Fundamentals of Gas Dynamics
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Week – 05
Lecture - 18
Discussion on Variable Area Adiabatic flow & *reference quantities

In the last class, we had seen the star reference quantities, and we are also seen how on what condition you can use Bernoulli's equation or what is the pressure ratio or the mach number below which you can use the Bernoulli's equation.

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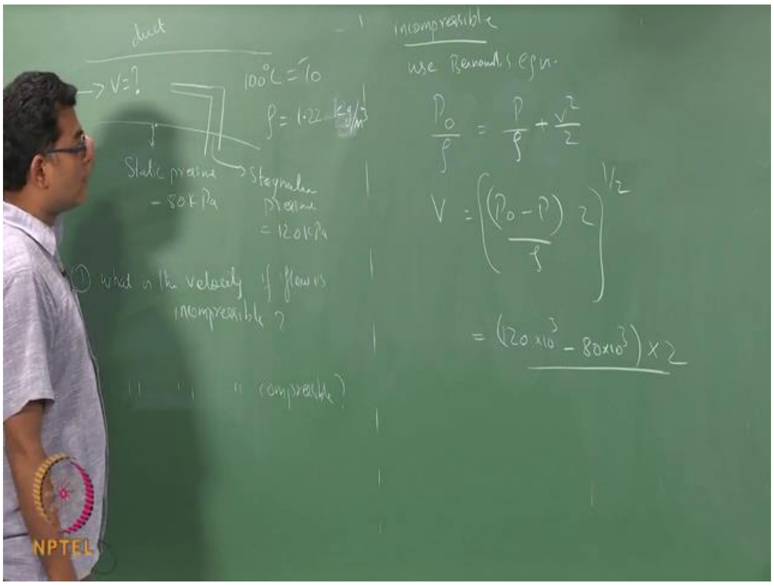

We will do demonstrate that with the couple of examples. So, I have a numerical problem here. So, I have some duct probably area changing duct or whatever. So, here is some velocity. I have a static pressure measured here at the wall. We will discuss how a static pressure and stagnation pressure are measured probably in the classes that comes later, but for the time being do what is the measured at the wall is the static pressure which is 80 kilo Pascal. And I have a pitot tube here, which measures my stagnation pressure; again we will discuss these things at a later point of time which measures around 120 kilo Pascal. So, the temperature at which this flow is at under 100 degree Celsius which will be a static temperature.

So, question one, what is the velocity if the flow is incompressible; what if this velocity if the flow is compressible. So, I have given you the static pressure, stagnation pressure and my stagnation temperature associated with this flow. Question is to find the velocity. If the flow is incompressible use Bernoulli's equation to get P 0 by rho equals P by rho plus V square by 2. So, I get my velocity as P 0 minus P by rho into 2 to the power of 1 by 2. So, what is my stagnation pressure here 120 into 10 power 3 minus 80 into 10 power 3 multiplied by 2 divided by density. We can assume perfect gas, so density is 1.

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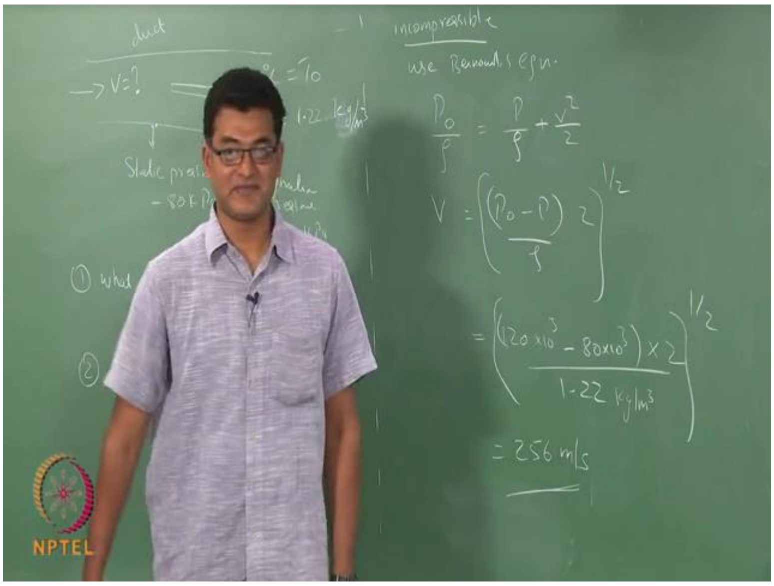

So, I assume. If density is not given, so you have density here.

