Fundamentals of Gas Dynamics
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Week – 05
Lecture – 17
Importance of stagnation temperature in relation to v

In the last class we have seen a reference quantity, the m star, so we will extend that to a new reference quantity and we will come back to the area variations and the changes of properties due to area variation in this lecture. The first thing is that we will introduce a new reference quantity.

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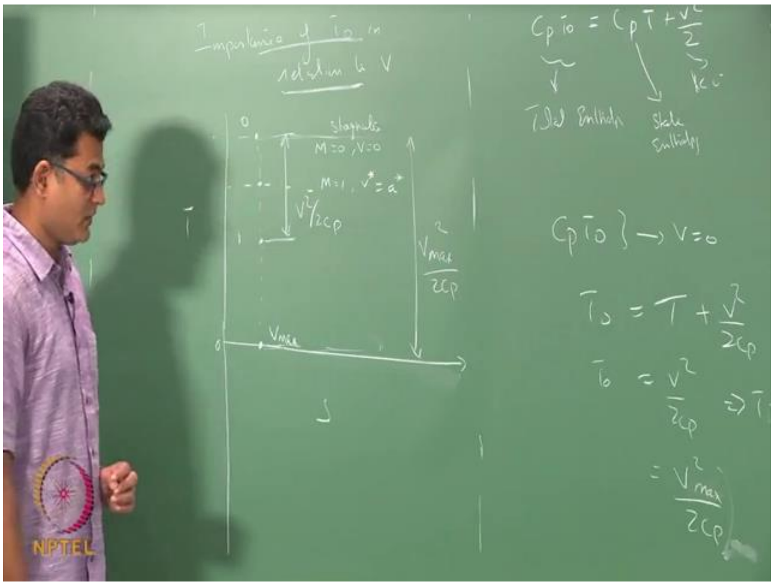

So, we will just try to see the importance of T 0 in relation to v. During this process of I am trying to understand the importance of T 0 we will introduce a new quantity. So, we will go back to the T S diagram. So, I had state 1. I isentropically made the velocity to be 0 at this stagnation point, so that is a 0 condition where my m is 0 or the velocity is 0. We have also seen how the m equals 1 condition produces a new reference quantity which we call it as v star which is equal to our velocity of sound a star at the same condition. And we look at the equations your C P T 0, C P T plus v 2 square by 2. So this is your total enthalpy, this is your kinetic energy and this is your static enthalpy.

So, what does this equation tell you? If I have a gas of this much total enthalpy that is balance between the kinetic energy and the enthalpy of this or to put it in the other way, I start with a gas of total enthalpy of C P T 0 where the velocity is 0. Now if I want to generate a velocity from this gas of with this total enthalpy I have to compromise here the balance is between the total static enthalpy and the kinetic energy. So the energy that is lost form here to generate kinetic energy is in the enthalpy, so some enthalpy is lost to get a kinetic energy.

Now, that is how you get this relation where we have seen this distance to be v square by 2 C P. If the whole of the energy is converted to kinetic energy what happens? If the whole of the total enthalpy is converted to kinetic energy obviously you get T 0 equals v square by 2 C P where the whole of this enthalpy is converted to the kinetic energy, implying your static temperature is 0. Or this is your v max that is the maximum possible velocity you can get from this. Or I should be writing this has the maximum we can get, but assuming C P to be constant during the process I can replace this has v max.

So, that is the maximum velocity you can get for a given T 0, so if I have a gas at T 0 equal some temperature there is a limit to which I can get the velocity.

