Fundamentals of Gas Dynamics
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Week – 05
Lecture – 16
*** Reference Quantities and their Relations**

So, this is going to be further on the star conditions, which we had discussed in the previous lecture. So, we will try to understand more on the A star, T star and the other quantities associated with the star condition.

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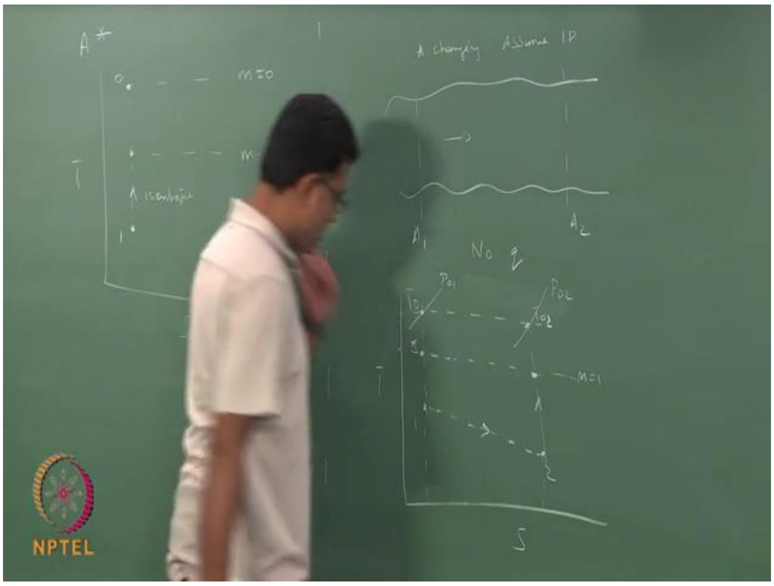

So, your A star is something that we had seen, is obtained when a point is reduced to a location where your Mach number is 1 and the process is isentropic as compared to your stagnation point where your Mach number is 0.

So, if I have a flow, arbitrarily shaped duct, so if my area is changing along one direction, assume 1D. So, I have section 1 and section 2, there is no q, there is no shaft work. So, I would draw the T-S diagram for such a process from 1 to 2. So, 1 has a star value somewhere here, 2 has a star value somewhere here. For the time being I am drawing it in the same line, we will see why it is in the same line.

So, at the point 1, I also have a state, which is the stagnation point, which is might is 0, 1. So, I have, with the point 2, I also have this stagnation point, which is the same as T 0, 1. Since there is no q, there is no shaft work, but we also know, that the pressure, stagnation pressure at, of 1 and stagnation pressure of 2 are different because there is a irreversibility associated with the flow. Now, we will see - what is the relation between the star states of 1 and star state of 2. So, our aim is to find the relation between A star 1 and A star 2.

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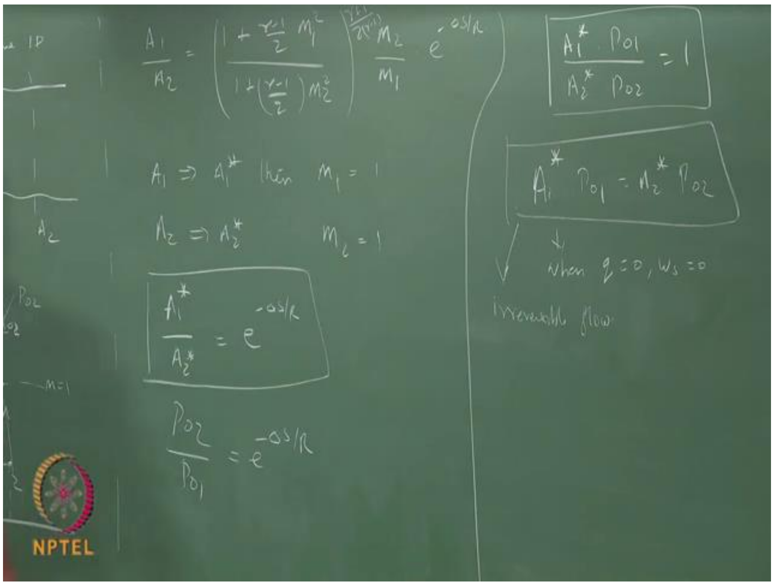

