Fundamentals of Gas Dynamics
Dr. A. Sameen
Department of Aerospace Engineering
Indian Institute of Technology, Madras

Week - 06
Lecture - 20
Converging nozzle

So, what we are going to discuss is converging nozzle with a special case of variable area problem.

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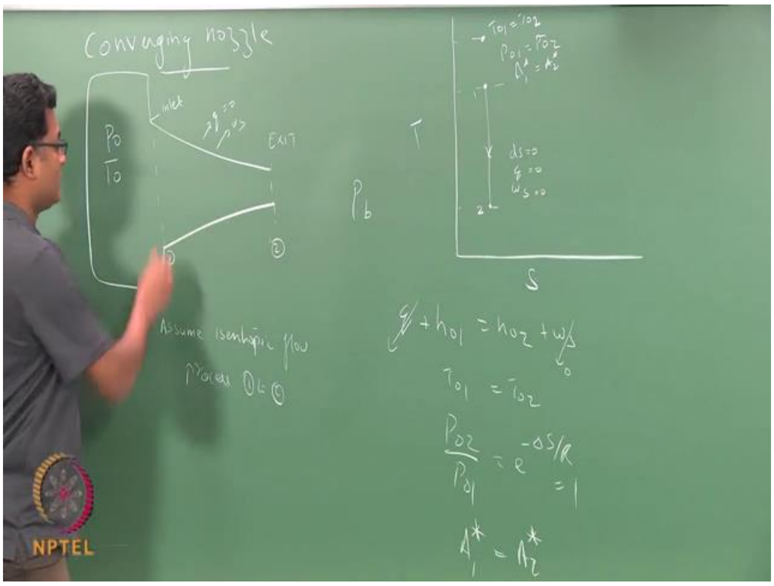

So, I have a nozzle that is converging this has some particular exit area. So, this is my exit. This is supplied from a tank, where the pressure is P 0, temperature is T 0. So, this is my inlet. So, this outside pressure is P b - back pressure P b. I will assume isentropic flow. So, whatever we are going to derive is strictly within this assumption. So, from 1 to 2, a flow is isentropic which means in the TS diagram from 1 to 2, I have an isentropic flow ds is 0 here, there is no q, there is no W s.

So, there is no heat transfer and there is no W s. So, associated with this there is a stagnation point. So, we have seen that this is equal to the stagnation state at the second point; the pressure is also equal. This is from your energy equation with the assumption that there is no q and no shaft work. So, I have my stagnation enthalpy same. So, the temperature - stagnation temperature is also same. P 0 2 by P 0 1 is e power minus delta S by R which is now 1. So, my stagnation pressure is also same.

Since, it is isentropic we have also shown A 1 star equals A 2 star. So, your A 1 star is also equal to A 2 star in this process whatever we are doing here in the entire section between any 2 sections in the entire stream here, we have A 1 star equals A 2 star. So, what we are trying to do is we will decrease our back pressure and see what happens to the slope.

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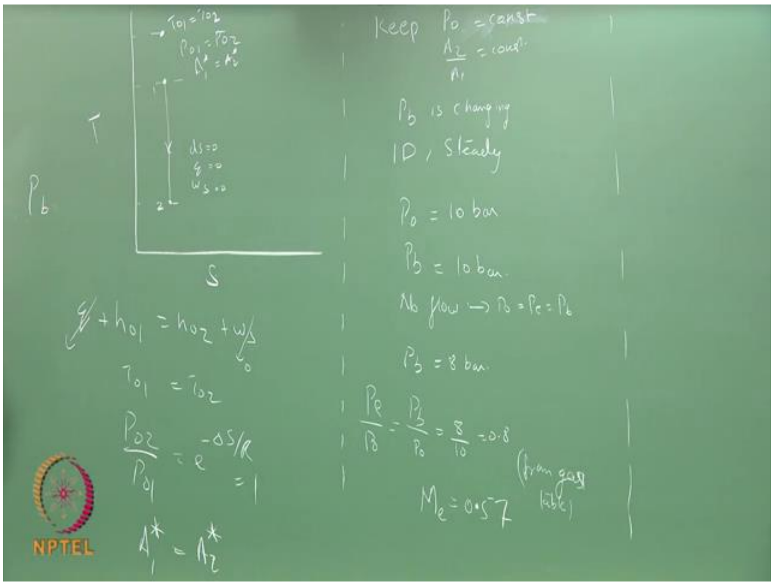

