Fundamentals of Gas Dynamics

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Week – 01
Lecture – 02
Basics of Thermodynamics

Welcome to the second lecture on Gas Dynamics. So, today we will discuss Thermodynamics.

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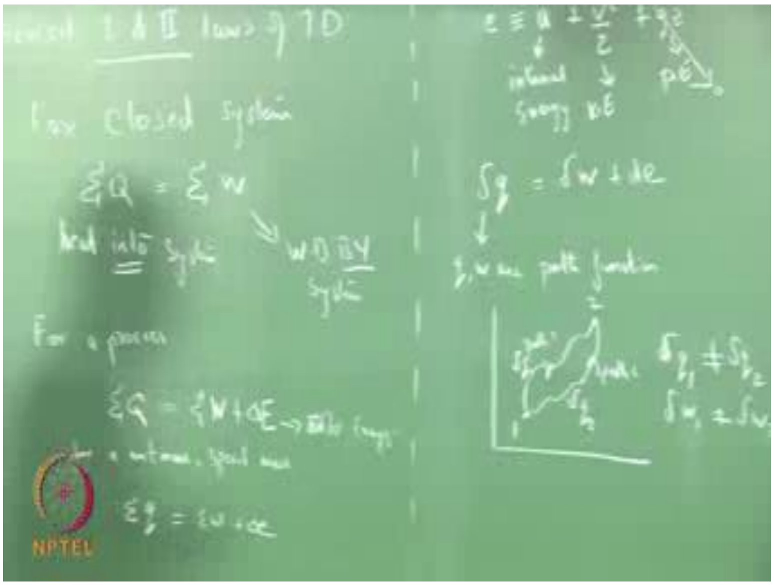

So, we will revisit first and second laws of Thermodynamics, whatever you have learned in your previous course. This is not going to be an elaborate discussion on thermodynamics and the subject, but just whatever we have required for Gas Dynamics. For a close system, the first law would tell you, the heat input is your work done. So, this is heat into the system, this is work done by the system. This is a standard sign convection that we use.

For a process, going from state 1 to 2, we have the heat input equals work done plus your internal energy or the total energy, in terms of a between state 1 to 2. So, this is your total energy. Now for a unit mass or as we call as specific mass, we use the notation delta q, sigma q plus sigma w plus sigma e; all in small letters for specific mass.

We will keep this notation, throughout the course a, when I write Q its for the whole mass, q is for unit mass. What is delta e or what is e? e numbers of constitutes u, which is your internal energy, plus v square by 2 which is your kinetic energy plus some potential energy. Now internal energy is something that is due to the molecular rotation, vibration and translations and other things of the fluid particles. This is due to the kinetic energy or the motion of the gas, and this is due to the potential of the gas. So, we will assume this to be 0 for the course at least.

Now, for an infinitesimal step process, for a small delta change from state 1 to 2, we write dq; delta q equals delta w plus de. This delta e is because q and w are path functions which essentially means, the work done and the heat input depends on the path that goes from state 1 to 2. So, if I have a state 1 and the state 2, the delta q and delta w. So, this is path 1 this is path 2. So, your delta q1 and delta q2 are not the same; q1 not equal to delta q2. Likewise the work done has also not the same, which essentially means this is a path function, and hence any incremental change is denoted by delta q and not d q. So, if the process is stationary; obviously, v is 0. So, your internal energy is just u. So, your delta q is delta w plus d u.