So, we have obtained this particular relation A 1 by A 2, which depends on the Mach numbers of 1, and station 1 and 2, be 1 plus gamma minus 1 by 2 M 1 square divided by 1 plus gamma minus 1 by 2 M 2 square divided by M 2 by M 1. So, this would be gamma plus 1 by 2 gamma minus 1 into e power minus delta S by R. For the process A is A 1 star and A 2 is A 2 star, then my M 1 is 1, M 2 is 1, which tells me A 1 star by A 2 star is e power... So, I have this relation.

I also know, P 02 by P 01 to be e power minus delta S by R, which tells me A 1 star into P 01 by A 2 star into P 02 is 1. So, my change in star is essentially my, due to irreversibility and changes in pressure, stagnation pressure are also due to irreversibility.

So, this is the relation between these two in a friend's points where I can write this, when there is no heat transfer, there is no shaft work and there is no potential also. So, this, there is irreversibility associated with this even when we write this relation. So, this is also applicable to irreversible flow. So, my star value at location 1 and the stagnation pressure at location 1 is related; the area and the stagnation pressure is related to another point in this particular pressure.

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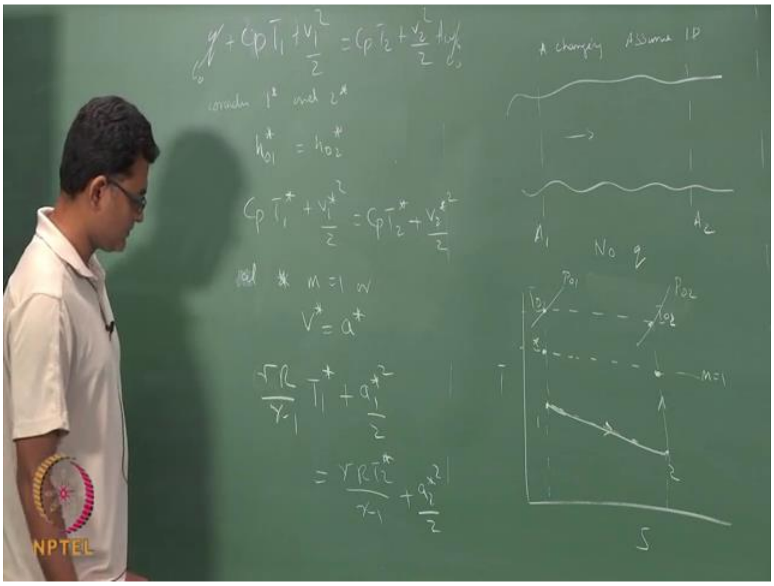

Now, moving further, I have the energy equation between point 1 and 2; between the point 1 and 2. Now, associated with 1 there is a star value, associated with 2 there is a star value. So, if I consider 1 star and 2 star, then h, this is h 01 star equals h 02 star. What I have written here if I substitute this star values, it is the enthalpy, stagnation enthalpy equating because the q here is 0, the W here is 0.

So, I can write T 1 star plus V 1 star square by 2 equals C P T 2 star plus V 2 star square by 2. Since it is a star quantity I can replace at, star, M equals 1 or my V star equals my A star. So, I can replace my V 1 star with A 1 star. So, what I will do is, C P I replace it as gamma R by gamma minus 1 T 1 star plus A 1 star square by 2 equals the other side, the 2nd point T 2 by gamma minus 1 star plus A 2 star by 2. So, this is the star condition at, the LHS is the star condition at 1, RHS is the star condition at 2.