So, first let us keep P 0 and T 0 constant, let us talk about P 0 alone to avoid confusion. So, I am keeping P 0 constant and my A 2 by A 1 also constant; I am not changing the area at the inlet or area at the exit. Now P b is continuously changed, P b is changing. So, I do an experiment with different P b, remember what we are going to do is a quasi 1-D and steady problem which means that the change is not happening in time, but we are changing the P b and then watching what happens.

So, I change my P b to some value and see what happens at this point, I am not changing P b continuously. So, there is no time variation, this is a steady process what we are going to do all this example that we are going to see is a steady process. So, these are all steady process. So, remember that. So, we will change this P b to a new value and see what happens without including the time variation.

So, let us start with some value P 0 at 10 bar and P b also10 bar. So, this is case where P 0 equals P b and hence no flow. So, this is the back pressure I have. So, the inlet pressure is stagnation pressure is P 0 which is giving you the gas, if the back pressure is also the same value, there is no pressure. So, your exit pressure P exit is also the same value there is no change. So, your P 0 equals P exit equals P b, no flow.

Now, I decrease this P b further to 8 bar, if as I told you I am not continuously changing this is a new experiment with 8 bar as my exit pressure. So, my P b by P 0 is 8 by 10 equals 0.8. So, the Mach number associated with this ratio for gamma equals 1.4 is around 0.75. So, the Mach number at the exit will be equal to around point 0.75, 0.8, 0.8 is around 0.57 from gas tables for gamma equals 1.4. So, here my P exit is again equal to P 0 because this is going to be subsonic flow. So, the pressure here is also going to be the pressure at the exit. So, the subsonic flow, there is no change in pressure between exit and the ambient pressure.

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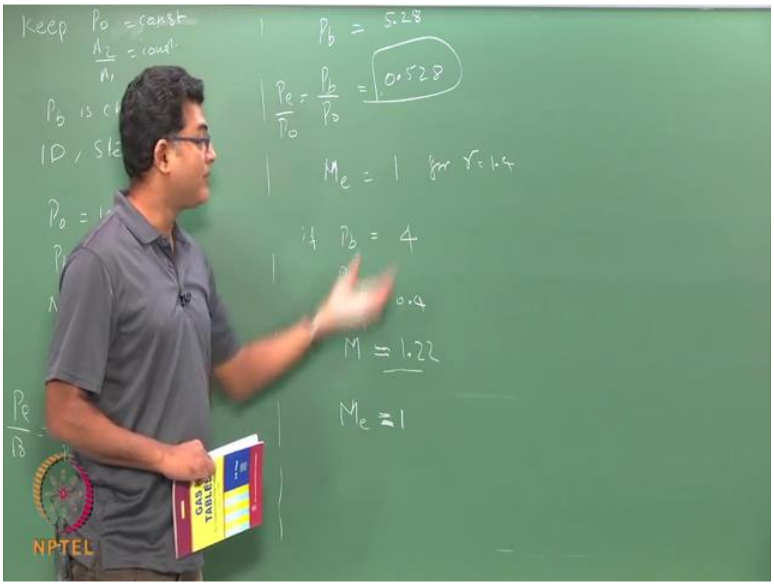

Now, I decrease my backpressure further to a 5.28, so that my P b by P 0, P exit by P 0 is 0.528. At this pressure, my mach number at the exit is going to be 1, for gamma equals 0.4. So, if I ask this pressure ratio my mach number at the exit is going to be 1. Now if I decrease my P b further to say around 4 my P b by P 0 is 0.4, the mach number associated with this is 1.22 around 1.22. But the problem here is we had seen in the other class, previous lectures to have a supersonic flow, the area should be increasing dA should be positive which is not happening here. Here the dA is always is negative. Now, dA is negative, inlet Mach number is subsonic, it has reached the M equals 1 for some pressure ratio.

