Fundamentals of Gas Dynamics

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Week – 06
Lecture – 21
Condition of Choked flow and associated properties

Continuing with the converging nozzle we are going to discuss choking and the relation with m star and m star with m and other things.

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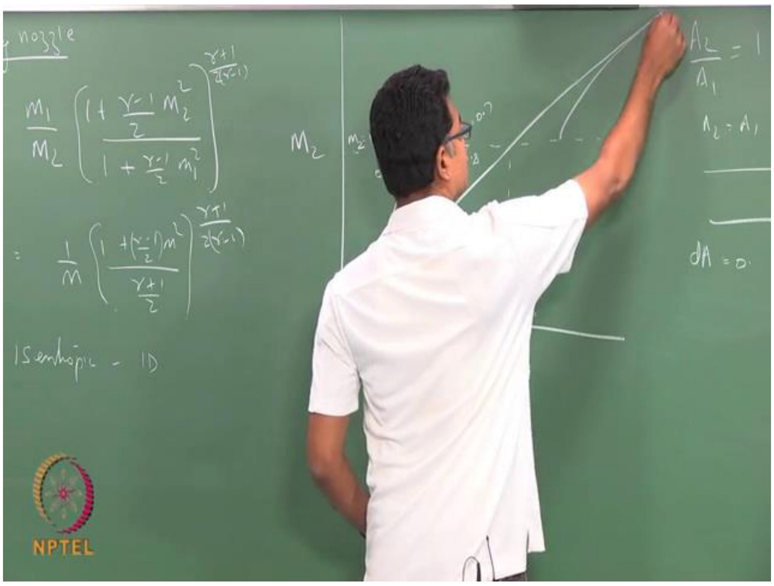

Before we start I will write down the equations for something which we have derived (Refer Time: 00:36). So, the continuation of saw what are we are discussed in the previous class. This is something which we are derived few classes earlier. So, I will start from this equation today and A by a star to be 1 by m into 1 plus gamma minus 1 by 2 m square divided by gamma plus 1 by 2 the whole rise to the same exponent. So, we will start form these equations try to see what we can get in relation to converging nozzle. Let us take the first equation.

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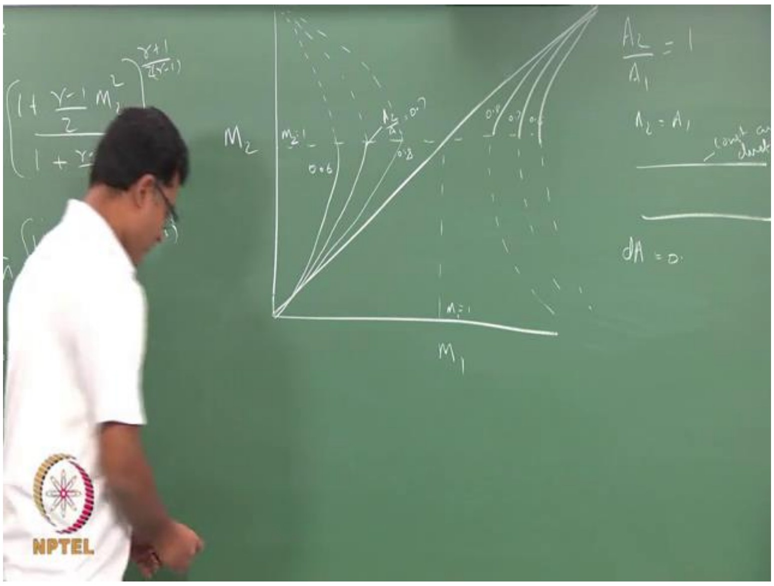

So, if I draw my variation m 1 and m 2, if my A 2 by A 1 is equal to 1 my A 2 equals A 1 which means I am talking about a constant area duct. Remember this equation is isentropic and cos 1D. So, with that assumption if I substitute this my m 1 is always equal to m 1 because there is no area change d A is 1, so whatever m 1 I have it same as my m 2. So, I would get 45 degrees line here where A 2 by A 1 is 1. If it is less than 1, so this is m 1 equals 1 and this is m 2 equals 1. For area ratio A 2 by A 1 less than 1 I would get a curve something like this for some value of let us takes 0.7, so this would be my 0.8, this would be my 0.6.