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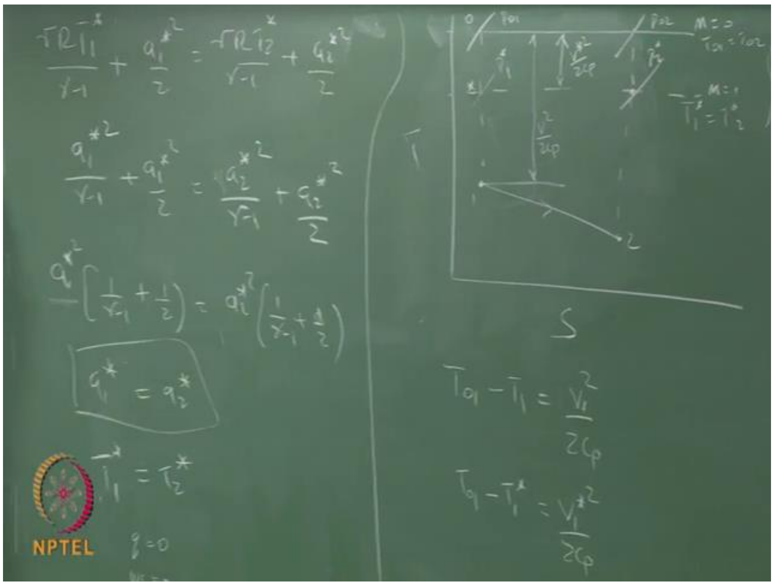

So, I will rewrite that whole thing here, is gamma R T star 1 by gamma minus 1 plus A 1 star by 2 equals gamma R T 2 star by gamma minus 1 plus A 2 star by 2. So, gamma R T 1 is the velocity of sound square. So, I can replace that with this. This would be A 2 star square by gamma minus 1 plus A 2 star square by 2. So, if I take A 1 outside 1 plus gamma minus 1 plus 1 by 2 equals A two star square with the same quantity, it just means A 1 star equals A 2 star.

So, if I have irreversible process from 1 to 2 and I take point 1 to imaginary reference, quantity reference point where my Mach number is 1, I take the point 2 to the reference state where M equals 1. The star velocity of sound, the velocity of sound associated with this star condition for both the points are the same, which also means, T 1 star equals T 2 star. So, only assumption that is taken here is q equals 0, W s equals 0. So, the process is still irreversible. So, the temperature here is also the same; like our stagnation temperature, the star temperature is also same. So, this will be on the straight line.

Now if you look at the plot again, I am enlarging that process, 2, 1 with S-T, this is my stagnation condition, somewhere here is my star condition. So, this is the star value, this is the stagnation value, what we obtain now is T star 1 equals T star 2, T 01 equals T 02 is something which we already know. Both this condition obtained when q equals 0, W s is equal 0. So, the pressure here is different, P 01, P 02, P star 1 and P star 2. They are also same if the process is isentropic.

We have also seen that this distance, this difference between the temperatures T 0, T 1, T 01 minus T 1. So, what is T 01 minus T 1? T 01 minus T 1 is nothing but V square by 2C P. So, instead of 1 if I take this star value, this should be T 01 minus T 1 star equals, I will replace the velocity V 1 with V 1 star square by 2C P. So, this distance is V square by 2C P, whereas this distance is V star square by 2C P.

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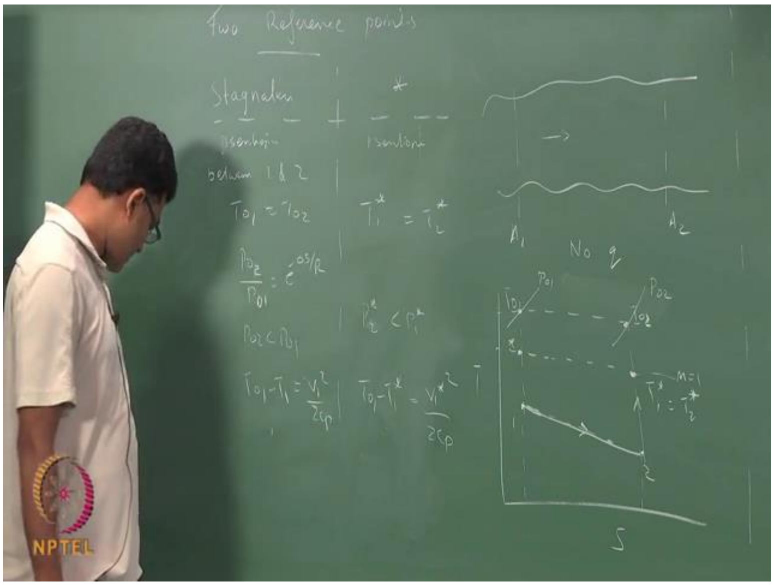