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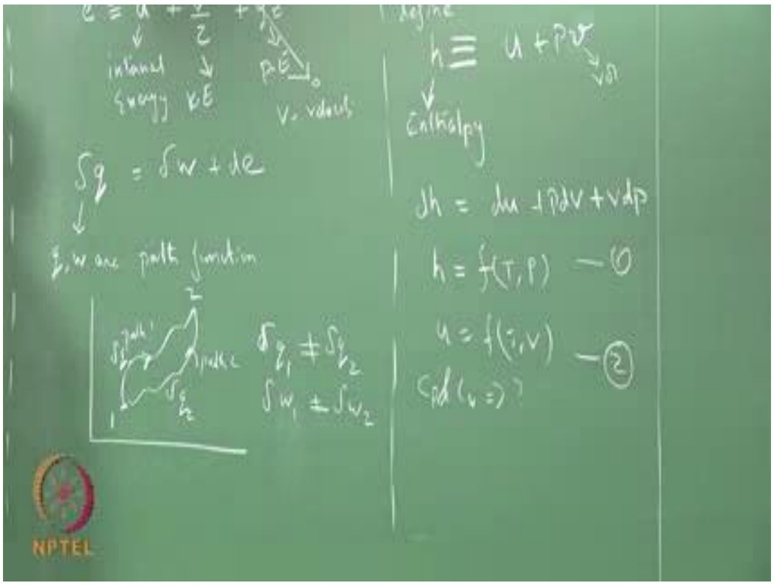

We also define something called enthalpy, which is u plus p v, this internal energy and your combination of pressure and volume. So, here v is volume. So, this v is velocity. So, to avoid confusion, I would write this as small v which is volume of unit mass, and V is velocity. So, I will again keep that particular notation. So, whenever there is confusion between these 2 notations I will try to refer that. So, the enthalpy is defined like this which essentially means, if I differentiate this for a small process this should be d u plus p d v plus v d p, which is called c p and c v.

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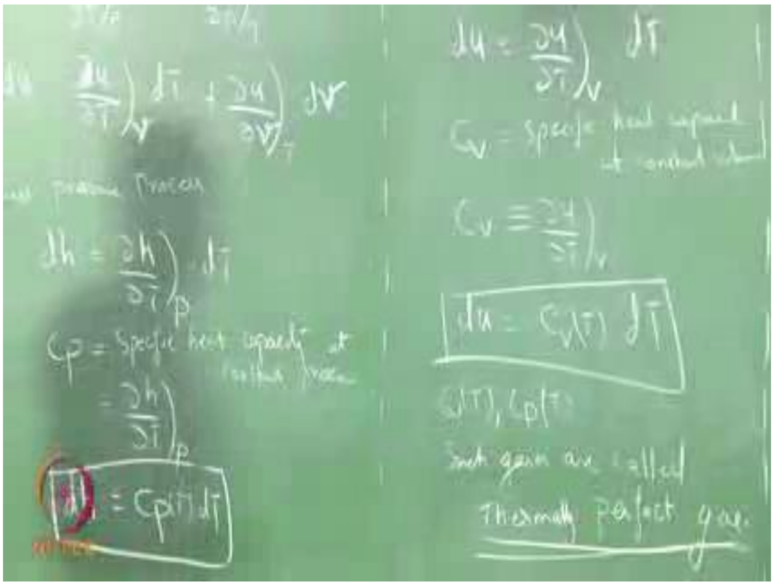

So, what I am going to do is, differentiating those equation, differentiating the first equation and get d h by d T at constant pressure into d t plus d h by d p at constant temperature into d p, d u is d u by. So, there is volume. Now if the process is, constant pressure process; obviously, the first equation is d h is d T, where we define the coefficient as c p which is specific heat capacity as dou h by dou t at constant p, or d h is c p d T, but remember this is also a function of temperature; c p is also a function of temperature.

Likewise the second equation, if the process is, process my d u is dou u by dou t at constant volume into d T, and now I define c v. So, it is specific heat capacity at constant volume. So, this is at constant pressure. So, my d u, there is c v again function of temperature into d t. So, for chemically inert gases, your cons specific heats of functions of this temperature. It is not going to be a functions of other quantities, such gases where c v function of temperature, c p functions of temperature; such gases are called thermally perfect gas. So, thermally perfect gas means, these equations are valid, and specific heats are just a function of temperatures. Now, if they are also constant, then you have calorifically perfect gas.

