Fundamentals of Gas Dynamics
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Week – 05
Lecture – 15
Variable Area Adiabatic flow (cont\'d)

This lecture is the continuation of variable area adiabatic flow. And we will try to see if the change in area can be represented in terms of Mach number, ok that is our aim. So, I will write the equations which we had seen in the other day.

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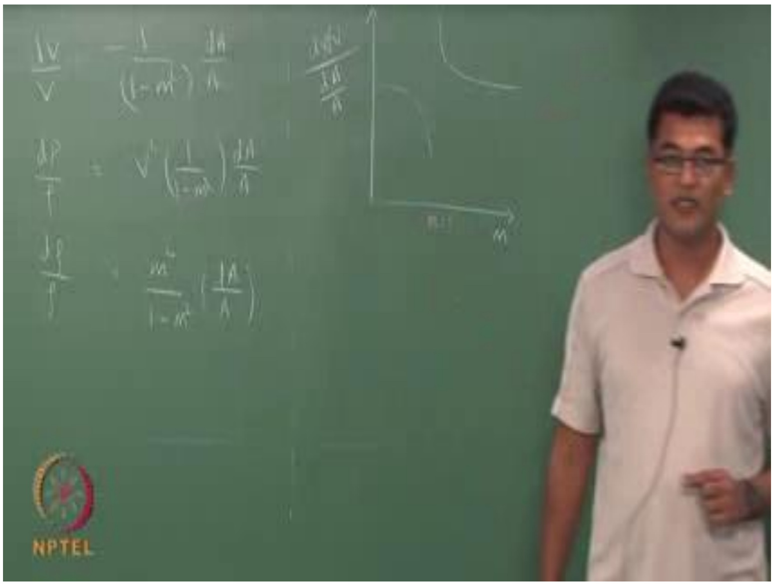

So, the first equation which we seen is this so will try to see how this quantity is varying with respect to mach number. So, if I take d v by v by d a by a, this will vary according to this equation, which is going to be, some discontinuity happens at m equal 1. So, you would see something like this. So, if I have m in my x axis, and this quantity in my y axis I would get something. So, at m equals 1 there is something, that is happening which we had seen at m equals 1 there is no area change. So, can we use that information to get a reference value, or can we represent the changes in area in terms of m, at that area where there is no change in area due to the velocities. So, that is our aim.

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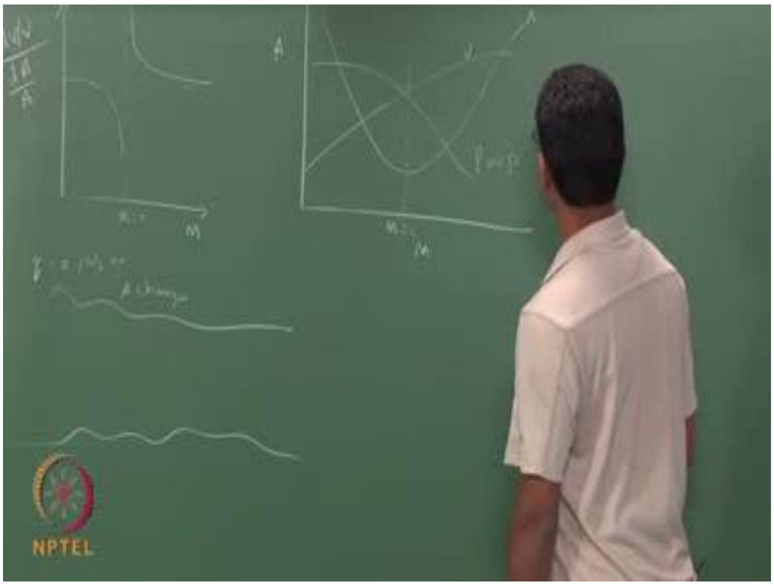

So, if I plot m versus a,, I would have a minimum at m equals 1 because that is the value where d a is zero. So, my velocity would also go something like this, and the density would come down something like this. So, this is my area, this is my velocity this is my density or pressure. So, we will use these information what all we have learned in the other class, to evaluate the changes in area. So, this happens for some arbitrarily shaped body, whose areas changing. We have assumed q to be zero, w is to be zero, and evaluating this, and these flows are, we have also used the energy equation to get this relation. So, with this condition is what we are going to evaluate the area changes. So, for some particular area ratio can we describe this in terms of mach number.

So, I would start for an isentropic flow.