Now, if I decrease it further you are getting a supersonic pressure ratio Mach number pressure ratio related to supersonic flow, but there is no area increase. So, a flow cannot go beyond M equals 1. So, happens that your M at the exit would still be 1, after you have reached this pressure ratio. So, whatever you do the backpressure whatever backpressure you decrease to after this pressure ratio, your exit mach number is still going to be 1, and it does not change, does not depend on further on your mach number. So, what is happening inside the nozzle, what is happening inside the nozzle after your P b by P 0 reaching that particular pressure ratio is not going to affect anything inside the nozzle, if you keep your P 0 constant and A 2 by A 1 constant.

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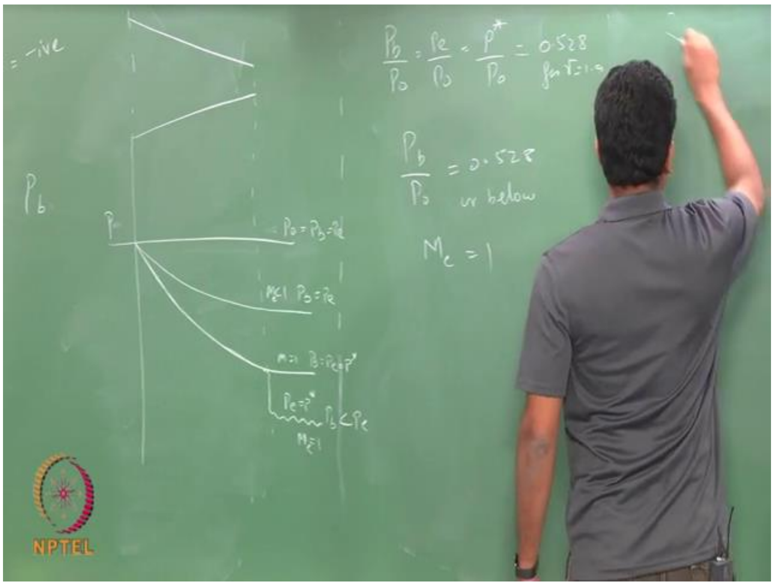

So, what did we do, we had the nozzle here, P 0 equals P b, there is no flow which is also equal to your P exit. If I decrease my backpressure a bit by a small amount, I would get a subsonic flow in the nozzle, which will exit out. So, the exit would be equal to P exit would be same as your P b, but it would be M less than 1 will be mach number at the exit is less than one.

But once I reach that ratio for m equals 1, my P b is equal to P. And if I call that as my critical pressure ratio, this exit pressure ratio is my P star such that my P b by P 0 P e by P 0 equals some P star by P 0, this is equal to 0.528 for gamma equals 1.4, if I use different gas I would get a different ratio. But for air, you would get if the pressure ratio is this you are going to get m equals 1 and I call that pressure as my critical pressure which is also our P star. So, M equals 1 is also our star condition which is at that area at this for this for this particular ratio is our A star which we will deal it a little later. For the time being m equals 1, P b equals P star.

Now, if I decrease my P b further nothing would happen here, my P b would be much less than my P exit, but P exit would still be equal to P star. So, this value is not going to change. So, this is M still equals m at the exit is still 1 the mach number at the exit is still one, but the flow does something here which again we will deal it after studying shocks and other things what happens outside the nozzle, at present we are dealing only with what is happening inside the nozzle.

There is nothing that is going to happen after P b reaching your P star. So, once that is done the point here is the mach number at the exit is not going to increase other than one, even if you decrease your P b, if your exit pressure is this or below your mach number at the exit is always going to be 1 for a converging nozzle if you keep your P b constant. So, you keep decreasing your P b no matter how much you decrease if this is there this is what you get.