Now, we further, so I have, two stag, two reference quantities, stagnation and the star condition. So, if the process is isentropic, this process is also isentropic. Between two points 1 and 2 I have T 01 equals T 02; here again, T 1 star equals T 2 star. P 01, P 02 by P 01 is e power minus delta S by R; P 02 is less than P 01. Likewise, here your P 0, P 2 star is less than P 1 star and here V, your T 01 minus T 1, that is, your velocity square by 2C P; here, T 01 minus T 1 star is your V 1 star square by 2C P. V 1 star square is also your A 1 star square, which you can write it in terms of T 1 star.

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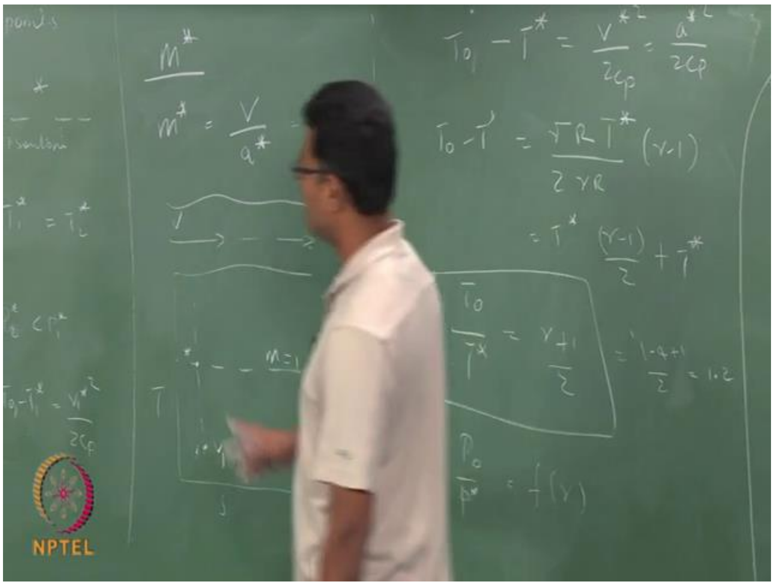

And we can write the entire quantity in terms of, so if I write T 01 minus T 1, let us remove the suffix. So, for any point T 01 minus T 1 star is V star square by 2C P, which is also your A star square by 2C P. So, T 0 minus T star, I can replace that with gamma R T star divided by 2 gamma R multiplied by r minus 1, which is nothing but T star into gamma minus 1 by 2 and I take this star here. So, your, T star by, T 0 by T star is gamma plus 1 by 2. So, this is nothing from the energy equation, the energy equation between the stagnation point and the star point.

Now, this is an isentropic process. So, I can relate this 2P 0 by P star as well because from star to stagnation is an isentropic process, I can use a isentropic relation. So, the ratio of this is again a function of gamma alone. So, if gamma is 1.4, so this is 1.4 plus 1 by 2. So, this is 2.4 by, 2.4 by 2, this is one point.

So, M star is the Mach number at the star condition. It does not mean M equals 1; M star is V by A star. So, if I have a fluid that is moving at some velocity V, for this particular V I have M star. So, I take the fluid, if this fluid at state 1 if I take it isentropically here, I have a star value, but at this point I also have a velocity V 1. So, this M is always 1 because that is the value. That is how we are defined the reference quantity. So, the M star is this V 1 divided by this A star. So, this is not equal to 1, it can be 1, but it, it is not equal to 1, okay. So, we will talk more about M star in the next lecture or so when we discuss converging, diverging nozzle and flow through converge, converging nozzles.

So, the message from this lecture is that we have two reference quantities, stagnation and star, which is very useful in defining this thing. And we have also defined something called M star because M is always changing with respect to temperature as well. And it is not just a function of velocity alone and at large velocities, M would be tending to infinity, whereas now we have defined a new quantity wherein, which we can scale down the whole process to something very small. We will discuss further on M star later that is.