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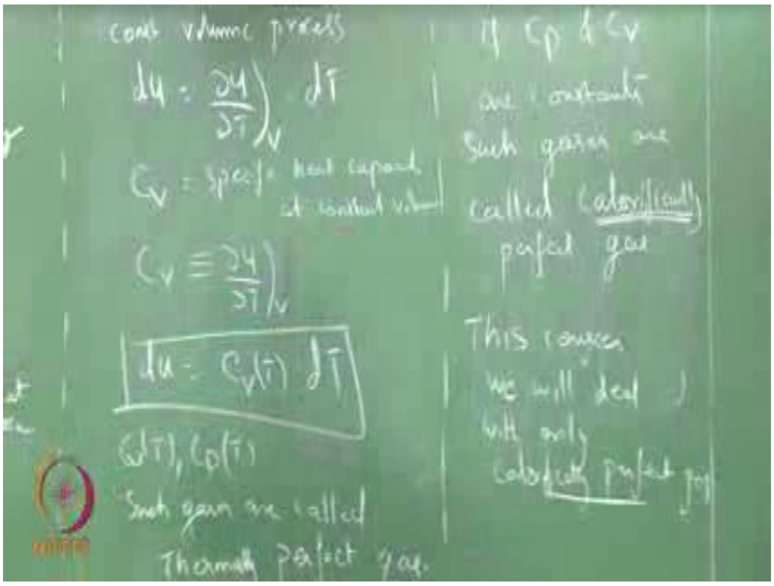

So, in this course we will deal with early calorifically perfect gas. So, for all the gases that we are going to consider in this course c p and c v are constant. Now we will continue with our laws of thermodynamics. Now we are defined enthalpy, we are defined our internal energy, and we have defined the first law, we have written the first law as this.

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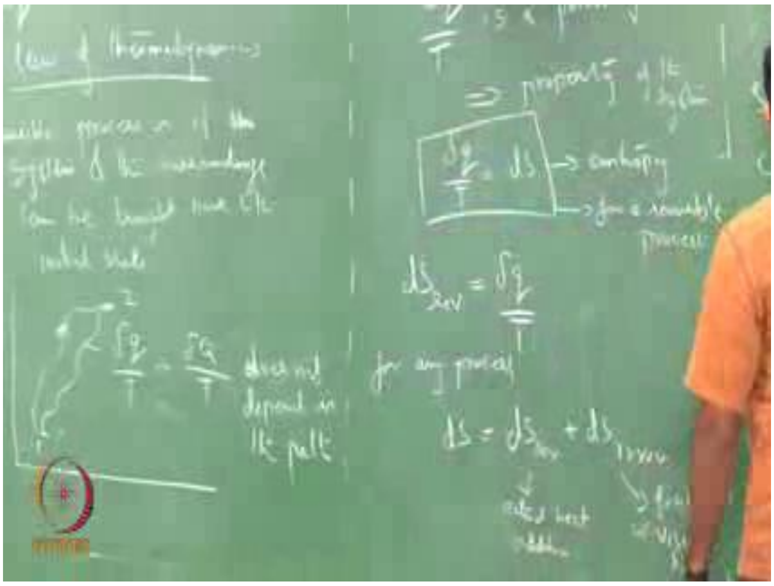

Now, we will discuss the second law. So, what is the second law? In reversible process you have say from its like. So, how do you define a reversible process? A reversible process is, if you can bring back, if the system and the surroundings can be brought back to the initial state. So, you do a process from point 1 to 2, state 1 to 2. if we can bring back the system and the surroundings from 2 to 1, then that process is a reversible process. Now, for a reversible process, from state 1 to 2 what is not appear is the delta q by t or delta q by t is, or does not depend on the path. So, whether I go from this path or I take another path, the delta q by t is going to be just a function of that particular state, that location of the state.

So, from state 1 to 2 delta q by t is going to be same, which means what, which means delta t by q is a point function, and point functions are; function delta q by t implies delta q by t is a property of the system or of the process or whatever. This is defined as your entropy. So, for a reversible process this is defined as process. So, your d s reversible is d q by t. So, for any process, your change in entropy is then d s reversible and d s irreversible. So, your irreversible entropy change is essentially due to, friction or viscous heating or viscous dissipation, and this is your actual heat addition, which is defined as, which is given as delta q by t.