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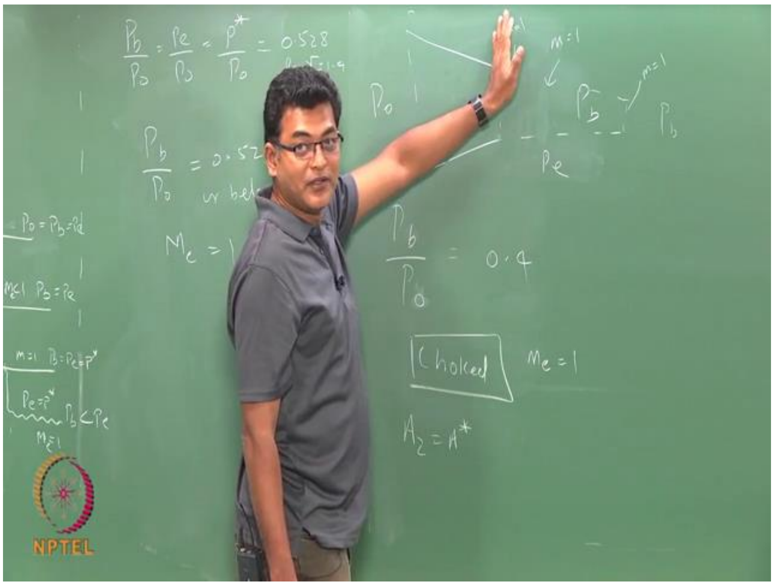

So, if I have a nozzle P 0, P b, P exit; now I have a nozzle P b by P 0 is say 0.4, P b by P 0 equals 0.4 which means that mach number here is 1. And I have something that is happening outside the nozzle. Now what I do is I cut this nozzle to this value keeping the same backpressure, if I do this if we look at whatever we have done here the area ratio does not come in the physics. So, whatever I do here keeping P b, P 0 constant that is very important. If we keep you have to keep P 0 constant and then do this process of cutting this nozzle into two since the area ratio is this you are still going to get M equals 1 here or if I elongate this again since P b by P 0 is this I am still going to get M equals 1.

So, this condition is not going to change. So, these are called choked condition. So, whatever you do the m exit is always 1 and your area at the exit becomes your A star. What does that mean, so I have some mach number here, I am doing an isentropic process to bring my mach number to 1, I am doing something to do take it to 1 and that value of the area is your A star which is precisely what we are doing. It is an isentropic process we are bringing it to 1. Now, when an I said for given mach number, I am reducing it to 1; and this would be your A star, but here I am cutting the A star still I am getting M 1.

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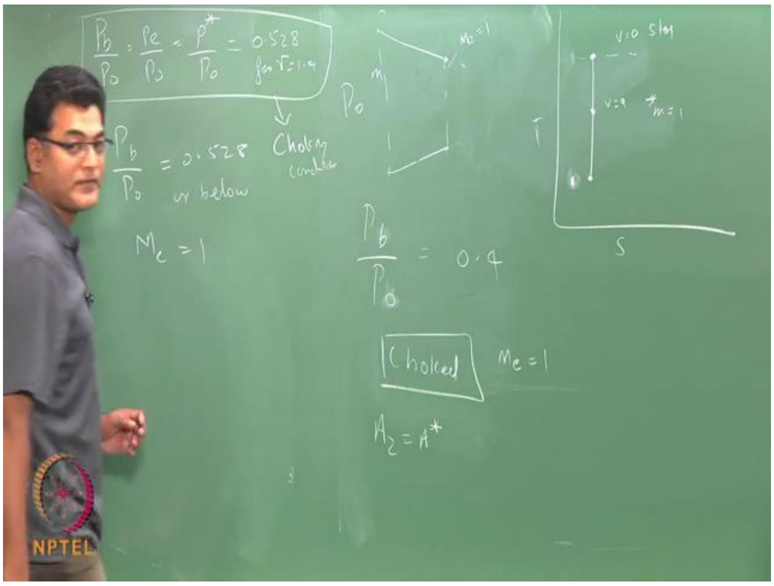

So, the process here is for example, what I have told you here is for a given mach number here, how did we define our star condition, we have the T S diagram. So, the process is from 1 to 2, so I have a state 1, I do something to take it to I do an isentropic process to take it to velocity 0, which is my stagnation.