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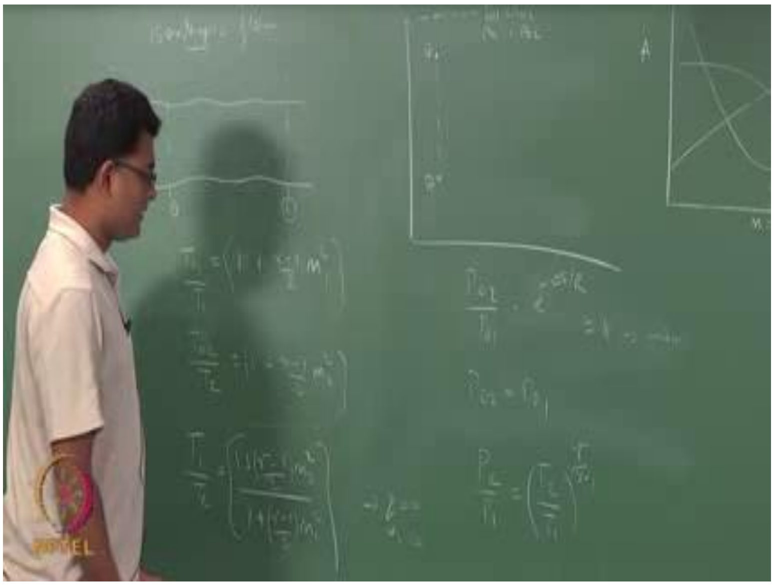

Some flow at section 1 section 2, how did we see the process in the ds diagram, because it is an isentropic flow, we have 1; say let say 1 has g1 to 2 isentropically, associated with 1 I have a stagnation point, which 2 also I have a stagnation point, since it is isentropic t 0 1 equal s t 0 2 and p 0 1 equals p 0 2. So, if I use that information t 1 we have t 0 1 by t 1 to be 1 plus gamma minus 1 by 2 m 1 square. Likewise point 2 has this relation. So, this is the isentropic process that happens at state 1, where the fluid is taken to a velocity 0 isentropically.

So, this relation holds. The same thing happens at station 2; this relation holds, because we know t 0 1 equals t 0 2 I can rewrite this equation in terms of t 1 and t 2 along. So, here we assumed w q equals 0 w as equals zero. and we have also seen p 0 1 p 0 2 by p 0 1 to be e power minus delta s by r which is now assumed 1, so your p 0 2 the same as p 0 1. So, this is isentropic. So, you can use isentropic relations here to find, say p 2 by p 1 equals t 2 by t 1 to the power gamma by gamma minus 1, which again you can write it in terms of m 1 and m 2, but now if it is not isentropic.

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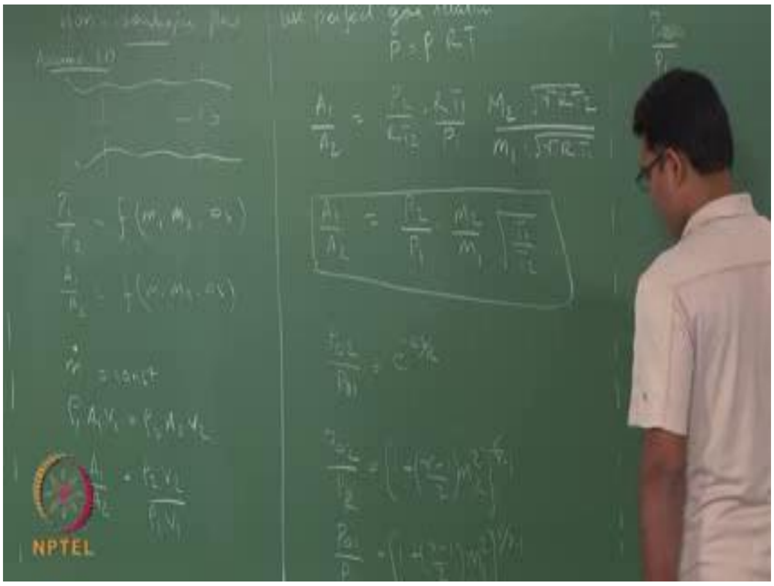

Let us consider a non isentropic flow. So, I have a flow here, my ratio p 1 by p 2 will be a function of m 1 m 2 and delta s and of course, gamma or r. If that is true then my a 1 by a 2 also depends on m 1 m 2 and delta s. My m dot is constant. So, I can write rho 1 a 1 v 1 equals rho 2 a 2 v 2, assume 1 d flow. So, if I write, if I want to find a 1 by a 2, I would write rho 2 v 2 by rho 1 v 1. Now I substitute rho 1 and v 1 from perfect gas relation. So, my a 1 by a 2 is replaced by rho 2, as p 2 by r t 2 multiplied by r t 1 by p 1 into v 2 is replaced as m 2 into a 2 is gamma r t 2 divided by m 1 dot gamma r t 1, which gives me a relation p 2 by p 1 into m 2 by m 1 into root of t 1 by t 2.