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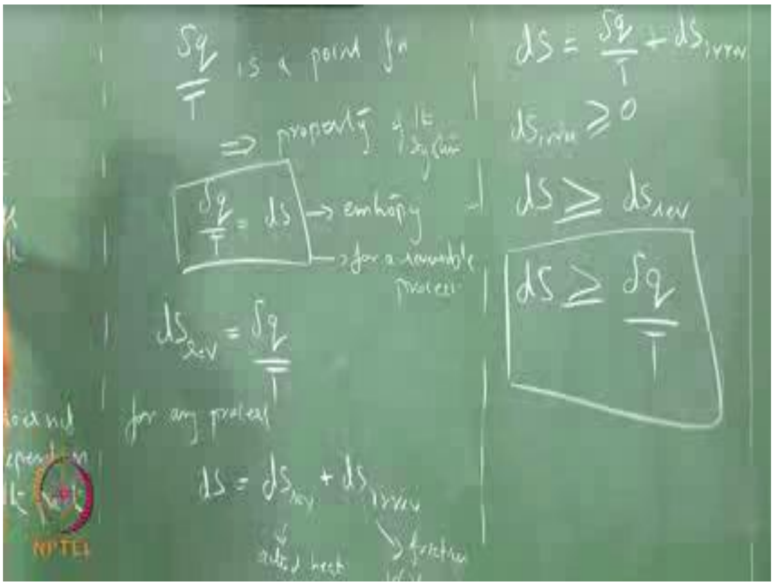

So, if I rewrite that equation. So, irreversible entropy is always going to be happen or always going to be there in any real flows. So, d s irreversible is always greater than 0.

So, which essentially mean that, the total entropy change, is always greater than d s reversible, or d s always greater than delta q by t, and this is the so called Clausius in equality. So, this gives you as given in most of the thermodynamic text books the Second law gives you the direction in which your delta q and delta w works. Remember delta q is the heat input into the system, delta w is the work done by the system.

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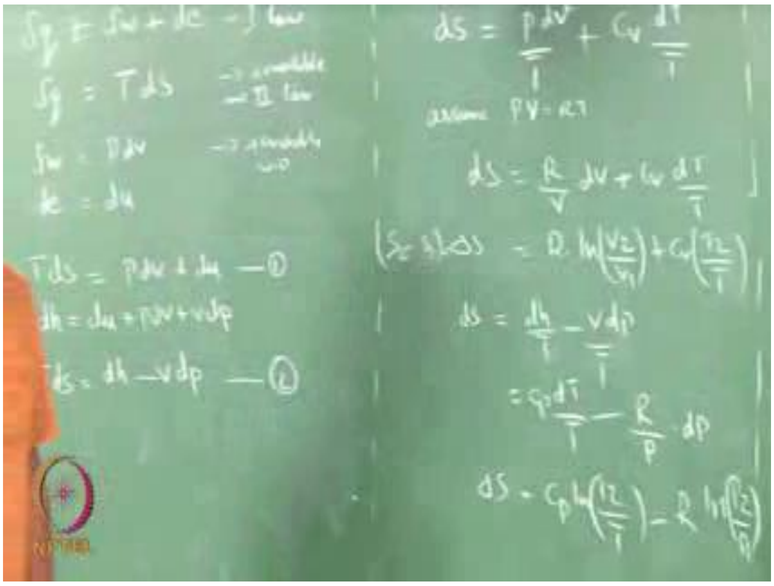

So, my first law gives me this equation. My second law gives me this equation for a reversible process. So, this is from your second law. This is from your first law. Now delta w, you can write it as p d v which is a irreversible work done, the work done due to change in your volume. And if I restrict my d e to just d u, which means my, it is a stationary process, and the potential let us assume 0. I can rewrite the first law as t d s equals p d v plus d u. we have also defined our enthalpy as this, from which I can substitute for d u, I should get t d s as d h minus v d p. So, this is equation 1, equation 2. So, these 2 equations; from these 2 equations we can derive few other forms of change in entropy.