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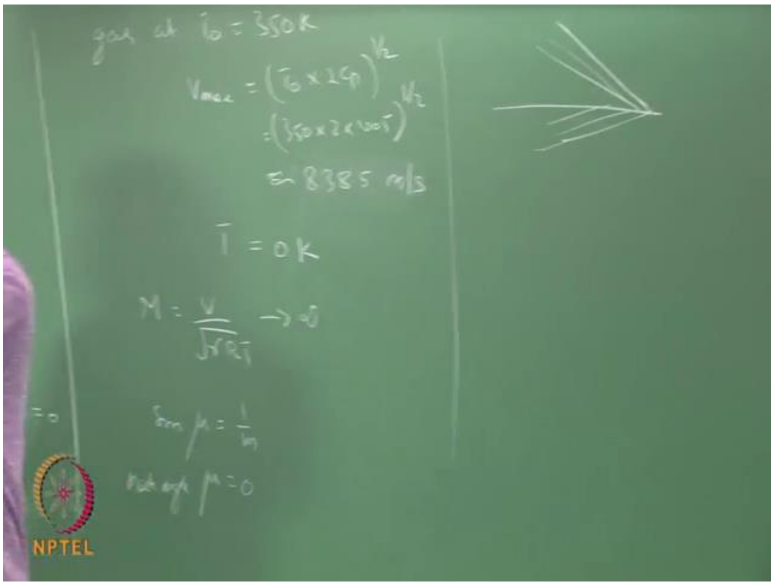

So the maximum velocity I can get here is T 0 into 2 C P which is 350 into 2 into 1005 which is.

Student: 1150.

11.

Student: 50.

Meters per second. 350 into 2 root.

Student: 838, sir.

So, this is the maximum velocity with which you can get if the stagnation temperature is this. You cannot achieve a velocity greater than this with this stagnation temperature that is the meaning of this. The total enthalpy of this gas is this multiplied by C P and that is converted completely to kinetic energy, this is the velocity. And in the process your static temperature reduces to 0 Kelvin, which means in the T S diagram here that status somewhere here along the axis; this is 0 my state is somewhere here on the axis, this is my v max, maximum velocity that can be achieved. So this distance would be v max square by 2 C P.

If that is the case what would be the Mach number, this approaches to infinity. This is something which you cannot achieve in reality that is a reference point as our stagnation point which we have done. We can use this quantity we max also has the energy associated with a gas. If m equals infinity your sin mu is 1 by m is now the Mach angle is 0. As we have seen earlier as the Mach number increases cone angle keep decreasing at m equals infinity your mu is now small very small. Now if that is the case what would be the P 0, I want to draw this in your PV diagram 2. What would be the P 0?

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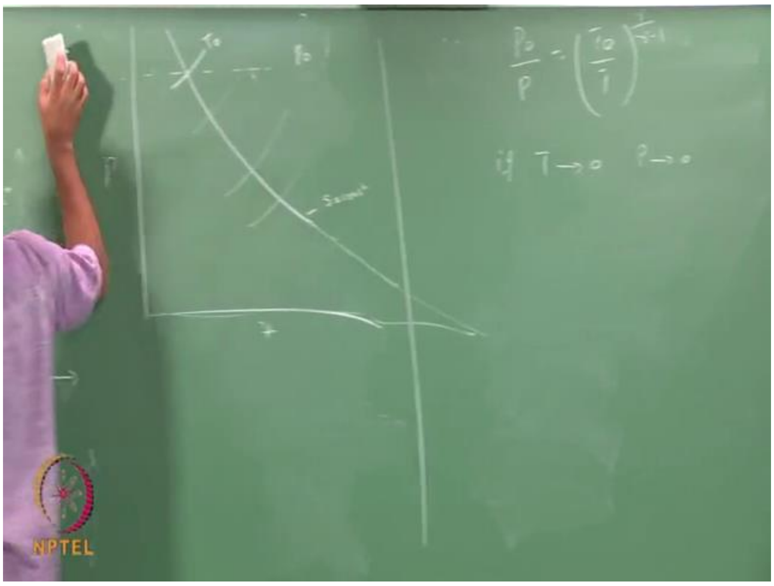

In the P V diagram I have a P 0, this is an isentropic process. So, I have an isentropic process here. This process 0, 1 and this max condition all lie in this particular line, so this is my constant entropy line. So, this point is my T 0. Now T 0 keeps coming down at v max I have my T equals 0, but what about p? P also keeps decreasing. But what would be the P at v max? Since, the process is isentropic you can write P 0 by P to be T 0 by T to the power gamma by gamma minus 1. For a given P 0 and T 0 if T approaches 0 your P also approaches 0, so your pressure also approaches 0. So, you would state it along this line.

If a have a large stagnation temperature I can get associated v max which is my new reference quantity. But before coming to the quantity variations due to area change which is what we have been doing in the last few classes we will see how this essentially varies with Mach number, which we have derived to be of this form gamma by gamma minus 1.

