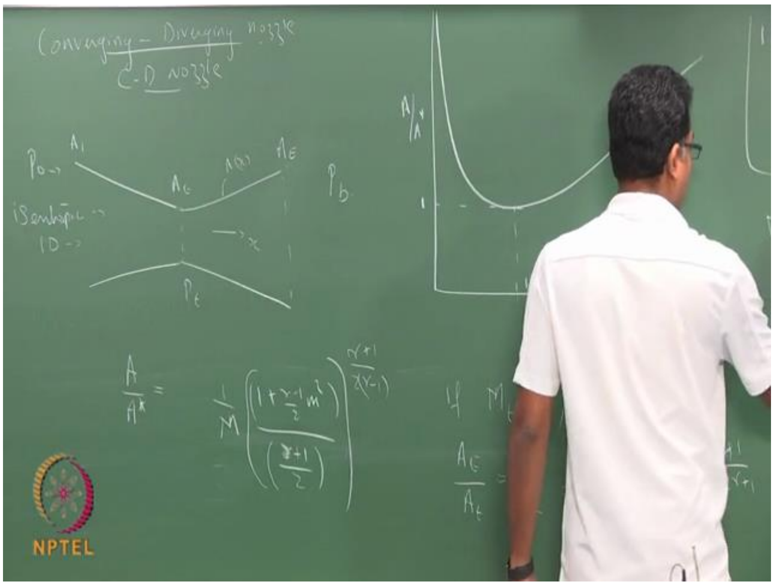

Now, what we will do is we will add a diverging part to a converging nozzle and see what happens. So, this is nozzle which is typically used in high speed flows where you need supersonic velocities. For example in rocket nozzle or in a supersonic aircraft or typically called as a C-D nozzle.

So, what we have is a converging section and then we have a diverging section. So, this is typically called as your throat, this is A 1 and this is your exit. So, this need not be straight line, it can be any form, so it is just that this is a function of x, where this is my x, so it can be of any shape. As long as it is 1-D - quasi 1-D our analysis would be true with isentropic assumption. So, again we assume no losses, no q, no shaft work, so it is isentropic flow and quasi 1-D. Within these two assumptions, we are going to analyze the flow as we have done before. So, there is a P 0 that is coming in there is a P b that is coming in the P b backpressure that is existing outside the exit of the nozzle.

So, what we have seen is depending on some conditions where the P here and P 0 decides the choking condition or the mach number at the exit or the converging nozzle which is here the throat area. So, once you have that particular condition decided, so if the mach number here at the throat is 1, then you are going to have a supersonic flow provided you have enough pressure ratio that is what we are going to see on what conditions you are going to have a pressure ratio here or whether you are going to have a supersonic flow here or a subsonic in the diverging section.

So, M equals 1 means the mach number equals 1 here after this the flow can decelerate or accelerate depending on the pressure ratio. So, we will see what it is. So, if I write the equations of A by A star which I will write it down here 1 by M 1 plus gamma minus 1 by 2 M square divided by 2 plus gamma plus 1 by 2 whole to the power gamma plus 1 by 2 gamma minus 1.

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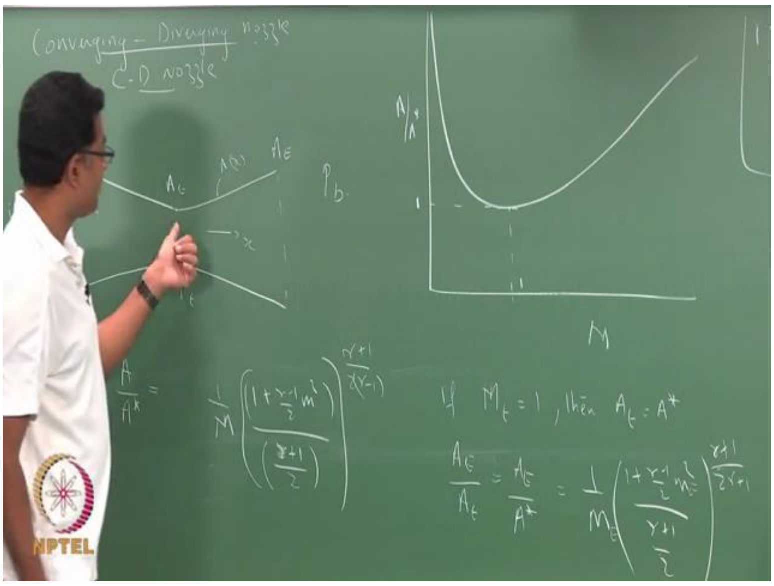

And this plot also has a curve of this form A by A star. So, this is the A by A star which depends only on your mach number. Now A star is a condition when the mach number is 1. Now we are suppose we have Mach number at the throat as 1, if M at throat is 1, then your A throat can be considered as your A star. Or I can replace this equation by A at the exit divided by A at the throat equals since we have already assumed isentropic flow my A star is going to be constant.