For example let us take equation one. So, your d s is p d v by t plus d u I can write it as c v d t by t and if I assume p v equals r t. So, these v are all volume v s if I assume p v equals r t my d s r by v d v plus c v d t by t. So, my delta s which is s 2 minus s 1 between 2 points is r into l n v 2 by v v 1 plus c v into t 2 by t 1. So, your change in entropy is this particular. Now, if I take the other equation t d s, I will take the t the other side d h is by t d h is c p d t by t minus v by t is r by p. So, my delta s is c p l n t 2 by t 1 minus r l n p 2 by p one. So, I can also write this.

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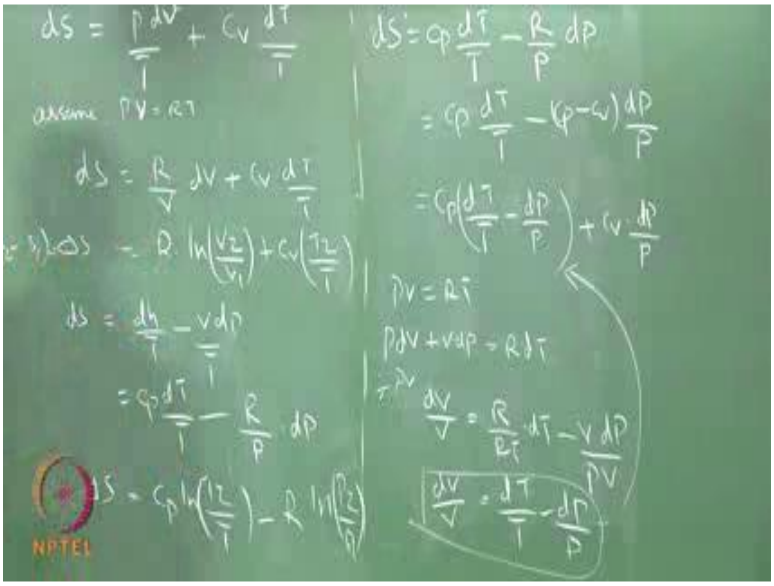

So, c p d t by t r I write it as c p minus c v d p by p. So, I take c p out d t by t minus d p by plus d p by no minus c p by p plus c v into d p by p. So, if take p v equals r t I differentiate this p d v plus v d p equals r d t. Now I divide this equation by p v. So, my d v by v equals r by r t r by r t would be 1 by t into d t minus v d p by p v. So, this is v d v equals d t by t minus d p by p which I substitute here. So, my delta s or d s is rewritten as c p d v by v plus c v d p by p.

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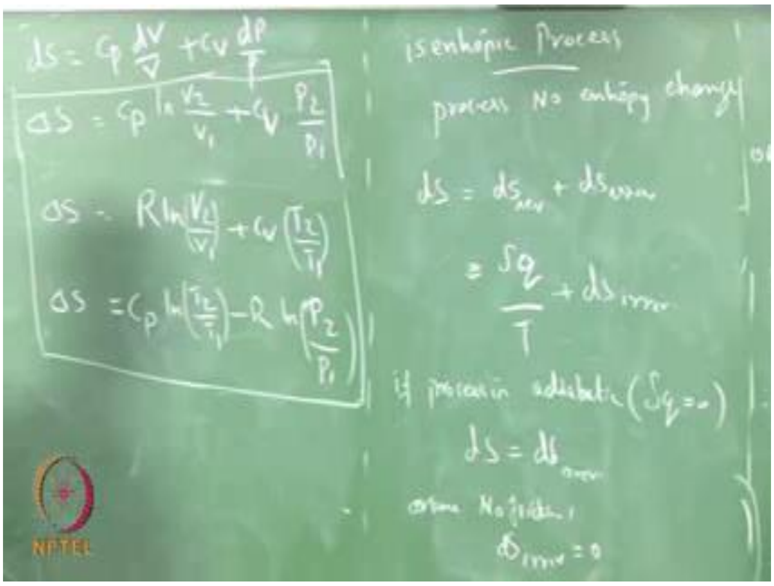