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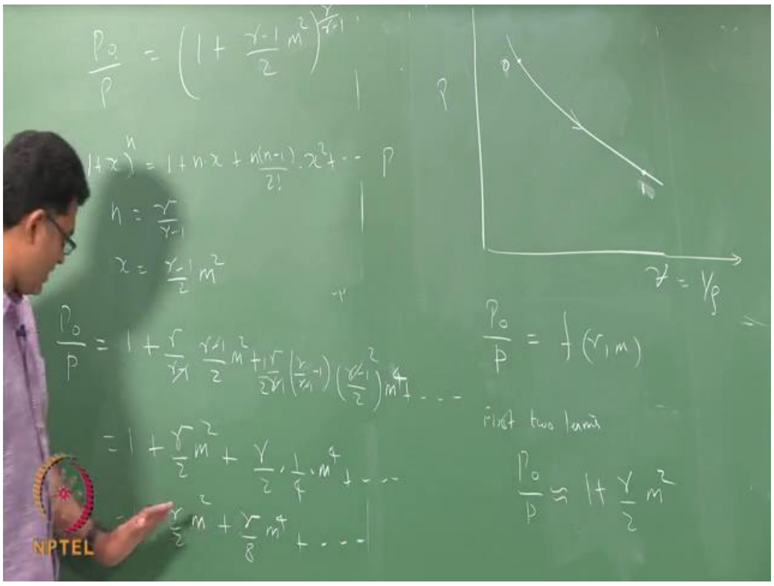

This is the relation that we have derived for the pressure change as the fluid undergo some process between stagnation point and state 1 in this particular diagram. Now this variation depends on Mach number. So, what I am going to do now is I will expand this using binomial theorem as the following 1 plus n into x plus n into n minus 1 by 2 factorial into x square and so on.

So here my n is gamma by gamma minus 1 and my x is gamma minus 1 by 2 m square. So, my P 0 by P is 1 plus gamma by gamma minus 1 into x is gamma minus 1 by 2 into m square plus I will write one more term which is gamma by gamma minus 1 into gamma by gamma minus 1 minus 1 into 1 by 2 into x square is gamma minus 1 by 2 whole square into m square plus. Which is now 1 plus gamma by 2 m square plus gamma by 2 into 1 by 4 into m power 4 plus. This is nothing but 1 plus gamma by 2 m square plus gamma by 8 m to the power 4 and goes on.

So, what does this equation tell you? Let us take P V diagram or 1 by rho diagram. From state 1 to 2 if I have an isentropic process from 1 to 2 or because I have written here P 0 by P I will assume 1 as my stagnation condition and 2 as my static condition of state 1. I have process between 1 and 0 or 0 to 1. So, I have a process from 0 to 1 which is not your condition where we have v max the some point, some state in the flow. Now during this process my P 0 by P goes in this particular fashion which is the function of gamma and m, if I take only the first two terms. Let us take only the first two terms I am truncating my series after the second term. Now if this is the case, this equation can be reduced to in the Bernoulli's equation.

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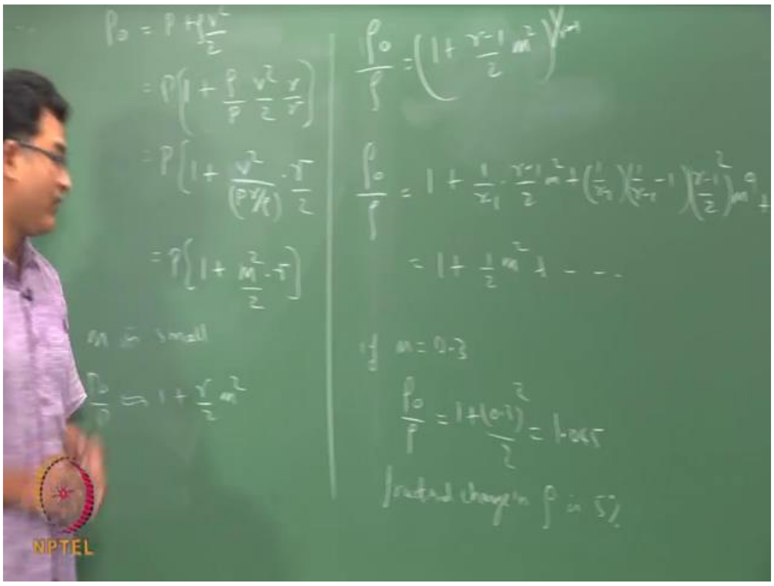