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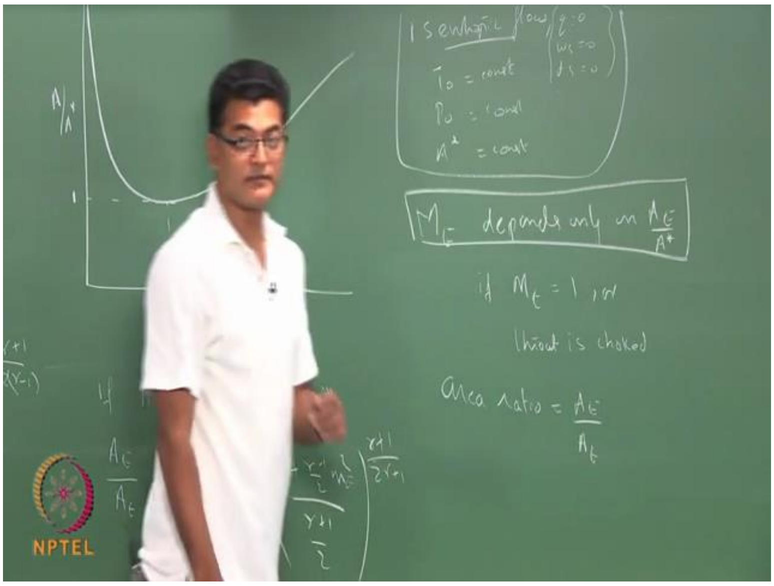

So, the moment I assume isentropic flow, I write three things, what are they, T 0 equals constant, P 0 equals constant, and A star equals constant, under the assumption of isentropic flow with no heat, no shaft work, no entropic change. So, under these assumptions, these three are always true which we will assume it here. So, your A star when mach number at the throat is 1 then we can call A throat as my A star which means I can write A E by A throat as A E by A star which now only depends on your mach number.

Or we put it the other way round gamma plus 1 by 2 gamma plus 1. Or to put the other way round the mach number at the exit is going to depend only on this ratio. Your mach number at the exit depends only on this ratio A E by A throat. A star, if your mach number at the throat is 1 or if the throat is choked. If the throat is choked I can make this statement, M exit depends only on my area ratio, which is A exit by A star. So, when I say area ratio for a C-D nozzle, it always mean A at the exit divided by A throat. For a given area ratio if the flow is choked my exit mach number depends only on my A E by.

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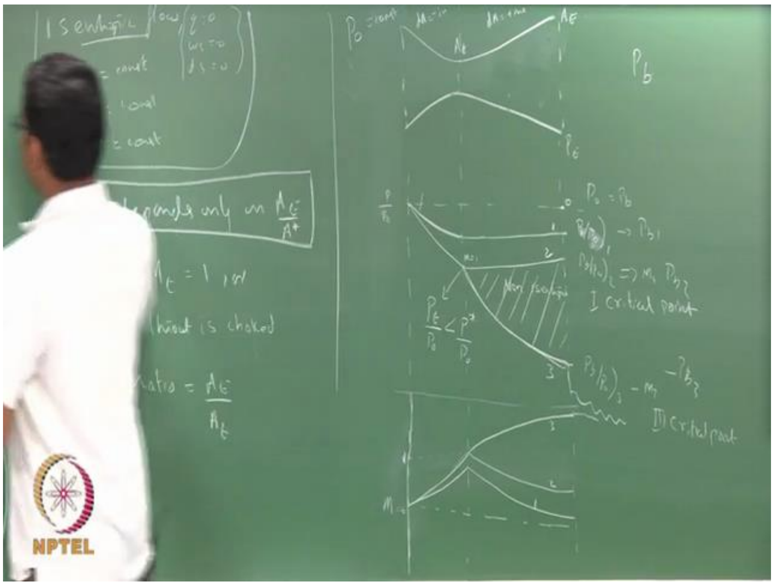