So, my delta s is now defined as c p l n v 2 by v 1 plus c p c v p 2 by p one. So, along with along with this equations r l n v 2 by v 1 plus c v t 2 by t 1, and this equation c p l n t 2 by t 1 minus r l n p 2 by p 1. So, these are important relations from which we are going to derive further inferences about several processes. Now, let us look at isentropic process, what are isentropic process, process with a, now entropy change. So, we have any way defined our d s to be d s reversible plus d s irreversible, which is delta q by t plus d s irreversible if the process is adiabatic.

What is adiabatic? Your delta q is 0, there is no heat transfer if; that is 0 then your entropy change is essentially, or irreversible entropy change. So, all irreversible entropy change associated with an adiabatic process, is the total entropy of the system, because there is no heat addition to it. Now, in addition to this if I also assume no friction or viscous heating or etcetera. My irreversible entropy is also 0. So, in such cases you will end up with.

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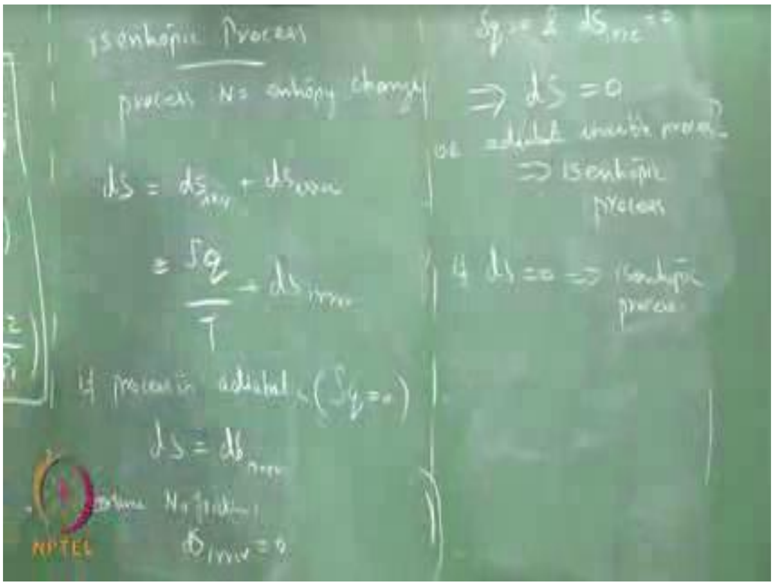

isentropic Process

$dS = dS_{rev} + dS_{irrev}$

$= \frac{\delta q}{T} + dS_{irrev}$

if process is adiabatic $(\delta q = 0)$

$dS = dS_{irrev}$

No friction

$dS_{irrev} = 0$

$\delta q = 0$ & $dS_{irr} = 0$

$\Rightarrow dS = 0$

or adiabatic reversible process

$\Rightarrow$ isentropic process

if $dS = 0 \Rightarrow$ isentropic process

So, delta q equals 0, and delta s irreversible equals 0, implies my total entropy is 0, or adiabatic reversible process is isentropic process. If d s is 0 somehow whether it is adiabatic or irreversible if d s is 0, then it is a general way of defining isentropic process, what you should remember here is, all adiabatic reversible process at rev isentropic process, but the converse is not true. Now, let us see what are these in terms of that. So, if the process is isentropic what will happen to this set of equations?