I take it to a state where velocity is same as my sound velocity which is my star condition, which is my M equals 1 condition, but that is precisely what I am doing here I am taking this state to a state where my Mach number is 1. So, this is my star condition, but I have also told you even if I cut this I am still going to get M equals one. So, this does not mean that I have two A star here, but rather when I do this process something is happening to the inlet mach number, but I am keeping my P 0 constant. So, something is happening which that something is not what we are going to study that is a non-isentropic process which we will not deal with.

So, whatever that is happening here is 1 and that is associated with the choked condition. So, for this P 0 if the pressure ratio is below 0.528 that is it that the choked condition. So, this pressure ratio here is my choking condition for the converging nozzle. If it is below than that it is always going to be, so in choked condition what would be the mass flow ratio.

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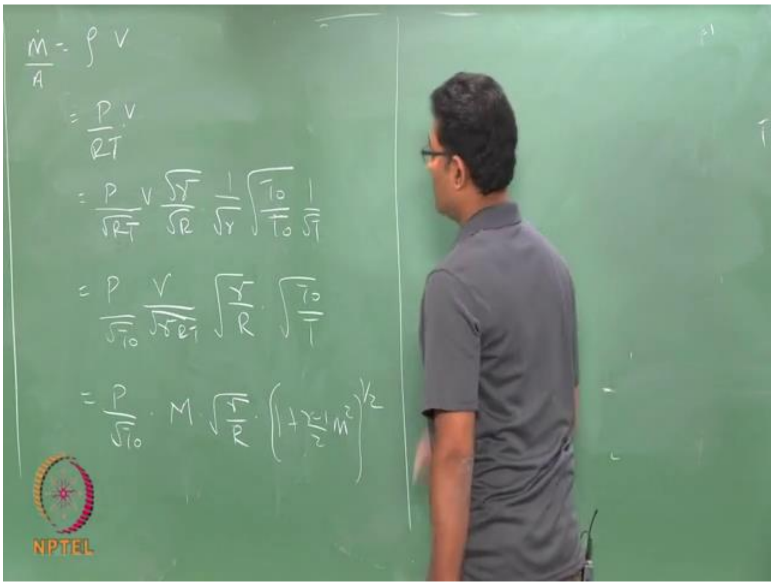

Let us evaluate the mass flow rate at the exit; M dot is or M dot by A is rho into V P by R T into V. Now P by R T then here v multiplying root gamma by root R, so there is root gamma here then I multiply and divide by T 0. I what I have done here is I have taken one root T here there is one root T missing, there is 1 by root t. What I had done here is I have taken one root T, here another root T here multiplied and divided by root gamma multiplied and divided by root T 0. P V by root gamma R T that would be my M, I write gamma here by r and then root T 0 by T and I take that root T 0 here. So, this would be my P by root T 0 into M, because this is my velocity of sound into gamma by R P 0 by T is 1 plus gamma minus 1 by 2 M square. Now there is a root here, it will be 1 by 2.

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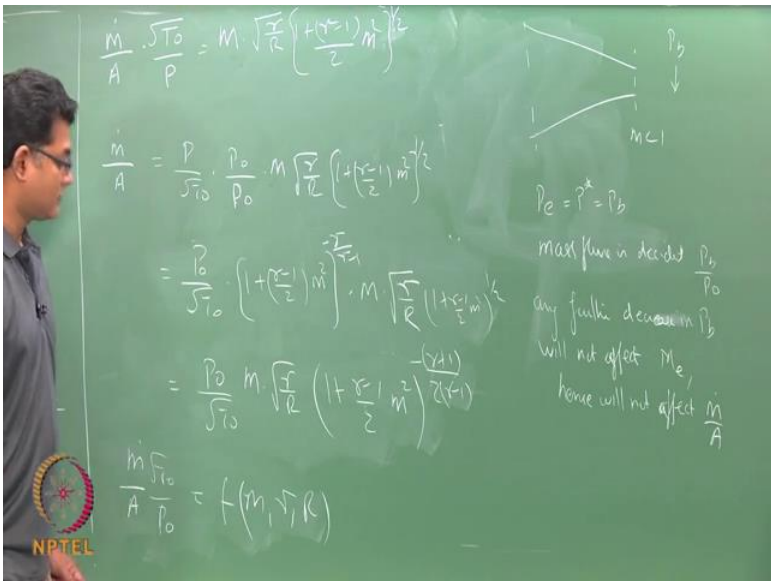