I can write this as 1 plus or I will start from Bernoulli's equation and reduce to this particular form. I write P 0 equals P plus v square by 2 rho, P plus rho I take P outside so this would be 1 plus rho by p v square by 2 I multiplied this both numerator and denominator by gamma, so this would be P into 1 plus gamma P by v square by P gamma by rho into gamma which is nothing but 1 plus m square into gamma by 2, which is precisely what we have here. So, Bernoulli's equation is a truncated equation of the full compressible flow equation. So if your Mach number is very small I can assume P by P 0 is only the first two terms. So the Mach number is small my P by P 0 approximates to 1 plus gamma by 2 m square.

How small is small something we will see now in terms of density. So I can rewrite this in terms of density where this is 1 plus gamma minus 1 by 2 m square to the power 1 by gamma minus 1. I do the same expansion here, so rho 0 by rho is 1 plus n is gamma minus 1 into gamma minus 1 by 2 m square plus. This again if I take only the first two terms this is 1 plus 1 by 2 m square. Now if my m is let us say 0.3 then rho by rho 0 is 1 plus 0.3 square by 2 this is around 1.045 it is approximately 5 percent.

So, your fractional change in density due to your speed of the fluid or the velocity of the fluid is just 5 percent; is approximately 5 percent if it is 0.3. If I use that 0.3 here then my fractional change in pressure when I do this process that is equivalent to what you get from the Bernoulli's equation instead of using this that is the point. If I decide I cannot accumulate 5 percent I can only account only 1 percent or I can extend it to say 20 percent then this would be quite large, instead of 5 percent here this would be one point say 3 or if you can live with it you can live with it, but generally what we do is we take this as demarcation where we say the flow is incompressible, if the Mach number is less than 0.3 and if it this more than 0.3 we assume that to be fully compressible flow.

Meaning at 0.3 we can truncate our series after the first two terms, otherwise you have to take the full series which is essentially this equation. So that is what you see when the gas is expanded or to a point from a given energy content, so the given energy content is now decide by T 0 and P 0 and with that energy content your getting a velocity which is less than v max as we have seen so that v max to get that v max some energy is been taken from the total energy content. And in the process the pressure change that you see here, if Mach number is less than 0.3 it is incompressible, greater than 0.3 it is compressible where we need to use the full set of equation.

Now that is how the properties change during the expansion process or the total energy of the gas is now producing some kinetic energy. Now what we need to do now is in the area relation that we are been working on the previous classes how does this written there, how does the property change P 0 or d P by P or some fractional change that is happening how does it change according to the Mach number or the velocity.

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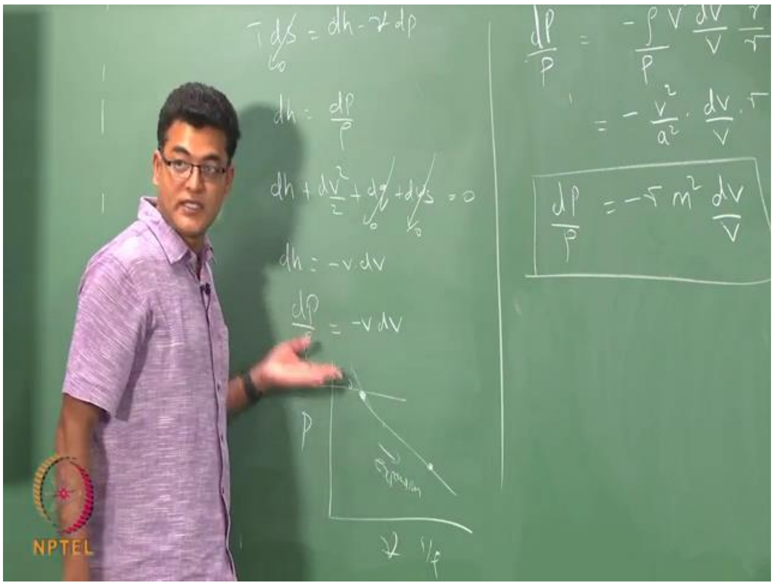

So, let us start from the equations where we have left in the previous class. We had written T d s equals d h minus v d P, we assume to be the process to be an isentropic one, so this is 0 so my d h is d P by rho. Now from the pressure energy relation we have d P by rho or from the enthalpy relation d h plus d v square by 2 plus d q plus d w s equals 0 we assume no heat no shaft work, we will end up with d h equals minus v d v which we substitute here to get d P by rho to be equal to minus v d v.