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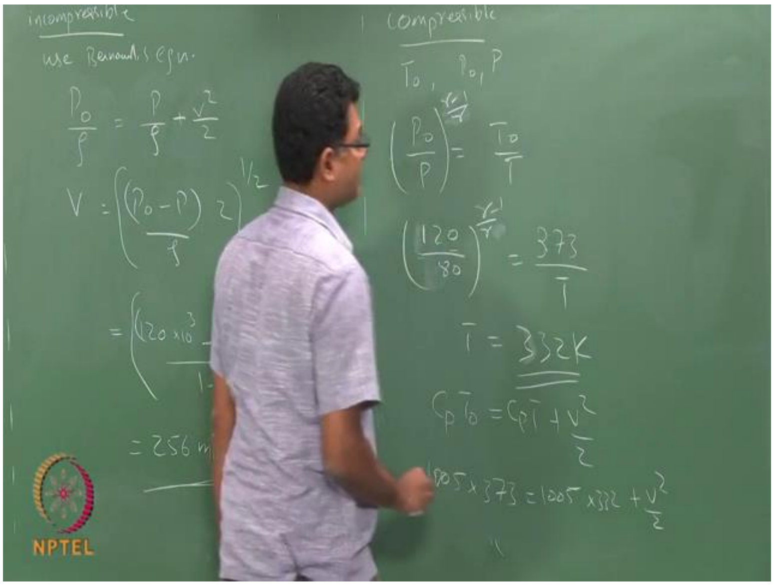

Now, if the flow is compressible, what do you do. We have T 0, here P 0 p. So, from P 0 by P is related to T 0 by T as gamma by gamma minus 1 and this particular form for a fully compressible flow, assuming no loses, no heat transfer, no shaft work. So, this is our compressible flow relation. So, P 0 by P is a P 0 is 120 by 80 to the power gamma by gamma minus 1 equals T 0 is 373 divided by T. So, T would be once you have the T, you can use the energy equation to get your velocity.

Students: (Refer Time: 07:12).

332? So, I use this value of T static T to find my velocity. So, my C p is 1005 joule per kg Kelvin into T 0 is.

Student: Sir, (Refer Time: 07:42).

P naught by P.

Student: (Refer Time: 07:48).

P naught.

Student: (Refer Time: 07:53).

So, this is gamma minus 1 by gamma, so this gamma minus 1 by gamma. So, this value is 332 Kelvin. So, C P into T 0 is 373 equals 1005 into 332 plus V square by 2 from which you would get your V to be 287.

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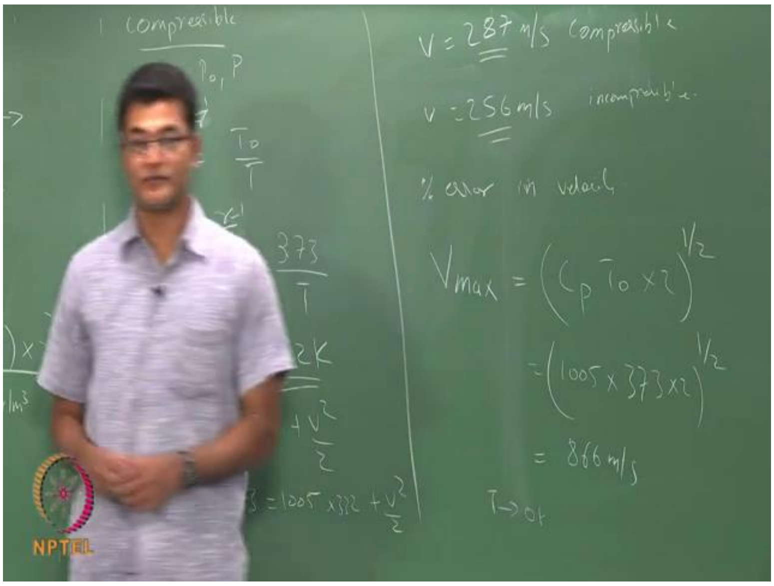