And the once you have this exit Mach number my P exit by P 0 T exit by T 0 everything now depends on your M E, which is depend on area ratio. So, area ratio decides all these factors if the flow is choked. Now we are going to see what is this condition. Like what we are done before we will try to see when you change your P b what happens for a given area ratio. So, I have a in A throat, and there is a area exit there is a P 0 which is constant, there is a P exit, there is a P b, P b is my backpressure. Like, what we have done in our converging nozzle, what we will do here is we will keep this constant for a given nozzle; I will decrease my P b backpressure.

So, I will draw a line. Again we are going to consider only flow inside outside what happens is not being consider at present which we will discuss it in the following lectures after discussing normal shocks and oblique shocks. So, when P 0 equals my P b, there is no flow. So, I have a fabricated nozzle in my hand, there is no flow, there is no pressure difference. Let us take P by P 0.

Now I decrease my P b to some value; such that the flow at the throat is not choked. So, I would have a subsonic flow in the converging nozzle, the flow as not reached M equals 1. And now the subsonic flow faces a diffuser, so it again decreases my velocity. So, here this region dA is negative, and in this region your dA is positive. Now I have a subsonic flow, so dA is negative so my velocity increases, once it reaches your throat condition it faces a diverging section so velocity decreases. So, this would go to some value like this. So, this is some P b which is does not create the choked condition.

So if I decrease my P b again, I may get M equals 1 at the throat. Now once I get M equals 1 at the throat I have this condition, it can either decelerate or accelerate. So, let say for a given A by A star, now since it is already choked, I can call this as my A star and this as my A. So, for given A E by A throat, I have two values of Mach number. So, this is my subsonic mach number, this is my supersonic mach number. So, one of these values would happen here. So, either the flow will decelerate or the flow will accelerate. So, this is my P by P 0 is kept constant. So P 0 by P b by P 0 at say so this is line 1 this is 1 P b by P 0 at 2 this is P b by P 0 at 3 So, this is choked.

So, this value here P throat by P 0 is less than your P star by P 0 so that the converging nozzle is now choked. So, this flow is choked. So, we say the throat is choked; once the throat is choked I have these 2 conditions for the area ratio mach number 1 is subsonic, mach number 2 is supersonic, so I have two solutions something that happens here either subsonic or supersonic. Now whether it is subsonic or supersonic, let us take M 2, depends entirely on your pressure ratio, and this is the isentropic solution. So, this is isentropic curve and what I am describing here is the isentropic solution.

So, in between what happens is - the non isentropic solution, which we will discuss it later. For the time being, once the flow as choked it can have two solutions subsonic and supersonic as I have described here in this particular plot where A by A star gives you two solutions that precisely that thing happens here. I can have two solutions either the flow will accelerate or the flow will decelerate depends on your backpressure. This mach number again M 1 or M 2 entirely depend on the area ratio which you had just now seen, but which solution will be picked decided by your pressure ratio.

Now, we will go further and draw the Mach number variation. I will draw it a bit, so for the case or let us put line 1, line 2 and line 3. So, remember here between 0 and 2, you have infinite number of isentropic solution, but once the flow is just choked, it has only two isentropic solutions one is this, one is the subsonic case. In between, you have non isentropic solution which is typically the shocks and other things which again as I told you, we will discuss it later.

Now we will try to see how the mach number variation is so let us a start from a mach number equal 0 line. Let me draw it slightly lower M equals 0.

So, let us draw for the case 1. So, the flow is accelerating, but it has not reached mach number 1 at the throat. So, let the throat condition it is here, so the flow is accelerating and it does not reached 1. And then the flow sees a diverging section, so it decelerates, so the mach number reduces. And depending on your pressure ratio, you would have your exit mach number, because the flow is not now choked, so your area ratio will not decide your exit mach number only your pressure ratio will decide your mach number, so this mach number is decided by your pressure ratio P b by P 0.

Now, you have a condition where the throat is M equals 1. So, you have some value here which is 1. So, this is your axis. So, the flow because of that some pressure ratio your flow increases to mach number 1 at the throat, now the flow is choked. Once it is choked, it has two solutions; one is supersonic, one is subsonic decided by your area ratio. So, it goes here or it goes to some value here. So, this is my curve one, this is my curve two, this is my curve three. These two values once the flow is choked at the throat these two values M 3 and M 2 will be decided by your area ratio and which one is picked is decided by your pressure ratio.