Now we will start from that equation I want to find d P by P, or which I have taken rho to the other side we d v divided by P, I also multiply and divide by v, I also multiply and divide by gamma. So this would be P gamma rho is my velocity of sound so that would be v square by a square into d v by v into gamma this is nothing but gamma m square d v by v. For some process that is happening the pressure change due to your velocity change is what is written here, we will come back to this. Now let us look at the temperature change.

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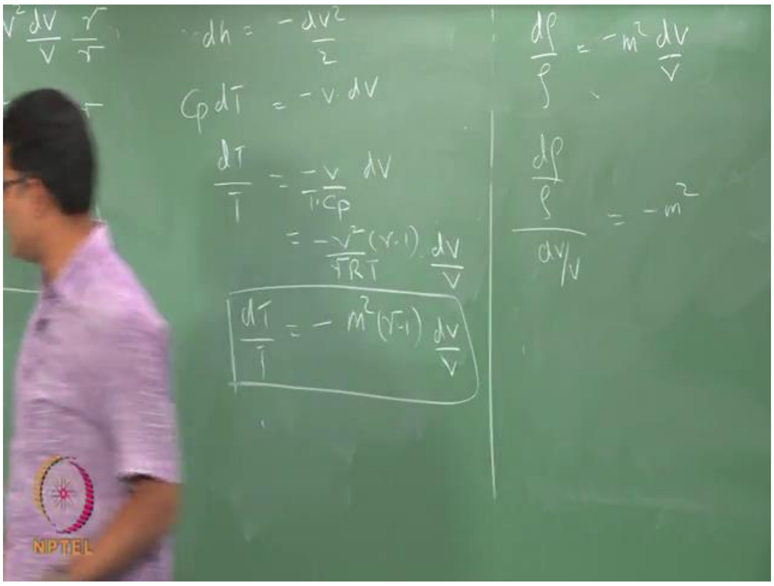

So your d h equals minus d v square by 2 this is C P d T equals minus v d v, so I divide d T by T I take C P here I replace C P has gamma r I divided by t so there is a T here gamma r by gamma minus 1 into T into d v, I multiply and divide by v. So, I have taken C P to the other side which is now written has gamma r by gamma minus 1 divided both sides by T so there is a T here then multiply and divide by v, so that would be v square. So this is minus m square gamma minus 1 d v by v.

So, this is the fractional change in temperature due to the velocity change. Likewise what we have derived in the other lecture was d rho by rho is minus m square d v by v. This is your fractional change in density due to fractional change in velocity which typically goes as minus m square. If there is a positive change in velocity there is a negative change in density. To have a velocity increase you need to decrease your density which is what you had seen in the previous P V diagram, where if this is my isentropic process this is P V for my isentropic process from stagnation state to some static state where you have generated a velocity the volume has increased which means the density has gone down.

So, this is an expansion process, this is an isentropic expansion where you are getting your velocity from your total enthalpy which we had seen to be C P T 0. So, that enthalpy is going to produce a fractional change in velocity in the process you are going to have decrease your density, whereas here temperature also decreases your pressure also decreases, because there is a minus a associated with this.

Now we include this along with our area change. This process has nothing to do with your area change, but now given the total energy and you know how to produce a velocity and now we know how the pressure temperature and density changes due to the fractional change in velocity, you relate that to now area change. Like in the previous class we will gather these equations and write it down here and try to see what it means.