So, depending on the area ratio I would have curve like this. And if my m 1 is supersonic again I would get a curve which would become like this and this equation does not distinguish between supersonic and subsonic flows. So, if I continue this curve this would be something like this and this would be something like this. This again for area ratio A 2 by A 1 say 0.8, 0.7, 0.6, the shape of the curve would be something like this.

Now let me redraw this just for one case for explanation purpose.

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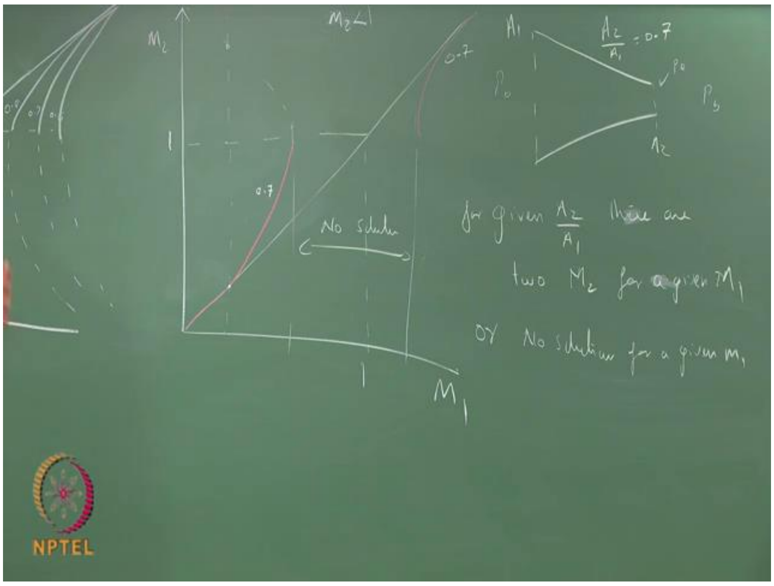

I have A 2 by A 1 and this would be my m equals 1, m 1 m 2 and my A 2 by A 1 0.7. Let us take this as 0.7, for m 2 less than 1. So, I am considering only my subsonic exit case. So what is this figure, this is A 2 this is A 1, A 2 by A 1 is fixed. If you look at this for a given m 1 is less than 1 or m 1 greater than 1, m 1 subsonic or m 1 supersonic I have two solutions here. For a given A 2 I have two solutions A 2 by A 1 there are two m 2 solutions for a given m 1 or no solution for a given m 1. So if I have either two solutions or no solution.

For this m 1 I have two solutions; one is supersonic, one is subsonic or I have for these range of m 1's I have no solutions for this particular case of A 2 by A 1. I did not talk anything about P 0 or P b or P exit all I am talking here is in terms of mach number, if my mach number inlet mach number is between these value for this particular range of A 2 by A 1 I do not have a solution. We will see what exactly that is. Now you are trying to understand what this picture is. So, let us do this small exercise.

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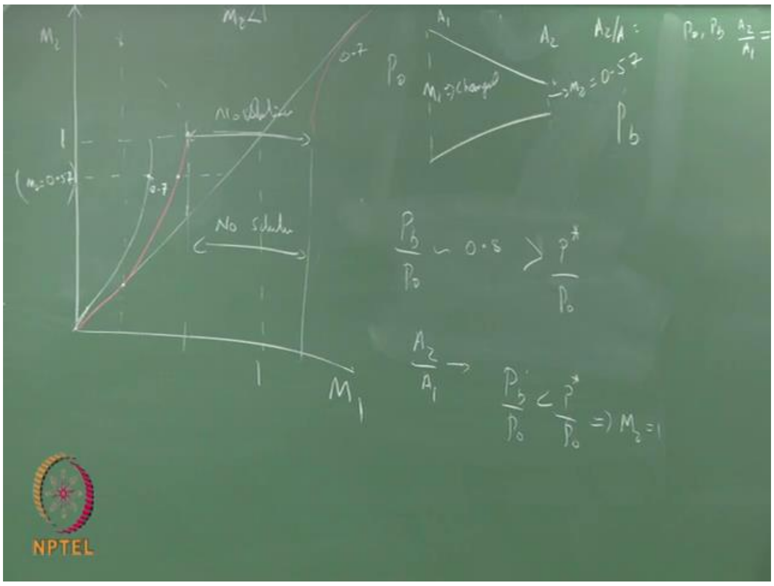