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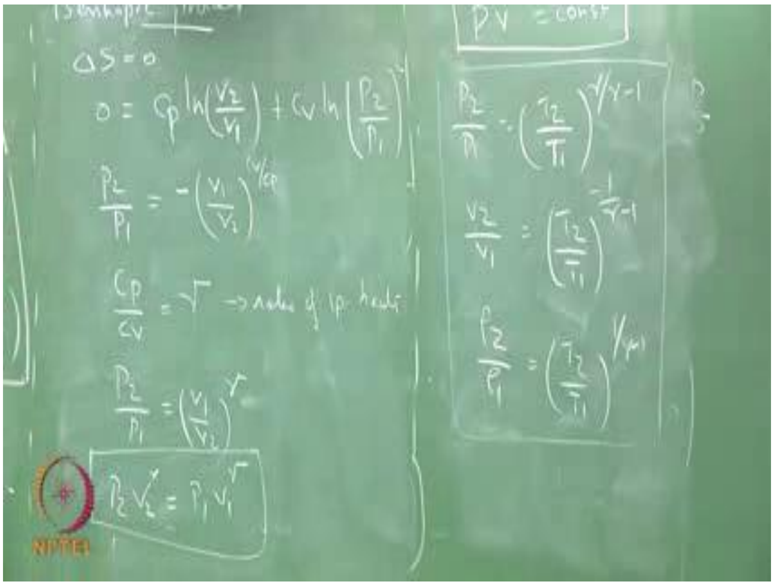

$\Delta S = 0$

$0 = C_p \ln\left(\frac{V_2}{V_1}\right) + C_v \ln\left(\frac{P_2}{P_1}\right)$

$\frac{P_2}{P_1} = -\left(\frac{V_1}{V_2}\right)^{C_v/C_p}$

$\frac{C_p}{C_v} = \gamma \rightarrow$ ratio of sp. heats

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$

$\boxed{P_2 V_2^{\gamma} = P_1 V_1^{\gamma}}$

$PV = const$

$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$

$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)^{-\frac{1}{\gamma-1}}$

$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}$

So, we are going to discuss isentropic process where delta s is 0. So, if delta s is 0. So, let us take this equation; p and v. Let us take the first equation 0 equals c p l n v 2 by v 1 plus c v there is an l n here there is an l n here too that is missing l n p 2 by p 1 which I can write it as p 2 by p 1 equals minus v 1 by v 2 c v by c p or p 2 c p by c p by c v is defined as gamma which is your ratio of specific heats. So, your p 2 by p 1 equals v 1 by v 2 to the power gamma or p v 2 is the power gamma is p 1 v 1 to the power gamma gamma equals constant I will write it here.

So, you can also write other forms from the other set of equations you can write p 2 by p 1 equals t 2 by t 1 power gamma by gamma minus 1 and v 2 by v 1 equals t 2 by t 1 to the power minus 1 by gamma minus 1 which I can rewrite it as rho 2 by rho 1 into t 2 by t 1 to the power gamma minus 1 1 by gamma minus 1 from this the other 2 equations. So, you can arrive at these relations. So, for the process from 1 to 2 for isentropic process from 1 to 2 you could get these relations from your first law, and second law of thermodynamic, which is been rewritten in this particular form.

As I told you in this beginning we are going to extract what is essential for this particular gas of course, from thermodynamics. So, we are not going to discuss any other issue; that is discussed in thermodynamic courses. Now, I can look at. So, p v equals gamma p v power gamma is constant for an isentropic process.

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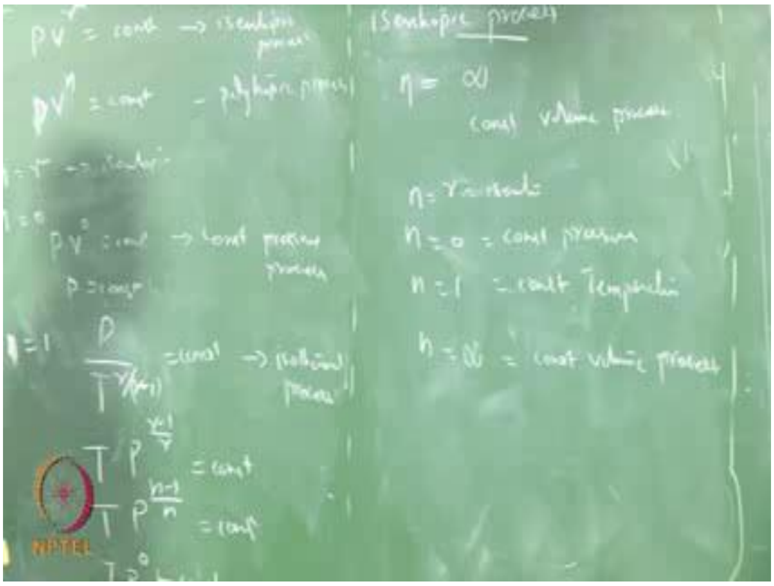