We disperse again so that is a difference in a velocity if you have assumed incompressible flow, so that is the error. So, you can actually find the percentage error in velocity, if you had assumed the wrong behavior. So, this is for your incompressible flow this is for your compressible flow this is what you use air flow at very low speed a mach numbers less than 0.3 or water flows or liquid flows or flow with the any constant density and this is what you used for the complete compressible flow. Now, we will do another problem. We can also find the maximum possible velocity with this T 0. So, you can continue doing this V max equals for given T 0, we can find C p T 0 into 2 power 1 by 2. So, 1005 into T 0 is 373 into 2 power 1 by 2. So, this is?

Student: 866.

8.

Student: 66.

866 meter per second. So, this is the maximum possible velocity that you can achieve in this scenario with this T 0 for a given T 0, this is a maximum possible. So, you have converting the entire heat content to the kinetic energy that is what this tells you, but it is not achievable it is not possible, because we know the temperature T 1 would reduce to 0 Kelvin. So, hence it is not possible.

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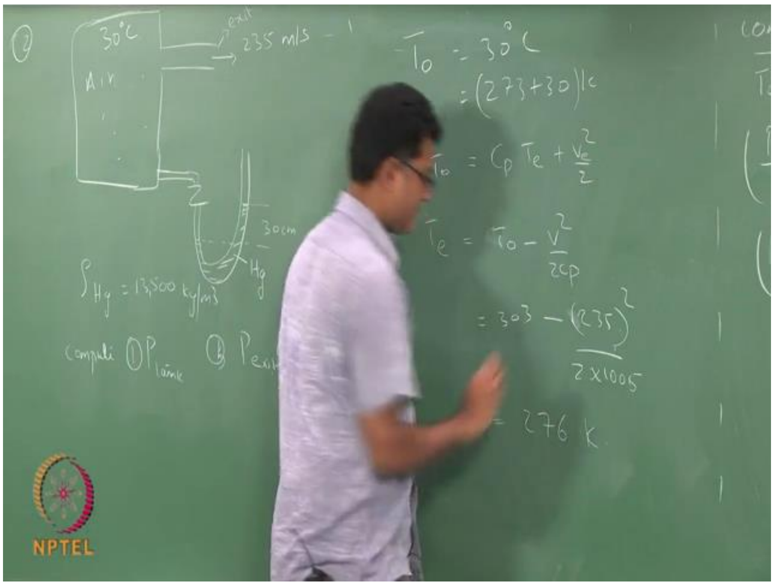

So, we will go to the second question. I have tank, the velocity that is coming out of this tank through a nozzle or orifices. And I connect this to a u-tube which is mercury, and a difference in height is around 30 centimeter. So, somehow I have measured my temperature here 30 degrees; now I am going to measure the pressure inside the tank using a u-tube manometer, which uses mercury as your fluid. And the distance the head difference is 30 centimeters.

So, you will want the density of mercury to be this. If that is a case, find P of the tank, P exit, so this is my exit, so P at the exit, m at the exit. So, I have a tank here, the fluid is 30 degrees air at it ejecting through an orifice at 235 meter per seconds. The tank is measured the pressure of the tank is measured using a u-tube manna meter with mercury that difference is thirty centimeters find P of the tank P at the exit, P m at the mach number at the exit.

So, this is the tank which contains lot of fluid particle in it. The velocity is assumed to be 0. So, the temperature you are measured there is T 0. So, the T 0 is 30 degree Celsius, which is 273 plus 30 degree Kelvin. If that is a case, apply my energy equation to get temperature at the exit plus the velocity at the exit. So, velocity at the exit is given, stagnation temperature is given. So, you can find your T exit to be T 0 minus V square by 2 C p which is 303 minus V square is 235 the whole square by 2 into 1005.

So, this enthalpy is used to generate this particular kinetic energy while doing that the enthalpy of the error has reduced, which means the temperature has reduced to 276 that is the meaning. So, it is a balance of energy. So, this energy is converted to kinetic energy, some energy heat content is lost. So, you get a reduced temperature. So, it is an expansion process where the temperature is reducing, but the question is to find P tank, what is P tank.