So, for any pressure ratio between these two, you have a non isentropic solution, or any pressure ration after this is you also going to have a non isentropic solution outside the nozzle, so that is my next curve - curve four. So, I have reduced my P b further than this. So, here what happens at mach number 1, the throat condition is still mach number 1, so it can go to subsonic flow if their pressure ratio is this. But now we have reduced it much further, so the pressure ratio would be this line plus, so it would take the same line as this and then do something at the exit to reach the or reduce mach number pressure reduce P b. So, like that it will go through the same curve here up to this point, and then you do something outside the nozzle to do get a different curve.

So, I repeat what the process we have done is we have a nozzle with some area ratio and P 0 is kept constant I am decreasing my P b, so when I decrease my P b, I start getting a flow. So, as I slowly increase, decrease my P b, there is some point at which you will get M equals 1 at the throat, the exit mach number it is again subsonic flow, because it does not reached pressure ratio where you can get a supersonic flow. So, you have a point at P 2.

Now, if you increase it further down, you would get supersonic flow, which is an isentropic solution; in between these two isentropic solutions, you have infinite number of non isentropic solution. So, any pressure P b between say for example, P b 2 and P b 3, you are going to have a non isentropic solution. So, this is for P b 1, this is P b 2, this is P b 3; pressures between P b 2 and P b 3 is non isentropic solution inside the nozzle. After P b 3, if I reduce P b backpressure smaller than P b 3 I am going to have no change inside the nozzle, but outside there will be some changes which we will discuss it after discussing shocks.

So, the exit mach number is going to be what you have decided by your area ratio. So, this P b 2, we call it as first critical pressure or first critical point; and 3, we call it as the third critical point in between comes your second critical point which again we will differ it to discuss sometime after discussing normal shocks.

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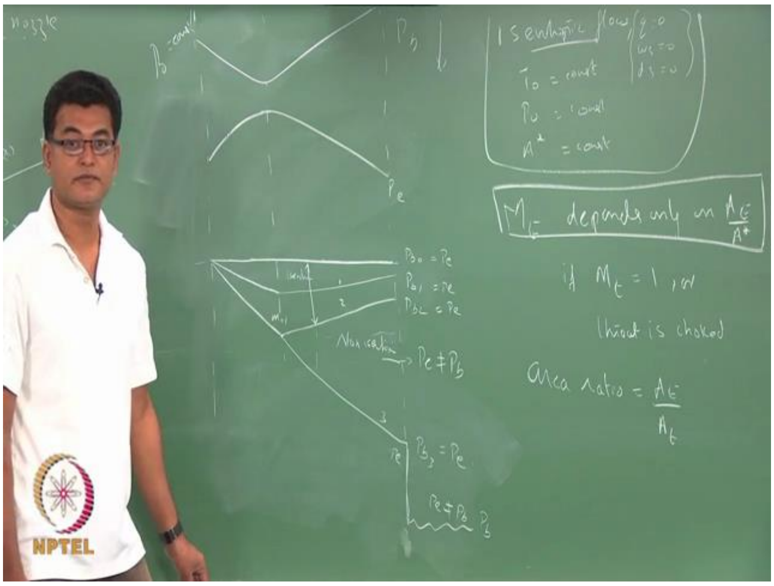

So, I repeat, I will draw a new curve I will repeat the whole process again. So, I am going to repeat whatever we had done now. P 0 is kept constant, P b is decreasing, I have a P exit, when P 0 is same as P b no change there is no flow. Now, I decrease P b to some value which would give me a subsonic flow at the exit.

So, this is P b 0, this is P b 1. P b 1 gives me a subsonic value at the throat if I decrease further my P b, I would reach M equals 1 at the throat, but P b is not enough reduce to get a supersonic flow at the exit. So, this value is isentropic flow. So, this increases the velocity and then suddenly it decreases because your flow is facing in diffuser side.