So, my M dot by A into root T 0 by P is nothing but M into root gamma by R into 1 plus gamma minus 1 by 2 into M square to the power 1 by 2. So, this quantity is going to depend only on your mach number at the exit and these things. So, your mach number at the exit is going to depend on these ratios I can change P also in terms of mach number by multiplying and dividing it by P 0, and I write this quantity.

So, this is equal to P by root T 0 into P by P naught is 1 plus gamma minus 1 by 2 M square to the power into there is an m here, there is a root of gamma by R into 1 plus gamma minus 1 by 2 m square to the power 1 by 2. So, this I will reduce this to T 0 m root of gamma by r 1 plus gamma minus 1 by 2 m square to the power minus gamma plus 1 by 2 gamma minus 1, and there is a bracket here. So, my M dot by A into root T 0 by P 0 equals function of M gamma and R. So, my mass flux is now depending only on my stagnation temperature and pressure.

So, if there is a change that is needed you have to change your stagnation pressure or stagnation temperature. So, when I have a nozzle, my P b will decide the pressure ratio P b will decide my mass flow rate till M less than 1 that is when my P exit is P star equals P b. my mass flux is decided by P b by P 0. But once it has reached this stage any further decrease any further decrease in P b will not affect the exit mach number.

Hence will not affect my mass flux, if I want to change my mass flux, I need to change my P 0 and T 0, so that is your choked condition, so that is what typically happens when you have a converging nozzle with the decreasing P b. And remember this is an isentropic solution that we are doing and the stagnation temperature and pressure is kept constant during this exercise.

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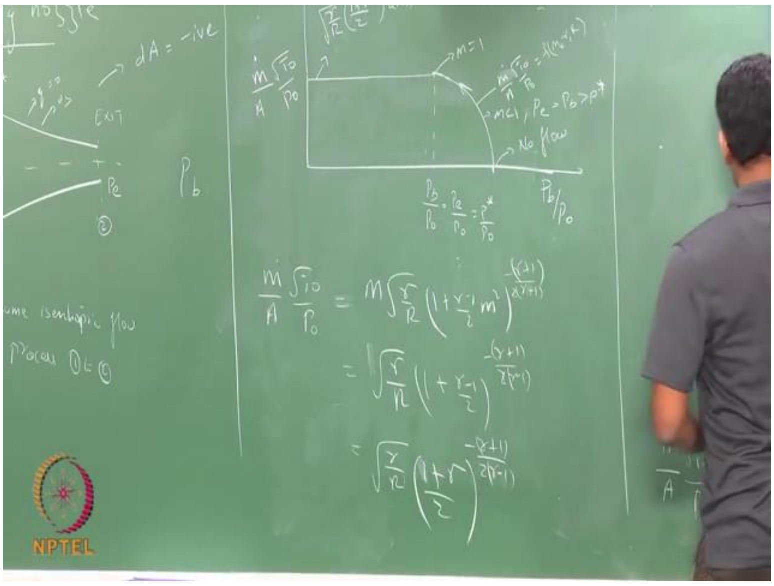

So, if I plot m dot for m dot by a into root T 0 by P 0. So, this is my P b by P 0. So, when P b by P 0 is 1, no flow. When P b is reduced, there you will get a mass flow rate according to this equation, which we have just now derived. So, you would get a mass flow rate according to this equation which would be like this. So, this would be according to this equation which is M dot by A root T 0 by P 0 is a function of Mach number R and T 0.