Now, I can have other processes where this power is not gamma; such process are called polytrophic process, where gamma, instead of gamma we have (Refer Time: 40:42) to the power n equals constant. So, instead of gamma you replace n in all these equations. So, if n equals gamma, we know its isentropic process. If n equals 0, what is this process? I put 0 here. So, p v to the power 0 equals constant, this is constant pressure process. Likewise, if my n equals 1, if my n equals 1 what is this process, its constant temperature process, which is constant or t to the p t p to the power gamma minus 1 by gamma equals constant.

So, if it is a polytrophic process t n minus n is a constant. So, if n equals 1 constant. So, n equals 1 is an isothermal process. So, if n equals infinity, then you will end up with constant volume process. So, n equals gamma is isentropic n equals 0 is constant pressure process, n equals 1 is constant temperature process, n equals infinity is constant volume process. So, if you look at this process in the p v diagram, how does it look. So, I have p v.

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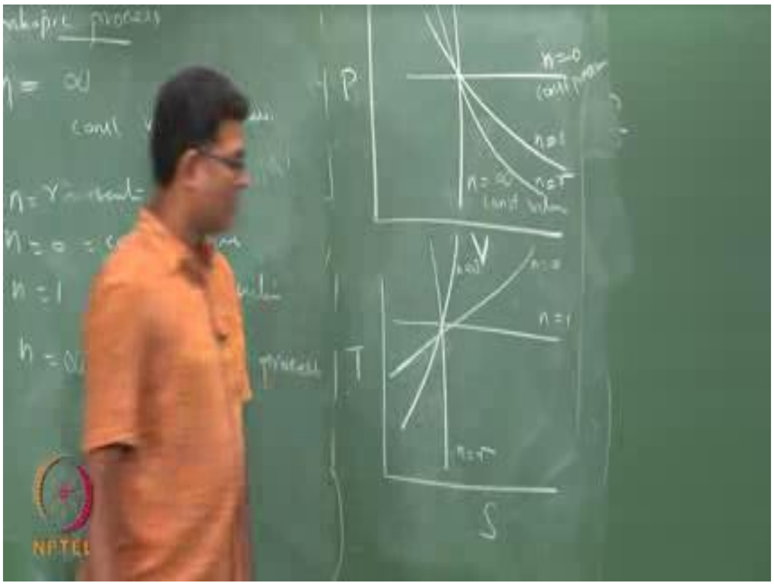

So, I am going to, if the process is from point 1 to 2. So, first lets draw the easiest 1 n equals 0 is constant pressure line. So, it is a constant pressure line. So, n equals 0 it is a constant pressure line, constant pressure. Now n equals infinity now its constant volume. So, it is something like this; n equals infinity constant volume. Now you have n equals 1 and n equals 1.4 or 1.3 or whatever. So, you have this n equals 1 and you have another process n equals gamma. So, n equals 1 is a constant temperature process and n equals gamma is your isentropic process.

Now, you can draw the same thing in t s diagram. So, the easiest 1 is constant temperature. So, straight line constant temperature is n equals 1 and constant entropy is n equals gamma, so you have n equals gamma. Then you have the other 2 here, here, n equals 0 and n equals infinity.

That ends this capsule of the course. what we are learned is the first and second law of thermodynamics, and the constitutive relations from which we have derived this process for isentropic flow, or the equations for isentropic flows, and we will be using these equations in the subsequent classes. We will have a tutorial on this particular aspect along a couple of questions, and we can expect the assignment based on this course, whatever we had discussed today.