Now, if I decrease P b further then P b, now this is 1, this is 2. If I decrease my P b further, I will have other solutions here, which are non isentropic, but once you reach P b 3, so I am decreasing. So, I decrease P b to some small value so I have a solution here which is non isentropic then I have another solution here, so there are infinite number of non isentropic solutions here between P b 2 and P b 3. But once the pressure is reducing to P b 3, I would have a supersonic solution which is isentropic. So, your P this is your P exit this would be your P exit, this will also be your P exit, and this will also be your P exit; in between your P exit is not equal to P b in this region, where you have non-isentropic flow which you will discuss it later.

Now, if I reduce my P b further there is nothing that would change inside the nozzle, but outside the flow will do something to reduce your P exit to P b. Here again your P b is not equal to P exit. These lines are the isentropic solutions in between here 1, 0 and 2 you have infinite isentropic solutions, this region you have infinite non-isentropic solution and this is the line that demarcates between the isentropic and accordingly your mach number also changes. Now, we will do it with the small example, the same thing what we are discussed now we will do it with the small value, so that this would become clear.

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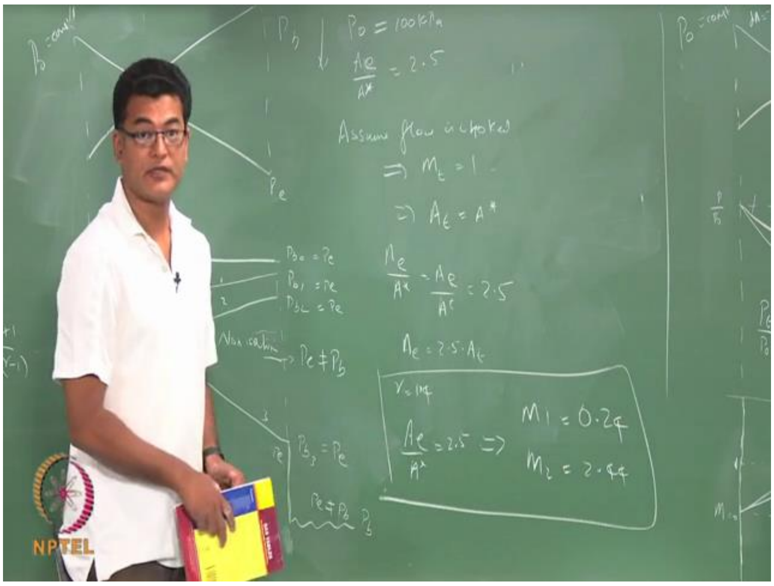

So, I have a P 0 some P 0 of so let us take 100 kilo Pascal. And I have decided my A exit by area to be 2.5. Assume flow is choked meaning M at the throat is 1, Mach number at the throat is 1, my A throat equals A star, so my area exit by it can write it as A E by A throat equals 2.5, which means my A area is 2.5 times of your A throat. So, your area A exit is 2.5 times of what you have at the throat.

Again I assume isentropic flow and other things your A star is going to be same as here throughout the flow. Now, we have said once the flow is choked, this will decide your exit mach number. So, you look at the table for gamma equals 1.4, look at the isentropic table A e by A star equals 2.5 gives me two mach numbers - M 1, M 2. So, M 1 is going to be 2.5, M 2 is going to be 2.44. M 1 is going to be, so I have two isentropic solutions for this particular area ratio if the flow is choked. Now, which one will this flow take?

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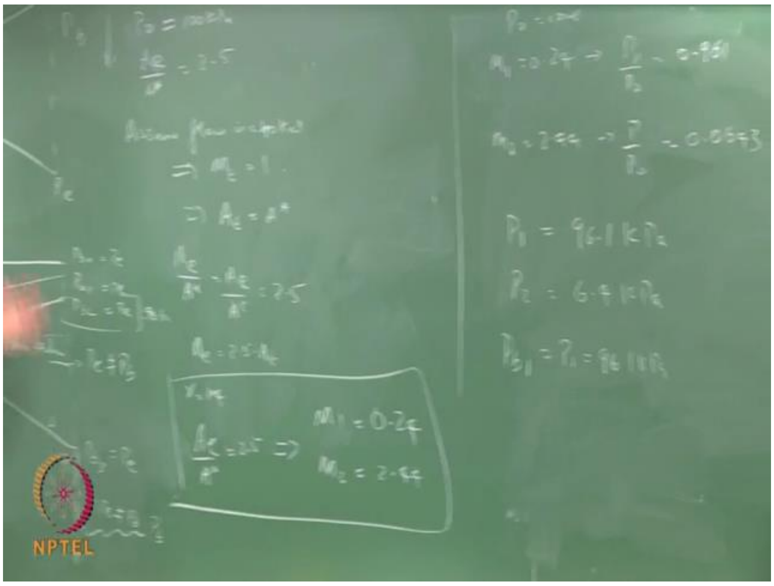