Mach number at the exit, because you do not know what is the Mach number at the exit. So, this is Mach number less than 1 your P exit is not yet equal to your P star. So, your P exit is same as your P b which is greater than your P star, but once your P b reaches P star. So, your P b y P 0 equals P exit by P 0 equals P star by P 0 you would get M equals 1. And once you reach M equals 1, if I substitute M equals 1 in this equation what I would get is M dot by A root T 0 by P is M into root gamma by R 1 plus gamma minus 1 by 2 M square minus gamma plus 1 gamma minus 1 into 2. So, when M equals 1 I substitute this, this should be root of gamma by R 1 plus gamma minus 1 by 2 to the power minus 1 plus 1 2 gamma minus 1. So, this is gamma plus 1 this would be root of gamma by R into 1 plus gamma by 2 to the power minus gamma plus 1 by 2 gamma minus 1 brackets.

So, once you have reached m equals 1 this is your value. And after that whatever you do with your P b, keeping your P 0 constant, your mach number is not going to increase at the exit, hence your mass flow rate is also going to be a constant after this. So, I am doing this process, I am decreasing my P b. So, after that my M dot is constant and this value is this root gamma by R into 1 plus gamma by 2 to the power whatever that is written here. I will write it neatly here.

This value would be gamma by R 1 plus gamma by 2 to the power minus gamma plus 1 divided by 2 gamma minus 1, so that would be the value here. So, this is a case when we keep P 0 constant. So, P 0 is kept constant we are changing the P b. So, that this ratio is reduced reducing and it is reducing initially and that decides what is the mass flow rate till it reaches mach number 1, further then that the mach the mach number at the exit is not reduced hence the mass flow rate is not reduced if you have this condition. So, this is the maximum flow rate, maximum mass flow rate it can go through that particular area because area is here. So, your mass flux this is 0.

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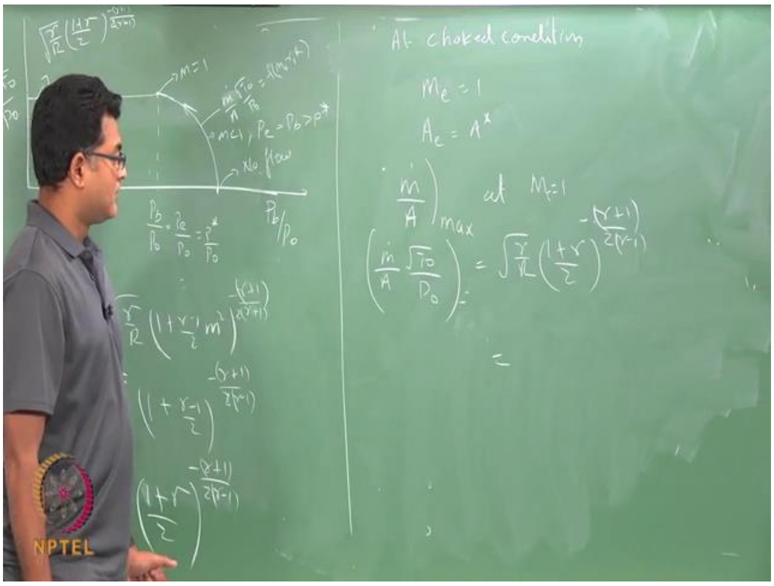

So, at M equals 1, M exit equals 1, your A exit is your A star and your mass flow rate or mass flux is maximum at M equals 1, which is now just a root of gamma by R into 1 plus gamma by 2 to the power minus gamma plus 1 by 2 gamma minus 1. So, this is M dot by A into root of T 0 by P 0 which would be maximum at this rate. So, this is a constant value. So, mass flux is maximum this ratio and this is a constant value. So, it depends only on your fluid. So, again I will conclude with explaining this particular plot again using the setup we have just now shown. So, what I am going to do is I decrease my P b keeping my P 0 constant.