I have a P 0 I have a P b, let us assume P b by P 0 like what we have done in the last class. P b by P 0 is somewhere around 0.8 which is greater than your P star value P star by P 0. We know that there exit mach number is not 1, so this is somewhere been less than 1, and I have A 1 here A 2 here, A 2 by A 1 is some value where I have a solution. Now whatever I do to area or inlet mach number my exit mach number if it is this value if you are already reach your critical pressure the exit mach number is always going to be 1, but now this is a case where it is less than 1.

So, let us if my P by P 0 is 0.8 the mach number here is going to be I look at the tables and what is the mach number f P b by P 0 is 0.57. So unless I change this pressure ratio this exit mach number is not going to change. I have an exit mach number of 0.57 which depends only on your pressure ratio P b by P 0. Now given this situation now I am going to change my A 2, so I decrease my A 2. So, I am decreasing my A 2 it means I am progressively decreasing this ratio. So, I should be in this curve where this is A 2 by A 1 decreasing so would go from this curve to this curve.

Now my m 2 is fixed, so my m 2 decided by pressure ratio is fixed. So, whatever I do should it should be on this line. So, I have changed A 2 to a lower value which means that from this point I have a new value that is here, so from this value now I have a new value. So my A 2 by A 1 is reduced, my m 2 is still the same, so what is happened here is my m 1 has changed. If I keep my stagnation pressure same and the back pressure same my m 2 will not change and in this particular case if I change my A 2 I do something so that my m 1 is changed. So, if I change my area my exit mach number is not going to change but my m 1 change, which means that I am essentially changing my static quantity at the inlet. And that is what this curve tells you.

Whereas, if I keep my m 1 is consent and I am not worried about how P b and P 0 change and I keep decreasing my A 2 I would reach a value here, so I have the flexibility changing these pressure ratios. So the next example is P 2, P b and I am progressively increasing my A 2. So, I have no control on P 0 and P b it can change any value, and let us assume it is going to be some condition where P b by P 0 is less then P start by P 0 which means that the exit mach number should always we 1 implies your m 2 is always 1. So, I have this line, my exit mach number is always 1.

Now I keep m 1 constant I do something which means I can take any value. So, if somehow I force m 1 to be some constant value and I change my A 2 I see that for some value of A 2 I have this case in between I have no solution I see that there is no solution to this isentropically. There is some non isentropic process that is going to happen which will automatically change my m 1. So, this is not a possible way of doing it. So, my m 1 will somehow be changed. There will be pressure waves that are going in this direction which will change my static quantities and change the pressure values.

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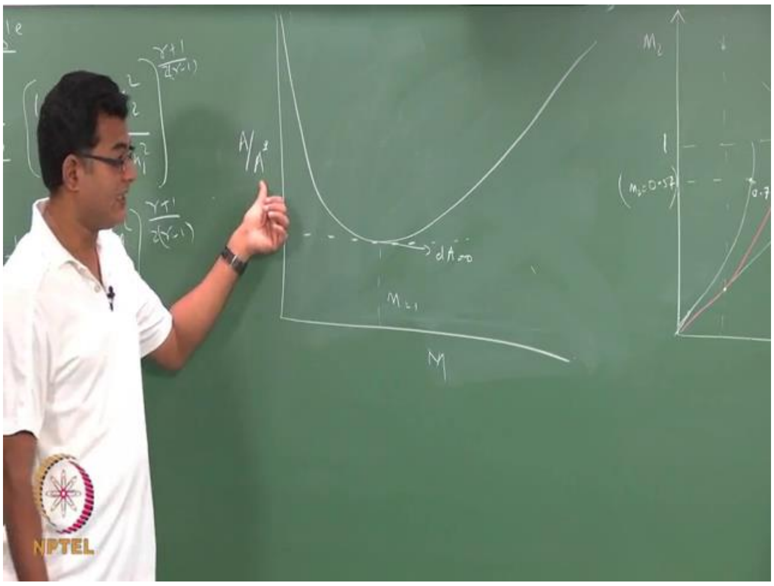

Now, if you look at this particular case where my A by a star, I will rub this plot. I always have some curve like this which gives me 1 here. This is when my d A is 0. At d A 0 I would have m equals 1 and hence I have a star condition here which is precisely what is happening here if my pressure ratio is less than the critical pressure ratio. So, the critical pressure ratio for gamma equals 1.4 is 0.528, if the pressure ratio is less than that I am always going to get m 2 equals 1 whatever you do.