So let us look at the pressure ratio since it is isentropic flow, we know P 0 is constant. So, M 1 - 0.24 gives me a P by P 0 of 0.961, 2.44 give me a P by P 0 of 1.4. So, I will make sure this is for gamma 1.4, so this is for gamma 1.4. So, in the problem, what we have defined we have kept our P 0 to be 100 kilo Pascal, and we have an area ratio, and we are assuming flow to be choked.

Now, these two solutions give me P 1 to be 96.1 kilo Pascal, and P 2 to be 6.4 kilo Pascal, so this is P 1. So, if my P b 1 96.1 kilo Pascal, which means I have a P 0 here P b here if P 0 is 100 kilo Pascal, P b 1 is a 96 kilo Pascal, I am going to get a subsonic flow which is this curve, no this curve which is your isentropic flow this curve. So, P b equals 96 kilo Pascal.

If it is reduced further to 6.4 kilo Pascal, I am going to get a supersonic flow. So, once the flow is choked area ration decides everything that is the main point from this discussion. So, once the flow is choked, area ratio decides everything and the between these two solution, the pressure ratio decides itself to get this. And for any other pressure ratio, area ratio any other pressure ratio less than P b 2, so greater than P b 2 that is inside in the subsonic side, you can have a any number of solution depending on your area ratio and the mass flow rate.

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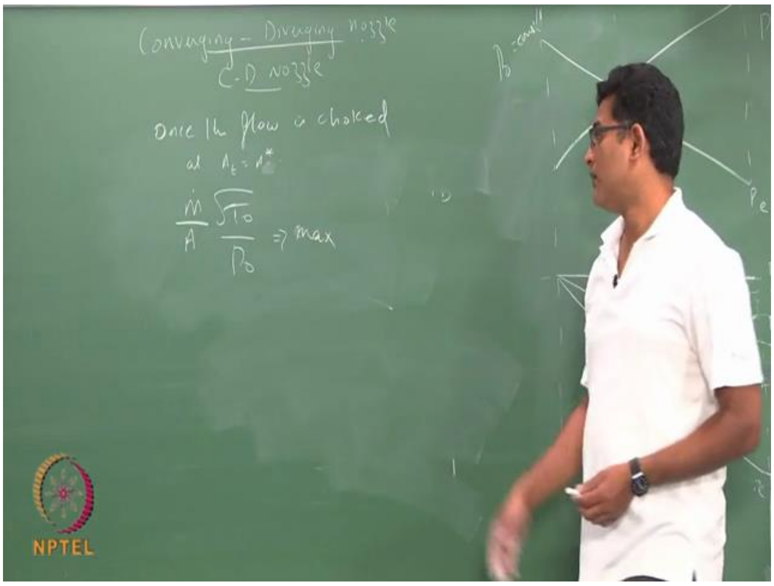

So, once the flow is choked, so again choked so at the throat A T at equals A star so for a fixed A E by A T. This quantity is also the maximum that can it can go through. So, the mass flow rate that is going through the nozzle once it is choked is the maxima it can take the maximum flow rate then comes from this particular condition which depends only on P 0 and T 0.

So, unless you change T 0 and P 0 your mass flow rate is not going to change so whatever you do your throat condition is same your M at throat is 1. So, your A E by A star is going to decide what is the mach number at the exit which would be one of these two curves. And given those conditions, you can find out the other quantities and your M star is the mass flow rate is again decided by your stagnation quantities which are not decided by your area ratio. So, this is the maximum mass flow rate you can go through.

So, we will do numerical problems related to whatever we are discussed now, maybe in a next class or class after that. So, what we have seen today is a C-D nozzle, the importance of area ratio which is the area at the exit divided by area at the throat thus decides the entire flow rate regimes, isentropic flow rate regimes in your C-D nozzle. And we have also seen that the two solutions isentropic solutions are possible once the flow is choked inside the nozzle. Again outside the nozzle we are not discussed anything we will discuss do that few classes later. So, the importance is the area ratio, C-D nozzle area ratio decides everything.

Thank you.