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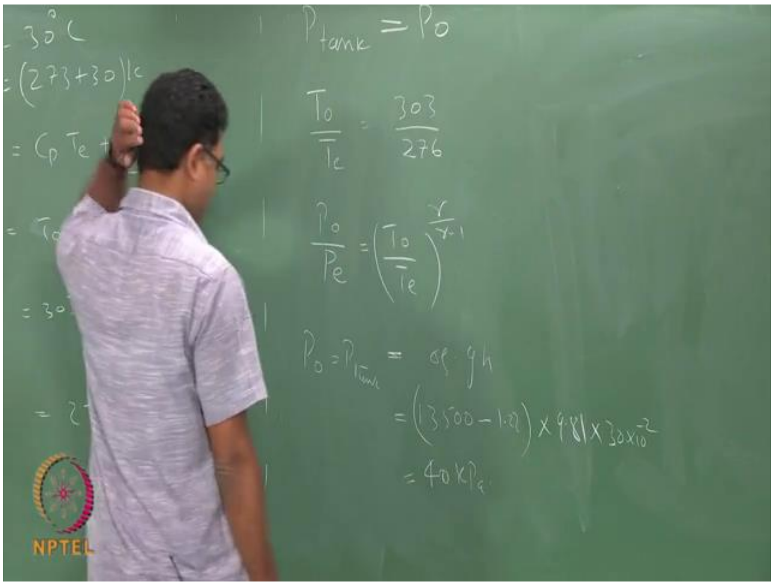

So, again your P tank is nothing but here stagnation pressure. So, now, you have static pressure and static temperature and stagnation temperature which is 303 by 276. Now this is related to your pressure. So, this P is at the exit, T at the exit. So, your P 0 by P exit is related to gamma by gamma minus 1. Now what is P exit, what is P 0? This is being measured by a u tube manometer here, which is this quantity. So, what is your; it is delta rho g h. What is delta rho density of mercury, which is given as how much?

Student: (Refer Time: 18:17).

Minus density of air which is around 1.22 into g is 9.8 1 into h is the height difference which is 30 centimeters 30 centimeters is 30 into 10 power minus 2 meters. So, this is?

Student: 40 (Refer Time: 19:03).

40 kilo Pascal, so the tank contains stagnation contains air at temperature 30 degree Celsius and 40 kilo Pascal.

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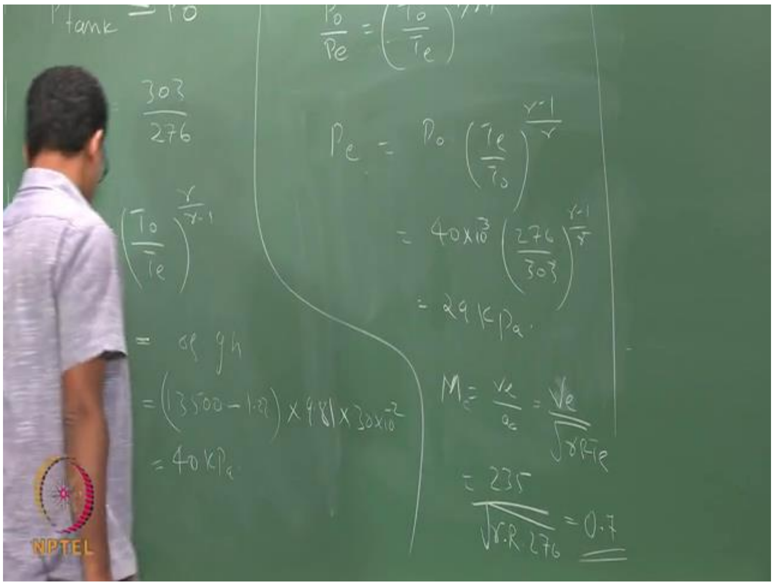

So, your T by P exit is now decided by this relation, so your P exit is nothing but P 0 into T e by T 0 to the power gamma by gamma minus 1 by gamma.

Student: (Refer Time: 20:07).