So, between the process p 1 and p 2, I have this. There is entropy change, between location 1, and 2, p 0 2 by p 1 is 1 plus gamma minus 1 by 2 m 2 square to the power gamma by gamma minus 1. Likewise I have p 0 1 by p 0 p 1 equals 1 plus gamma minus 1 by 2 into m 1 square to the power gamma by gamma minus 1. Now I use these three relation to write; p 2 by p 1. So, p 2 by p 1, p 2 by p 1 as p 0 2 by 1 plus gamma minus 1 by 2 m 2 square to the power gamma by gamma minus 1 into p 1 is 1 plus gamma minus 1 by 2 m 1 square to the power gamma by gamma minus 1 divided by p 0 1 which are replace it as p 0 2 by p 0 1 in 1 plus gamma minus 1 by 2 m 1 square divided by 1 plus gamma minus 1 by 2 m 2 square to the power gamma by gamma minus 1.

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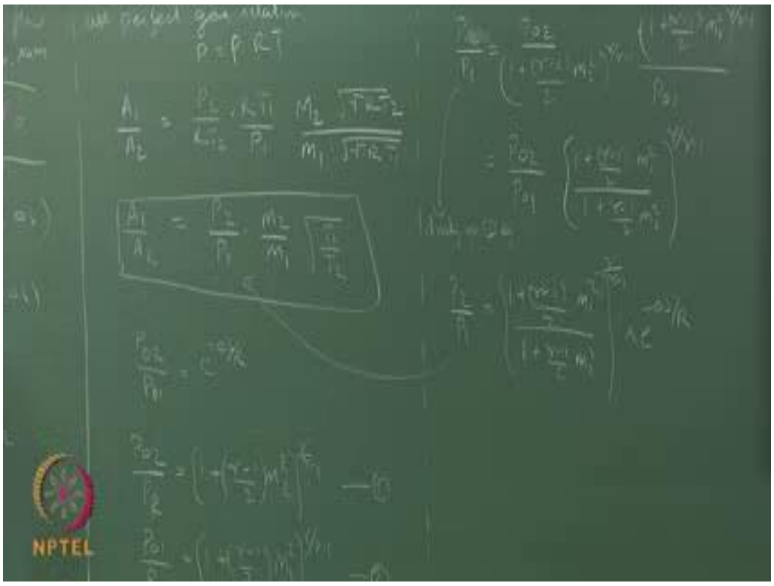

So, I taken this 2 relation, I divided 2 by 1. So, this is dividing equation 2 a 1 now I replace p 0 2 by p 0 1 with this relation. So, my p 2 by p 1 is 1 plus gamma minus 1 by 2 m 1 square divided by 1 plus gamma minus 1 by 2 m 2 square to the power gamma by gamma minus 1 into e power minus delta s by r. So, this is the pressure ratio p 2 by p 1 with this. So, if it is an isentropic flow, this is 1, and hence you would get this relation which is same as what you have got from your isentropic relation from t 1 and t 2. Now, I substitute this into this t 1 and t 2 anyway I can get it from that relation. So, there is there is no q, no w s. So, t 1 by t 2 has this equation, where you have obtained using isentropic relation, but p 1 by p 2 there is an reversibility irreversibility and hence you have to use this equation.

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So, I am rewriting that equation here; a 1 by a 2 is p. instead of p 2 by p 1 I replace that with this equation minus 1 into m 2 by m 1. now root of t 1 by t 2 I use this equation, which t 1 by t 2 is 1 plus gamma minus 1 by 2 into m 2 square divided by 1 plus gamma minus 1 by 2 into m 1 square, which into delta e power minus delta s by r. So, this I can reduce it further to 1 plus gamma minus 1 by 2 m 1 square divided by 1 plus gamma minus 1 by 2 m 2 square to the power to the power gamma by gamma minus 1, there is a 1 by 2 here there is a 1 by 2 there that is the root of t 1 by t 2. So, there is a 1 by 2 there. So, this would be minus 1 by 2. So, gamma by gamma minus 1 minus 1 by 2, so this would be gamma minus 1 into 2, 2 gamma minus gamma plus 1. So, this would be gamma plus 1 divided by 2 gamma minus 1. So, this exp1nt is gamma plus 1 divided by 2 gamma minus 1 into m 2 by m 1 into e power minus delta s by r which is my a 1 by a 2.