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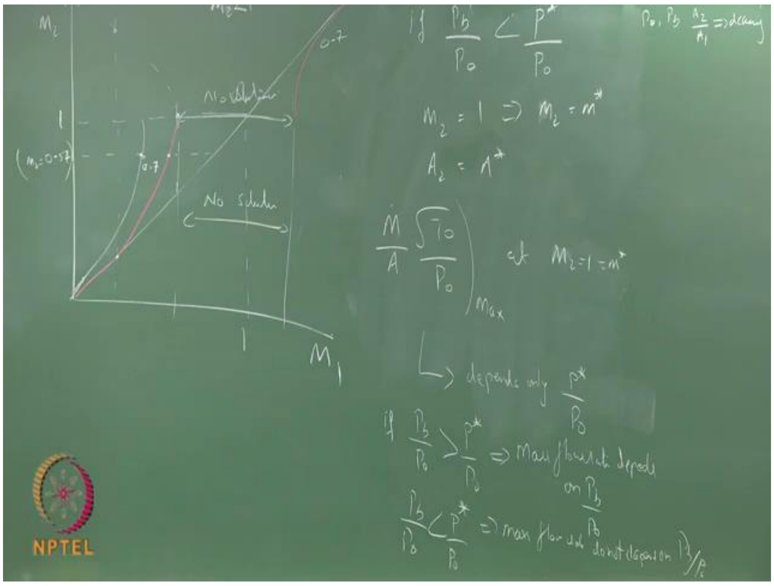

That means, form our previous plot. If P b is always or for any condition P b less than P 0 my m 2 is always 1 implying my m 2 is my m star and my A 2 is my a star because that is the condition in which your choking the flow. So, your m dot by A root of T 0 by P 0 mass flow rate would be maximum at m 2 equals 1 which is equal to your m star. So, this maximum flow rate you have achieving at some conditions which depends only on your pressure ratio. So, depend P star by P 0 which is a constant. So, if your P b by P 0 is greater than P star by P 0 than your mass flow rate depends on P b by P 0, is P b by P 0 is less than P star by P 0 then mass flow rate do not depend on P b by P 0. So, the maximum mass flow rate will depend only on your P star by P b.

So, now we are talked about the choked condition and the existence of m equals 1 at the exit it implying that the area at exist is now the star a star value we will try to find relation between m star and m.

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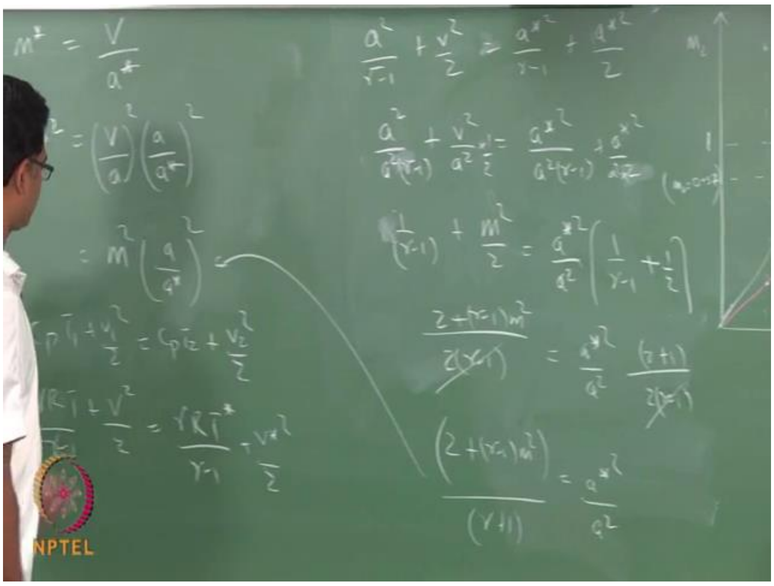

So, my m star is defined as v by a star which I would write it as a by a into a by a star. So, I would square this, so be m square into a by a star square. Now, we will get substitute a by a star from our energy equation which tells me c p T 1 plus v 1 square by 2 equal c p T 2 plus v 2 square by T 2 for an isentropic flow with no heat and now shaft work and I replace c p with any condition T plus and c p I would replace it with gamma R by gamma minus 1. So state 1 is any state, so I will remove the suffix 1 and 2 is my star condition so I would replace 2 with my star values. V star I can replace it as a star by 2.