29 kilo Pascal; since we already know the temperature, it is easy to find the mach number at the exit with this V exit by a exit with is V exit by a root of gamma R T exit, V exit is given 235 divided by gamma R. So, gamma of air and R of air is something which you should remember and T exit is 276. So, the mach number at the exit it would be 0.7. So, I will repeat the question here. So, I have a cylinder, I have a stagnation temperature and stagnation pressure is measured using this u tube manometer. So, now we have if the gas is going at 235 meters per second, my mach number is 0.7. Now, we will see what is the meaning of a star and other star quantities.

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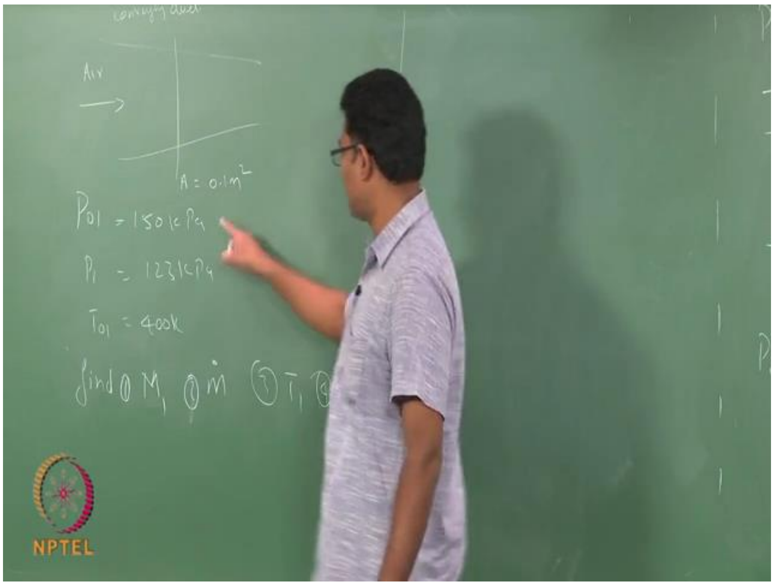

So, I have a flow through a duct or converging duct. So, the area somewhere at some location is 0.1 meter square, the P 0 1 is given as 150 kilo Pascal, P 1 is 223 kilo Pascal, T 0 is 400 Kelvin. Our usual assumption is there is no heat transfer, no Q, there is no shaft work, there is no w s, and let us also assume isentropic flow. So, you P 0 and T 0 on a going to be constant throughout the flow; with all those assumption, find your mach number at section 1, your M dot the temperature at section 1 and your A star. We will try to understand the meaning A star from this particular problem, assume air.

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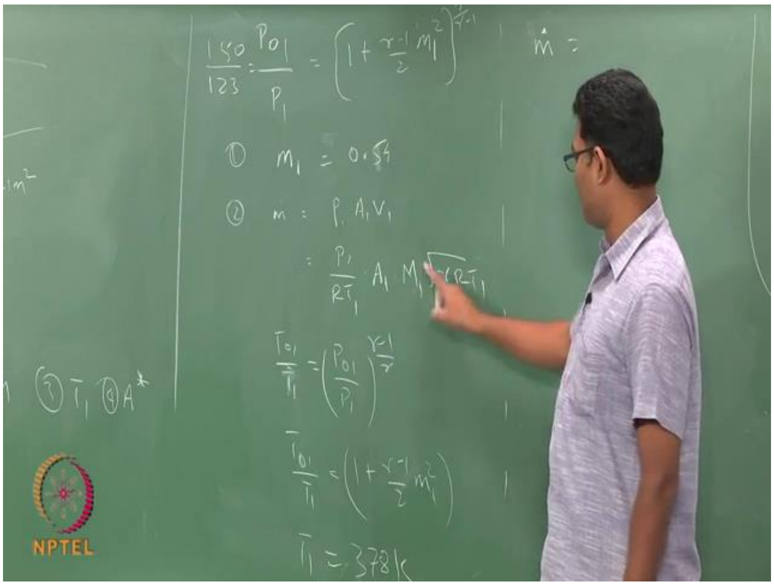

So, P 01 and P 1 is given, here you going to find the mach number. So, it is a straightforward substitution of this particular relation which is 1 plus gamma minus 1 by 2 M 1 square multiplied to the power gamma by gamma minus 1.

Student: (Refer Time: 24:30).

So, your M 1, you will get is as 0.44.

Student: 54.