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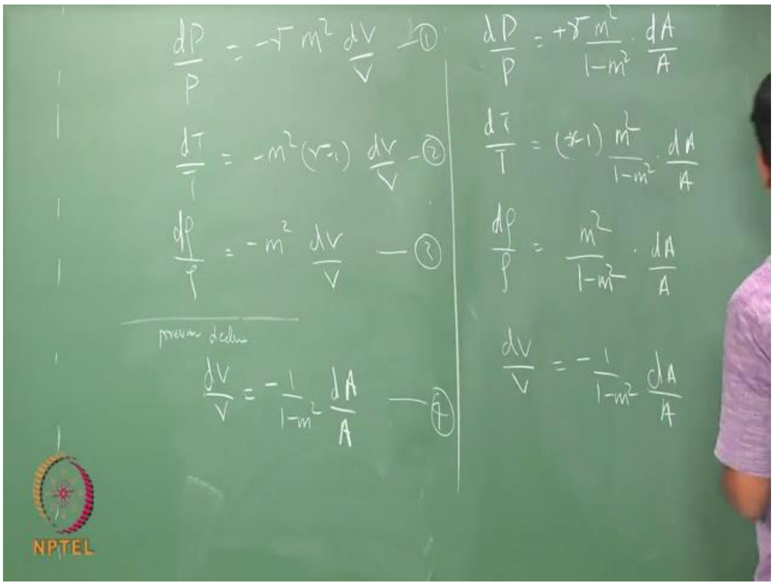

So, let me write the first equation d P by P equals minus gamma m square d v by v, d T by T is minus m square gamma minus 1 d v by v, and your density change d rho by rho equals minus m square d v by v. Now, what is d v by v in terms of area? From previous lecture d v by v as minus 1 by 1 minus m square into d A by A. Now if I substitute this quantity here. Let us say substitute equation 4, this is equation 3, this is equation 2, this is equation 1. So, I am going to substitute equation 4 in 3, 2 and 1. My first equation d P by P is now minus gamma minus and minus would cancel out so this would be gamma into m square by 1 minus m square into d A by A.

Now, d T by T is minus and minus cancel out, so this would be gamma minus 1 into m square by 1 minus m square into d A by A. Now let us look at the density minus and minus cancel out, so I would get m square by 1 minus m square into d A by A I will write this along with it so that we can compare. Now if you look at the left hand column equation these are all m square.

So whatever relation we have been talking about v and fractional change in velocity with fractional change in pressure or temperature or density holds for any Mach number, because it now this is a positive quantity. So, if there is a positive change in velocity there is a negative change in density there is a positive change in velocity there is a negative change in temperature or pressure, but whereas when it comes to area we have a problem because it is 1 minus m square. So, the relation is going to change depending on the Mach number. So, let us write first if m greater than 1.

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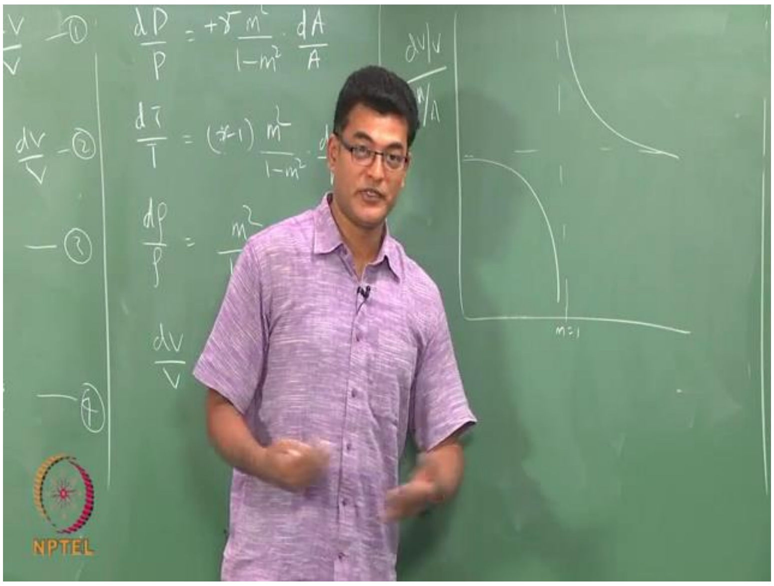

So, we have already seen how your d rho by d v change with d rho by d a change with Mach number. If I draw d v by v divided by d a by a then I am going to have a discontinuity somewhere here at m equals 1. So, these solutions what we have written here is not valid at m equals 1 because there is a discontinuity and we are not accounting discontinuity why, because we assumed isentropic flow.