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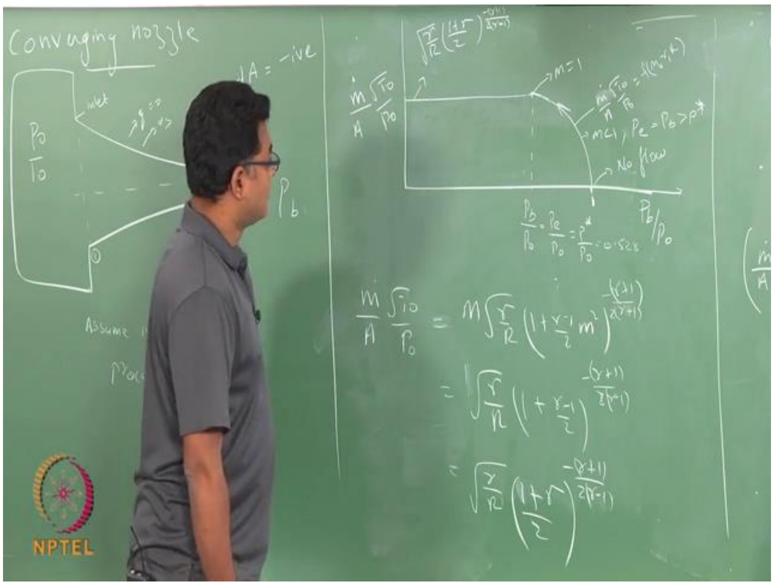

So, I keep decreasing my P b which means P b by P 0 is slowly decreasing from 1, when this is 1, there is no flow I keep decreasing so there will be a flow that is static that is all subsonic till it reaches the pressure ratio of 0.528 for air. Once it reaches, the exit mach number is 1; and once the exit max number is 1 whatever further you decrease mach exit mach number is not going to change. And hence if you look at the equation for the mass flow rate mass flow rate also is independent of the pressure ration what is there. So, the pressure ratio is not going to play a role in the mass flow rate and that is the choked condition because you are choking your flow with that particular pressure ratio.

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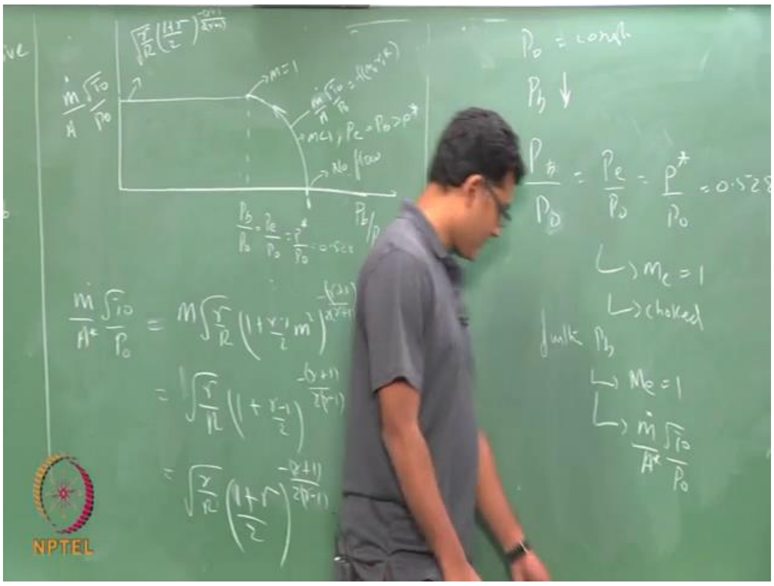

So, the message from today's classes, we have kept P 0 constant P b kept on decreasing. So, when we reach P b by P 0 equals P exit by P 0 equals P star by P 0, when you get this value you reach M equals 1 at the exit which is also your choked condition. Further decrease in P b has no effect on your exit mach number, and has no effect on you mass flux. So, if you want to increase your mass flux or decrease your mass flux, you have to change these values A or T 0 or P 0. So, when M equals 1, you can also call this as your A star. So, this can also be typically called as A star because M equals 1.

With that, I would end today's lecture; and we will take up the in the next class a continuation of this.