54? 0.54. If I use P 1 as 150 kilo Pascal divided by 123 kilo Pascal, so that is the answer to the first question. So, at section 1, my mach number is 0.54. Now M dot rho 1 A 1 V 1; rho 1 is P 1 by R T 1 into A 1 into M 1 into root of gamma R T 1. So, I have to find T 1, T 1 and P 1 is related using gamma minus 1 by gamma. So, I can find T 1; or since I know mach number, I can get from this relation. Since, I know already if know the Mach number at section 1, I can use this relation to get this or use this relation both are the same relation. And we are also going towards something called gas tables which we will do the same set of problems with the gas tables. The same set of problems we are going to do this with the gas table. So, now, you can calculate this and T 1 is.

Student: (Refer Time: 26:45).

3.

Student: (Refer Time: 26:47).

378 Kelvin, if that is case then I can easily find my M dot from this particular relation. So, P 1 is known, T 1 is known, A 1 is given, M 1 is known and T 1 is known.

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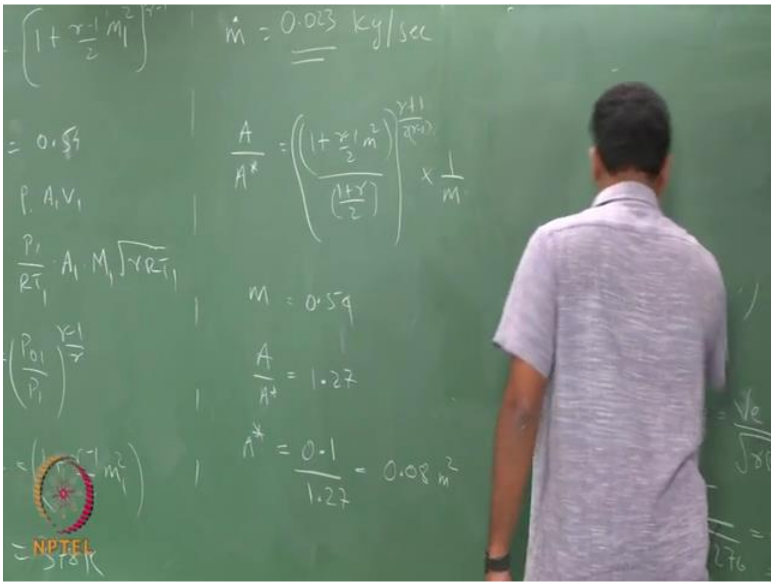

So, I would get something in kg per second. So, T 1 is my question 3, and m dot is my question 2.

Student: (Refer Time: 28:11).

0.023. What is the Mach number, Mach number 0.54, m dot is 0.03, so it is possible. So, A by A star relation in terms of mach number is.

Student: (Refer Time: 28:33).

So, now I know M, M is 0.54. So, if substitute M here my A by A star is 1.27 which means A star is A is 0.1 meter square divided by 1.27 which is.

Student: (Refer Time: 29:47).

0.08 meter square, now look at the meaning of A star.

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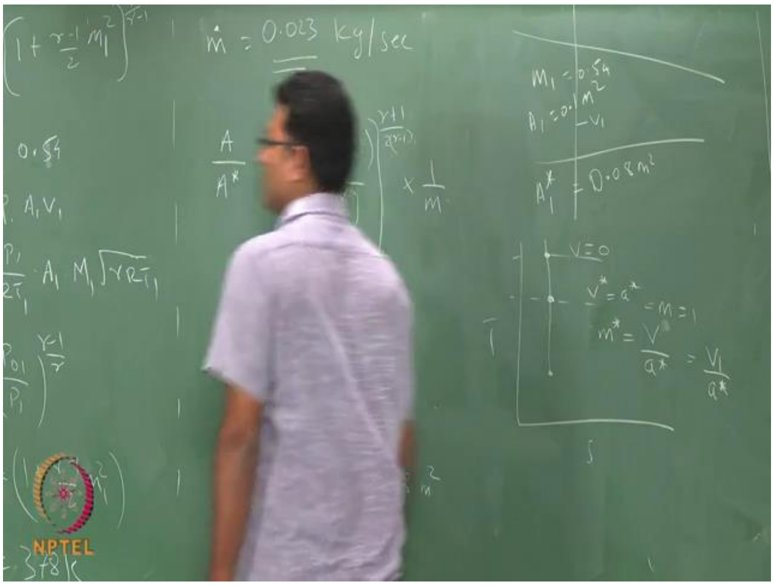

So, I have M 1 here to be 0.54, my A 1 is 0.1 meter square. Now the A star at this point, we found to be 0.08 meter square. Now what is my A star, I reduce I do an imaginary process, I reduce or increase the velocity to the velocity of sound. So, if I do this process here from state 1, I do an isentropic process, velocity 0 is my stagnation here is my V star equals A star.