Now I divide the equation with a star, so my a square by a star square plus v square by a square; ok so I will divided with a square gamma minus 1, this is 1 by gamma minus 1 plus a square is m square. So, this is 2 gamma minus 1 2 plus gamma minus 1 m square a star square by a square in 2 gamma minus 1 gamma plus 1, cancel the denominators. So, this would be 2 plus gamma minus 1 m square by gamma plus 1 equals a star square by a square.

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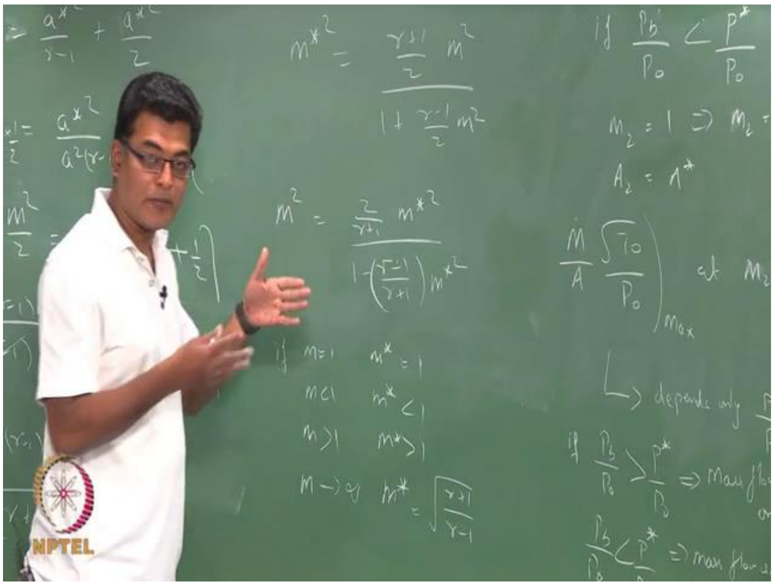

Which you can substitute here in this equation and you will end up with the square is gamma plus 1 by 2 m square divided by 1 plus gamma minus 1 by 2 m square or m square equals to plus gamma plus 1 m star plus 1 minus gamma minus 1 by gamma plus 2 m star square. So, when m equals 1 your m star is also 1, m is less than 1 m star also less than 1, m greater than 1 m is also greater than 1, but in m tense to infinity your m star tense to root of gamma plus 1 by gamma minus 1. Now that is advantage of having m star.

So, you talk everything in terms of m star. Associated with the any m you can have an m star, this is again as we have learned it is a typical reference quantity. So, what is this mean? If I have fluid that m equals some value I do an isentropic process to take it to m equals 1 and the mach number there is my m star.

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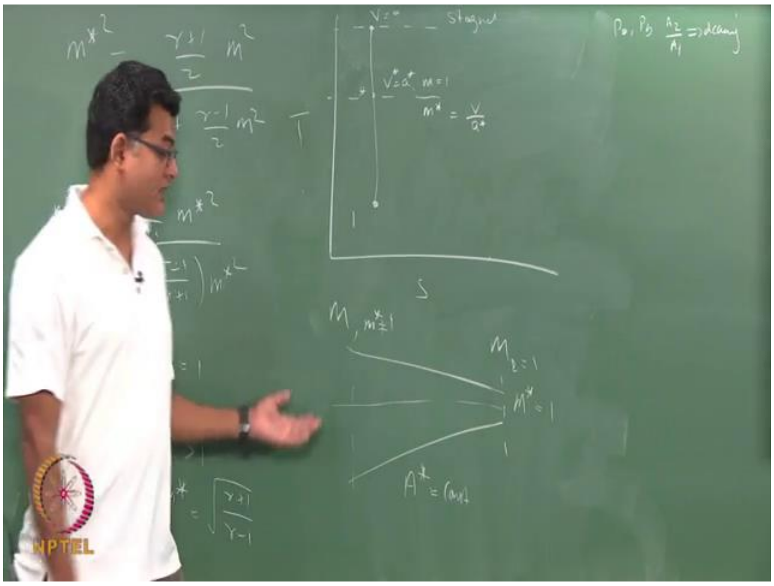