So, my area ratio is a function of m 1 m 2 gamma and r and delta s. So, it is depends on the fluid m 1 m 2 and your irreversibility. So, if this is the area ration, you can try to see if there is a reference quantity with this. So, I am going to introduce a new reference quantity which is called A star. The star condition is like our stagnation point star condition is when fluid is isentropically taken to m equals 1 condition, which means in the t s diagram from point 1. So, I have a state 1, I take it to velocity 0 is my stagnation point. So, if I take the point isentropically to v equals zero, then I get my stagnation point. So, this is my m equals zero. Now instead of m equals 0 if I take it to m equals 1. So, let say somewhere here; then I have my star condition. So, I take it isentropically that is very important.

So, if you can actually take the flow to m equals 1 in hundred different ways, but the 1 that is, that you are talking about is when it when it is reduced to m equals 1 or I mean or increased to 1 m equals 1 isentropically, if you do that is your star condition. So, what is your star condition? We will try to understand what is our star condition. So, we use this relation to evaluate further.

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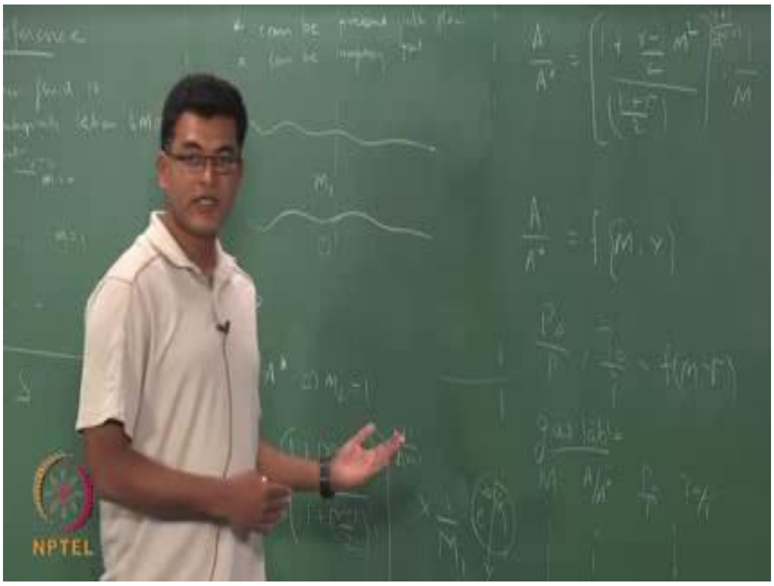

So, let say this star condition can be present in the flow, or it can be completely imaginary point. So, if it is a converging diverging nozzle, your star condition is the (( )) area where your d a is going to be zero, and your m equals 1. So, that is also your star condition. We will come to that in few lectures from now, but now let us concentrate on this arbitrarily shaped d a value. So, this is 1. So, along at there is a mach number associated with the station 1, this is a 1 d flow now I take this fluid to an imaginary value,, where m equals 1 and that value is your star condition and that area is your A star. So, the area associates. So, if I write a 1 by a 2 as this, and I take a 2 as my A star, meaning my m 2 is now 1.

So, I substitute that into our relation which we already know. So, I take a 1 by A star 1 plus gamma minus 1 by 2 m 1 square divided by 1 plus gamma minus 1 by 2 multiplied by 1 to the power gamma plus 1 divided by 2 into gamma minus 1 into m 2 is 1. This would be 1 by m e power delta s by r. Now I have taken it isentropically to star value. So, this quantity is now 1. So, that relation for any value, any location I can remove the suffix 1. So, my A by A star is a function of m and gamma, like what we had before, my p 0 by p or t 0 by t which is also a function of m and gamma.

So, if I know the mach number at any location I can find these ratios. So, for any given mach number I have 1 ratio, whereas, the converse is not true for any value of a by A star I can have 2 mach numbers, because now this is in m square. So, we will do some problem and see what this. What did you say?

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No.

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No it is contradict, so it can have 2; I did not mean negative values. So, contradict in my m. So, it can have 2 values. So, it has 2 values. So, 1 can be subsonic 1 can be supersonic we will see will when you do a problem we will see. So, this brings in something called a gas table, where I can write my m and then I can have my A star or p 0 by p and t 0 by t etcetera for a given gamma.

So, in the numerical solutions we will use the tables and try to evaluate these quantities to some problem similar to what we had d1 before. So, in this lecture I have introduced a new quantity, A star value, the star condition where the fluid is taken to imaginary point, where the mach number is 1, which means the velocity of the fluid is equal to velocity of the sound at that particular condition, this reference is something similar to your stagnation reference.

We will see in the next lecture, we will see further information on star conditions, how we can define what is the relation between t star v star and p star and the other things.