So, if I do this process, what is my area there that is what it is given? So, for the same set up, if I want to do isentropic process to reduce this velocity, this area has to changed to this area to get the velocity or sound as same as velocity of the fluid. So, I have to reduce this much in area to get my mach number to be 1 at this particular location. So, here is where when mach number is 1 fine that is the meaning of A star. So, what is M star is velocity of star by.

Student: (Refer Time: 32:05).

Velocity by A star, velocity if know the velocity here I can find my M star which will not be same as this, so that is the meaning of the star value the M star value.

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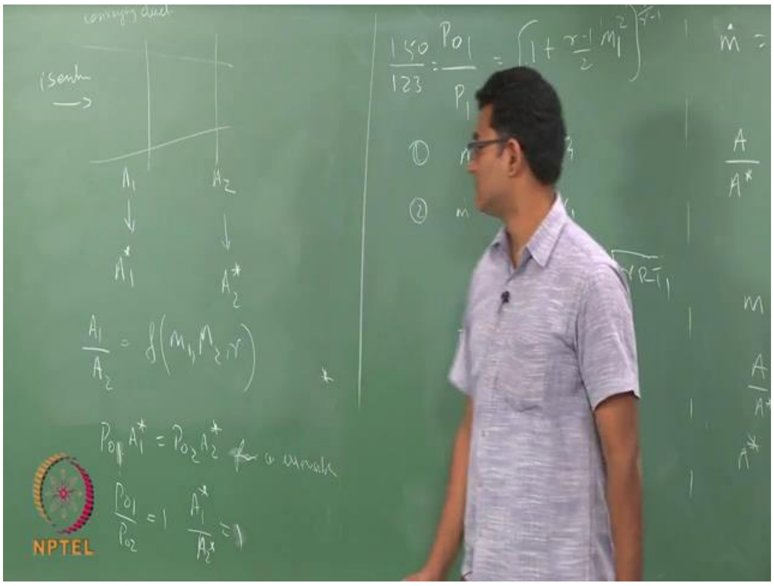

So, I can do modify the same problem if I have A 1, A 2 here. I can have an associated star value here; we can have an associated star value here. So, A 1 by A 2, I have a mach number 1 and mach number 2 the relation which we have already derived. If the process is isentropic, we know that P 01 A 1 star equals P 02 A 2 star equals, so if it is isentropic P 01 equals 1, and A 1 star by A 2 star equal 0. So, this is for any process for a irreversible process.

Student: (Refer Time: 33:54).

A equals 1, if this is isentropic process that is happening here my A star 1 is same as A star 2, in the same problem, which we have done before. If I have a section 2 here, I will have the same A star even though the mach number here may be different the a star associated would be different as long as. So, if the area would be different. So, the mach number associate would be here would be different, but the A star value is going to remain the same that is from this particular relation. So, what is this mean? So, this section at this flow if I have a fluid at this particular section A 1 it has to undergo a process which your area will reduce to A 1 star to get M equals 1, whereas here it will reduce to that same A star to get M equals 1.

So, if I have a large mach number to start with then what would be your M star what would be your A star how much will the area reduce or how much will you have to reduce the area to get m equals 1 that is the meaning of it. This is more implication when we discuss converging-diverging nozzle. So, what to should be your entrance area, what should be your inlet area, what should be your throat dia everything is decided by these quantities, which we will see it and the classes later on.

So, what we are learned now is the meaning of A star, how you can relate that to stagnation temperature Mach number stagnation properties relation everything, we will do the exactly the same problem using gas tables at a later point of stage. So, we will discuss something more on A star and then we will use the gas tables.