So, in the T S diagram I have a state 1 here I do an isentropic process such that the velocity is 0 is my stagnation state. Now I take it to a value where v equals a meaning m equals 1 that is my star condition, so my m star is my v by a star so this is my star value. So, my m star associated with this particular m is my m star. If m equals 1 I will have an m star equals 1 this is my star condition.

So, this is precisely the process what we are doing in a converging nozzle. I have an m here; I have an m here, so I bring this m isentropically to this value so I get an m here which is equal to 1 if it critically if it is less than critical pressure ratios. So, my m star here is also 1, but the m star here is not 1, because associated with this m you have a different m star. So, m star is not constant only your A star is constant in an isentropic flow your m star is not constant. So, I will end with whatever we had done previously.

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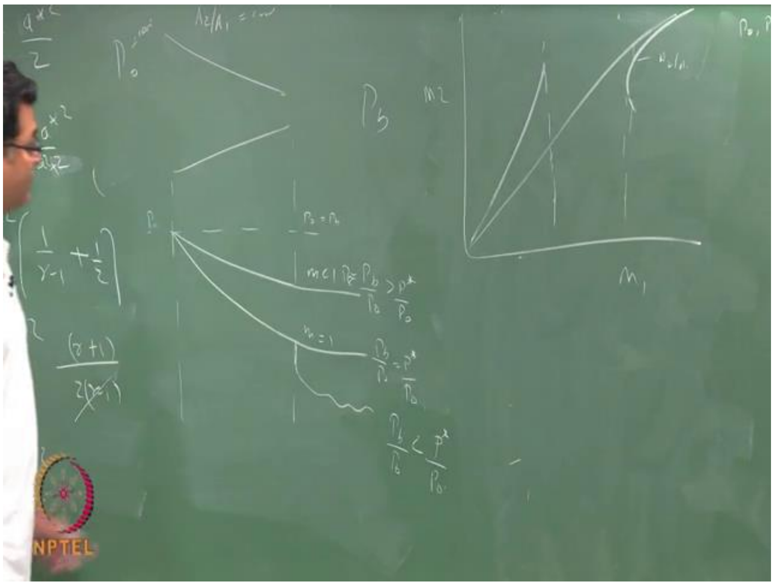

So, we will compile whatever we have done previously by changing your P b. So, I have typical area ratio, so P 0 is kept constant, this is also kept constant what we are done is we are decreased our P b. When P b is 0 equals P b there is no flow, otherwise you would get subsonic flow. If P b by P 0 is greater than your P star by P 0 your P b will also be equal to your P exit at this condition and when it reaches the critical value you would get your m equals 1. Any further degrees in your back pressure will not change the flow inside or at the exit, but after the exit there will some phenomena that is happening which we have not discussed here which will be discuss later. So this is what happens when P b is much lesser than your critical pressure ratio.

Now if I change the area ratio, the exit mach number is not going to change it depends only on your pressure ratio. We have also seen that from the plot there is a for a given m 1 there is a minimum contraction ratio where you have solution and above which the mach number does not give you a solution for that particular contraction ratio. So, to have a steady state flow of your m 1, if this is my m 1 this is m 2 this is for typical A 2 by A 1. For steady flow of your m 1 there is a minimum contraction ratio above which you are not going to have a solution. Likewise, if the flow is supersonic there is a minimum m 1 or a maximum contraction ratio will which you are not going to have solution. We will discuss that scenario in CD nozzles little later on.

For the time being this is more important will come back to those curves when we discuss converging-diverging nozzles and it is also important to realize that your relation between your m and m star associated with any m you will have an m star. So, this you will see also in your isentropic tables along with your P b by P 0 values associated with m there is an m star which is a reference quantity again.

Thank you.